

## Some observations on defects in nematic liquid crystals

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**Abstract.** After briefly describing the usually observed defects in nematic liquid crystals, we give a summary of our observations on high strength line defects and a regular network of point disclinations on the nematic-isotropic interface.

**Keywords.** Defects; disclinations; nematic liquid crystals.

Nematic liquid crystals are characterized by a long range orientational order of molecules with shape anisotropy (Chandrasekhar 1977; de Gennes 1975). For example, a compound like pentylcyanobiphenyl has rod-shaped molecules and exhibits a uniaxial nematic phase at room temperature. The local orientation direction of the medium is characterized by an *apolar* unit vector  $\mathbf{n}$  (figure 1). A non-uniform orientation of the director is characterized by an elastic energy density

$$F = \frac{1}{2} [k_{11} (\text{div } \mathbf{n})^2 + k_{22} (\mathbf{n} \cdot \text{curl } \mathbf{n})^2 + k_{33} (\mathbf{n} \times \text{curl } \mathbf{n})^2], \quad (1)$$

where  $k_{11}$ ,  $k_{22}$  and  $k_{33}$  are the splay, twist and bend elastic constants respectively. They are quite small in magnitude, usually  $\sim 10^{-6}$  dyne. For most practical purposes, it is sufficient to make the one-constant approximation, viz. assume that  $k_{11} = k_{22} = k_{33} = k$ .

The nematic order breaks the rotational symmetry of the isotropic phase and the allowed defects are either lines or points around which the director varies continuously. An attractive feature of liquid crystals is that they are generally transparent to visible radiation. Further they have strong optical anisotropy and it is a relatively simple matter to study the defects under a polarizing microscope. Indeed defects in nematic liquid crystals were studied long ago (Lehmann 1900; Friedel 1922). A thin sample of the liquid crystal ( $\sim 20 \mu\text{m}$ ) is taken between a slide and a coverslip and if there is no special prior treatment of the glass surfaces to align the nematic, the schlieren texture (figure 2) results. In this texture, there are several points from which two or four dark brushes are seen to emerge. The points are line defects viewed end on and are called disclinations of strength  $1/2$  and  $1$  respectively. The sign of the disclination is taken to be positive if the brushes rotate in the same direction as that of the crossed polarizers and negative otherwise.

If  $\mathbf{n}$  is confined to the  $XY$  plane and  $\psi$  is the angle made by the director with reference to the  $X$  axis, using the one-constant approximation the minimization of the free energy density (equation (1)) requires that

$$\nabla^2 \psi = 0. \quad (2)$$

The solutions are of the form

$$\psi = s\alpha + c, \quad (3)$$

where  $c$  is a constant and  $\alpha$  the angle made by the radius vector connecting the given point to the centre of the defect with the  $X$  axis. In the present paper we confine our attention to 'wedge' disclinations in which the axis of rotation of the director is parallel to the defect line. The director fields for various values of the 'strength' ( $s$ ) of

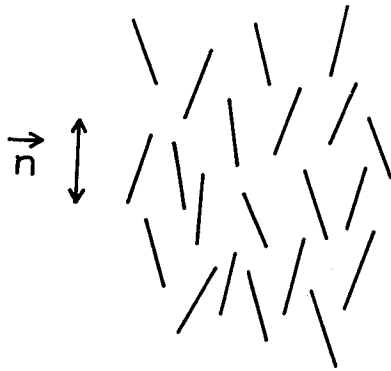


Figure 1. Schematic diagram showing the orientational order in the nematic state.

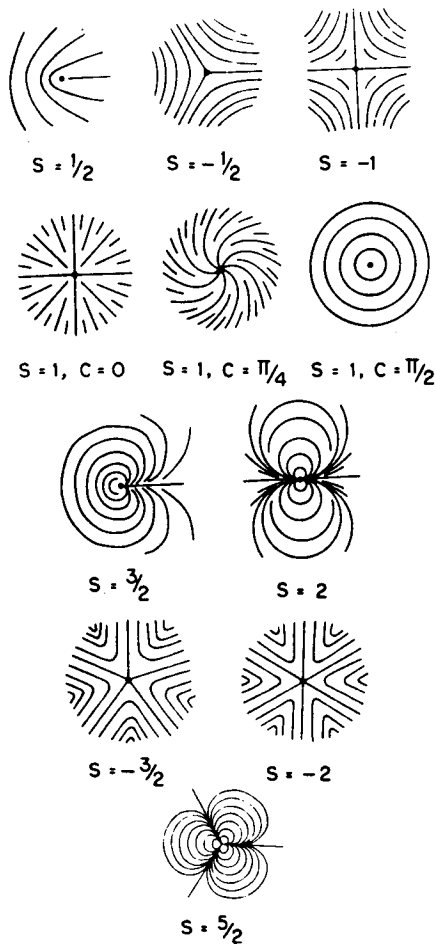


Figure 2. Disclinations of strength  $\pm 1/2$  and  $\pm 1$  seen in a room temperature nematic mixture. Crossed linear polarizers and illumination with sodium light ( $\times 600$ ). The dark rings around disclinations of strength  $\pm 1$  are due to the collapse of the director in the third dimension.

the defect are shown in figure 3, where  $2\pi s$  is the change in the angle of the director as one goes round the defect once. There is obviously a singularity at the centre of the defect and assuming that a 'core' region extends from the origin to  $r_c$ , the energy per unit length of an isolated disclination is given by

$$W = W_c + \pi k s^2 \ln(R/r_c), \quad (4)$$

where  $W_c$  arises from the core region and  $R$  is the size of the sample. It is clear that for a given size  $R$ , the elastic energy  $\propto s^2$  and that defects of strength  $|s| > \frac{1}{2}$  should



**Figure 3.** Schematic diagram of the director fields around disclinations of various strengths, according to (3).

spontaneously break up to form defects with  $|s| = 1/2$ . However, as seen in figure 2, defects of strength 1 are generally quite stable. This is simply a consequence of the fact that  $|s| = 1$  disclinations with planar configuration are topologically unstable: the director can continuously 'escape' into the third dimension as we approach the centre, and become parallel to the defect line at the origin, thus removing the singularity. Indeed detailed calculations of the elastic energy corresponding to the latter solution show that it has a much lower energy compared to the sum of the energy carried by two disclinations with  $|s| = 1/2$  (Meyer 1973; Cladis and Kleman 1972). The collapse of the director in the third dimension is clearly seen in figure 2 as a change in the effective optical path difference giving rise to the dark rings around defects of strength  $|1|$ . On the other hand, such an extensive collapse is not topologically possible for  $s = \pm 1/2$ , since the director rotates only by  $\pm\pi$  around such defect lines (figure 2).

Supposing now that we add non-mesomorphic impurities to the sample, for example  $\sim 10\%$  or more of leucoquinizarin to octyloxycyanobiphenyl. We immedi-

ately get defects of higher strength ( $\pm 3/2$ ,  $\pm 2$ )(figure 4) (Madhusudana and Pratibha 1982, 1983). Further, even *pairs* of such defects (for example,  $\pm 3/2$  or  $\pm 2$ ) can be seen in many preparations (figure 5). Indeed these high strength defects are very stable and do not break up to lower strength defects. Detailed observations have



Figure 4. A  $+3/2$  defect in a mixture of 20 wt% leucoquinizarin with octyloxycyanobiphenyl. Illumination with white light ( $\times 600$ ).



Figure 5. A  $\pm 3/2$  pair of disclinations in a mixture of 12 wt% leucoquinizarin and *p*-cyanophenyl-*p*-*n*-heptylbenzoate ( $\times 650$ ).

shown that the stability of high strength defects owes its origin to concentration gradients of the impurity in the sample. Further details about these high strength defects are published elsewhere (Madhusudana and Pratibha 1983).

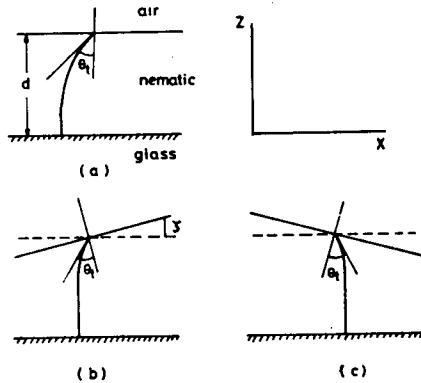
### Point defects

An easy way of visualizing a point defect in a nematic is to imagine a spherical drop in which the director is anchored radially at the surface. This results in a 'hedgehog' pattern, with a point defect at the centre of the sphere. In the following, we shall however be concerned with point defects which occur on the surface of a nematic sample.

de Gennes (1970, 1975) pointed out long ago that the shape of a free nematic surface gets distorted if the orientation of  $\mathbf{n}$  at the surface is antagonistic to that in the bulk. Consider a nematic film of thickness  $d$  spread on a glass plate treated for homeotropic (i.e. normal to the plate) alignment. If the tilt angle of  $\mathbf{n}$  at the nematic-air interface  $\theta_i \neq 0$ , the orientation of the director is not uniform in the nematic film (figure 6a). The corresponding elastic energy can be lowered by a tilting of the interface (figure 6b) so that the distortion in the director configuration is reduced. If we consider distortions only in the  $XZ$  plane, the tilt of the interface is given by  $d\zeta/dx$ , where  $\zeta$  is the height of the interface with respect to the original level (figure 6b). The tilting of the interface increases the surface area and hence the surface energy. Further, the non-uniformity of the height also increases the gravitational energy of the medium. The  $\zeta$ -dependent part of the net energy is given by

$$f_s = \int \left( \frac{A}{2} |\nabla\zeta|^2 - \frac{k\theta_i}{d} |\nabla\zeta| + \frac{g\rho\zeta^2}{2} \right) dx dy, \quad (5)$$

where only the leading terms are retained and  $A$  is the interfacial tension,  $\rho$  the density and  $g$  the acceleration due to gravity. Notice that the elastic term is negative and the other two terms are positive.  $\zeta$  cannot increase indefinitely but the system can lower its energy if neighbouring regions tilt in opposite directions as shown in



**Figure 6.** A nematic film with different orientations of the director at the two boundaries has a distorted configuration (a). The associated elastic energy can be lowered by a tilting of the nematic-air interface (solid line in b). If neighbouring regions (b and c) take opposite tilts,  $\zeta$  can remain small.

figures 6b and 6c, limiting  $\zeta$  to relatively low values. When two oppositely tilted regions meet, either cusps or dips are formed giving rise to disclination lines. If there is no preferred azimuthal orientation of the director in the  $XY$  plane, the surface is distorted in two dimensions, with conical cusps and dips which are associated with disclination *points* of strength  $+1$ . A large number of such defects can be expected to arrange themselves in a square lattice to minimize the total free energy of the sample. The pattern predicted by de Gennes is shown in figure 7.

The regular network has in fact not been observed on the free surfaces of nematics. For a typical nematic, the surface tension  $\approx 30$  dyne/cm and  $\rho \approx 1$  g/cc. On the other

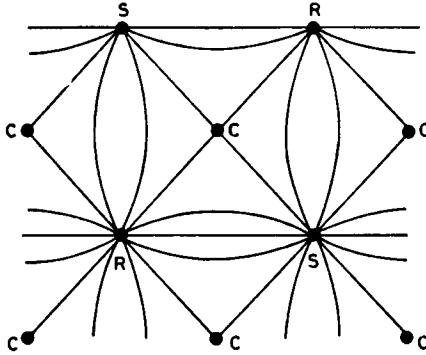


Figure 7. A regular square lattice of surface defects as visualized by de Gennes (1975). The lines are projections of the director at the interface. R and S correspond to  $+1$  point disclinations and C to  $-1$  point disclinations.

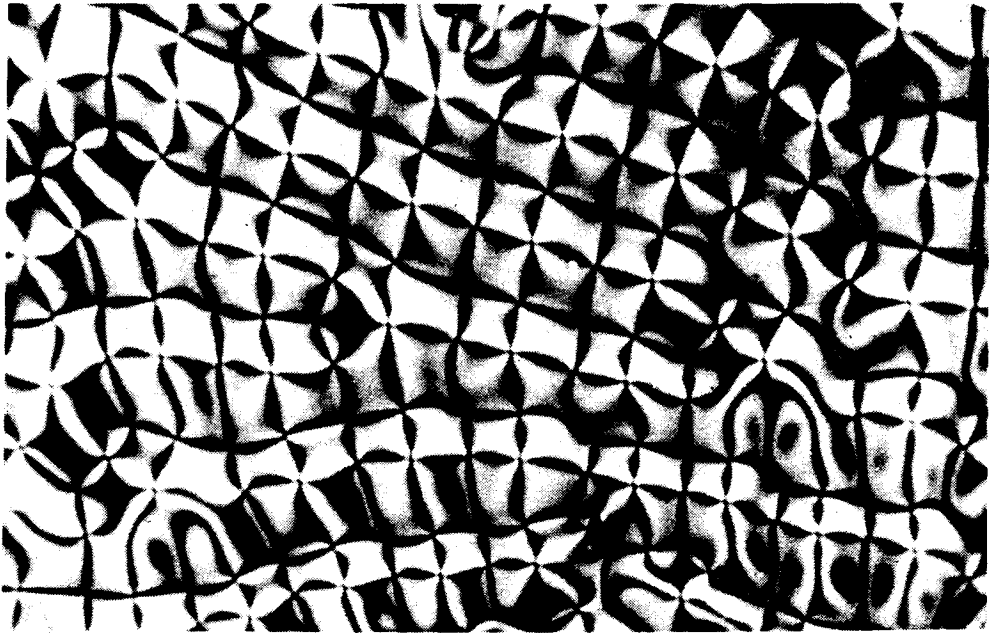


Figure 8. A regular network of point disclinations at the nematic isotropic interface of 4-cyanophenyl-*trans*-4'-*n*-pentyl cyclohexane mixed with a few per cent of heptyl cyanide ( $\times 400$ ). Note the dislocations in the defect lattice.

hand, the interfacial tension at the nematic-isotropic interface is  $\approx 10^{-2}$  dyne/cm and the density difference between the nematic and isotropic phases is  $\approx 10^{-3}$  g/cc. Hence the relative importance of the elastic term in (5) is enhanced by a few orders of magnitude in such a case. We recently studied such a sample and found the predicted regular lattice of surface defects for the first time (figure 8). The details of the experimental configuration can be found in our earlier publication (Madhusudana and Sumathy 1985).

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