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Has the Crab Pulsar Magnetic Field Grown after its Birth?

D. Bhattacharya *Joint Astronomy Programme, Indian Institute of Science, Bangalore 560012*
C. S. Shukre *Raman Research Institute, Bangalore 560080*

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Abstract. We investigate the evolution of rotation period and spindown age of a pulsar whose surface magnetic field undergoes a phase of growth. Application of these results to the Crab pulsar strongly indicates that its parameters cannot be accounted for by the field growth theories.

Key words: pulsars, magnetic field—pulsars, evolution—pulsars, individual

1. Introduction

Various authors (Woodward 1978, 1984; Blandford, Applegate & Hernquist 1983 and references therein) have suggested the possibility that the observed magnetic fields of neutron stars were generated after their birth by a thermally driven battery mechanism. In this picture, neutron stars are born with very low magnetic fields $\lesssim 10^8$ G and over a period of about 10^5 yr the fields are built upto $\sim 10^{12}$ G, the typical value inferred for pulsars. It is not clear if the necessary *dipole* fields can be generated in this manner (especially the 'vertical' components in the terminology of Blandford, Applegate & Hernquist 1983). However, assuming it to be possible we shall discuss here the implications for the evolution of rotation period and spindown age of the pulsar.

2. Definitions of relevant timescales

Several different timescales enter the following discussion. They are defined as,

t: the pulsar age

$t_c(t) \equiv P(t)/2\dot{P}(t)$ is called the spindown age or the characteristic age of the pulsar. Here P and $\dot{P}$ are the pulsar period and its time derivative respectively. The value of t_c for a pulsar at birth is t_c^0.

τ_m: the characteristic timescale for growth of the pulsar magnetic field.

t_{sat}: the time taken for the magnetic field to saturate from the start of its growth.

3. Evolution of the pulsar rotation

Assuming a simple 'magnetic dipole radiation' type of slowdown for the pulsar, we have the well-known relation:

$$P(t)\dot{P}(t) = kB^2(t) \simeq 10^{-15} B_{12}^2(t)\ \mathrm{s} \qquad (1)$$

where P is in s, $\dot{P}$ in s s^{-1}. B_{12} is the magnetic field B in units of 10^{12} G. Integrating Equation (1) one finds,

$$P^2(t) - P_0^2 = 2k \int_0^t B^2(t)\,dt \tag{2}$$

where P_0 is the initial period.

In the above-mentioned models, the magnetic field of the neutron star grows exponentially with time for $\sim 10^5$ yr and reaches saturation at $B \sim 10^{12}$–10^{13} G. It remains at this value till eventual decay sets in at the age of a few million years. In the growth phase, the initial field B_0 grows to

$$B(t) = B_0\, e^{t/\tau_m}$$

and thus

$$P^2 - P_0^2 = k\tau_m (B^2 - B_0^2) \tag{3}$$

$$= k\tau_m B^2(1 - e^{-2t/\tau_m}). \tag{4}$$

The spindown age is now

$$t_c(t) = t_c^0 e^{-2t/\tau_m} + \frac{\tau_m}{2}(1 - e^{-2t/\tau_m}). \tag{5}$$

The variation of t_c with t is as shown in Fig. 1.

After saturation has been reached, *i.e.* $t > t_{sat}$, the evolution changes and writing $B(t_{sat}) = B_{sat}$, we have

$$P^2(t) = P^2(t_{sat}) + 2k(t - t_{sat})\,B_{sat}^2 \tag{6}$$

and

$$t_c(t) = t_c(t_{sat}) + (t - t_{sat}) \tag{7}$$

which are the usual constant field evolution formulae.

Although one can discuss the various cases which can be distinguished by different relative magnitudes of the timescales involved, we shall consider only those which are physically reasonable. Obviously, the case when $t_{sat} \lesssim \tau_m$ is not of much interest as it corresponds to a pulsar being born with a magnetic field almost equal to its present

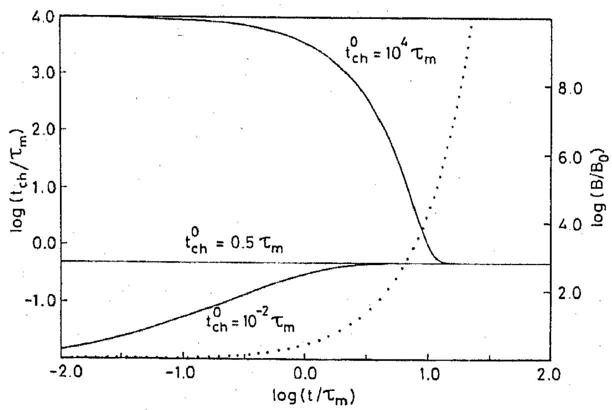

Figure 1. Evolution of spindown age of a pulsar as a function of time during exponential growth of its surface magnetic field. All times are in units of growth timescale τ_m of the magnetic field. Evolution is shown for three different values of initial spindown age t_c^0, namely, $10^4\,\tau_m$, $0.5\,\tau_m$ and $10^{-2}\,\tau_m$, as indicated beside the respective tracks. The spindown age approaches and saturates at the value $0.5\,\tau_m$. The dotted line shows the evolution of surface magnetic field, the corresponding scale being at the right.

value and thus defeating the motivation for the magnetic field growth. In the following we shall therefore assume that $\tau_m \ll t_{sat}$. Normally one also expects $t_c^0 \gg \tau_m$; *e.g.* for $P_0 = 10^{-3}$ s and $B_0 = 10^8$ G, $t_c^0 \sim 10^9$ yr. Even if $B_0 = 10^{10}$ G, t_c^0 would be $\sim 10^5$ yr. Thus we take $t_c^0 > \tau_m/2$, and find, using Equation 5,

$$\tau_m \leqslant 2t_c. \tag{8}$$

We shall consider two possible histories of the Crab pulsar in relation to the Crab nebula, since it has often been suggested that pulsars may be born in binary systems (see *e.g.*, Gunn & Ostriker 1970). Indeed, on the basis of observed pulsar proper-motions it has been concluded that almost all pulsars are released from binary systems (Radhakrishnan & Shukre 1985). Pulsars born in binary systems would be seen as single pulsars only if the binaries are disrupted. Such binaries are expected to undergo two supernova explosions, a few million years apart (van den Heuvel 1977). Also, due to mass transfer effects in such systems, it will be most likely the second explosion and not the first one which will disrupt them (van den Heuvel 1976). In general, after disruption a binary will thus release two neutron stars—one old and one young. Keeping this in view, we therefore consider in turn the two alternatives, *i.e.*, the Crab pulsar is the one born in either the first or the second explosion in a binary system.

4. The Crab pulsar as a young neutron star

If the Crab pulsar were born in the explosion of AD 1054 which created the nebula, its age is

$$t \simeq 930 \text{ yr}. \tag{9}$$

Also, at present

$$P \simeq 0.033 \text{ s},$$

$$t_c \simeq 1200 \text{ yr},$$

and

$$B \simeq 3.7 \times 10^{12} \text{ G}.$$

Using Equation 8 (and also 6 if $t > t_{sat}$), one finds, irrespective of the relative magnitudes of t and t_{sat}, that

$$\tau_m \leqslant 2400 \text{ yr}, \tag{10}$$

clearly at variance with the value of $\sim 10^5$ yr invoked for τ_m in the above-mentioned models.

Thus to reconcile the field growth picture with the present value of the Crab pulsar field one must then consider $\tau_m \leqslant 2400$ yr, as already noted by Blandford, Applegate & Hernquist (1983). In addition, we also impose the modest requirement that the present field is at least one order of magnitude more than the initial field. Now, in the case when $t < t_{sat}$ this implies $\tau_m \lesssim 400$ yr. Also, since

$$P^2 - P_0^2 \leqslant k\,\tau_m B^2 \qquad \text{(see Equation 4)},$$

substituting observed values for P and B, we get

$$P_0 \geqslant 30.4 \text{ ms}.$$

Such an initial period corresponds to a total energy loss of $\leqslant 3 \times 10^{48}$ erg since the birth of the pulsar and is at least an order of magnitude less than the energy budget of the Crab nebula. The nebula has $\gtrsim 10^{49}$ erg in the present relativistic-particle and magnetic-field content. We further require $\sim 10^{49}$ erg for post-acceleration and $> 10^{48}$ erg to account for the radiation since its birth (Woltjer 1958; Trimble & Rees 1970; Trimble 1971; Rees & Gunn 1974). As there are compelling reasons to attribute all of this energy to that derived from the rotation of the Crab pulsar, any value of $P_0 \gtrsim 20$ ms is ruled out.

In the other case, when $t_{sat} < t$, using Equation (6) one can show that it is possible to have $P_0 < 20$ ms, but then the requirement $B_{sat} \geqslant 10\,B_0$ implies that $t_{sat} \lesssim 180$ yr and $\tau_m \lesssim 80$ yr, which essentially amounts to instantaneous field build up.

5. The Crab pulsar as an old neutron star

The other alternative is that the Crab pulsar is older than the nebula and the companion of the star which became a supernova in AD 1054. The pulsar produced in this explosion remains undetected. Equation (9) now changes to

$$t \simeq 4 \times 10^6 \text{ yr.}$$

Since a change in t does not affect the inequality (10), it still holds. In addition, we also find from Equation (7) that $t - t_{sat} \leqslant 1200$ yr. Consequently, the field would have grown by an incredible factor of 10^{700} over 4×10^6 yr*, unless the stellar evolution timescale of the companion was $\sim 10^4$ yr, an unacceptably low value. We thus see that it is not possible in the case to explain both B and t_c simultaneously.

It may be argued in this connection that the field might have built up during a short-lived ($\sim 10^4$ yr) accretion phase. But as this accretion must have definitely stopped 930 yr ago, from Equation (7) it follows that the characteristic age at that time was 270 yr. This then is the upper limit on $\tau_m/2$ and we are once more led to an unreasonably fast growth of the field. Also, recent studies of known X-ray binary pulsars do not show any evidence of growing fields during accretion (Ray 1984).

We also investigated a power-law type growth for the magnetic field which may be relevant in the non-linear phase (Blandford, Applegate & Hernquist 1983). The details are not very illuminating but the conclusions are very similar to the above exponential case. For the case when the pulsar is young, it is not possible for both t_c and t to have small values. In the other case when the pulsar is considered to be an old object, again the field must grow in a very short time interval. Similar conclusions can be arrived at also by considering the variation of the braking index with time.

6. Effects of accretion torques

So far we have restricted ourselves to the slowdown of pulsar period due to radiation losses. There is, however, one other factor which affects the rotational history of a pulsar which must be taken into account when applying the field-growth formulae, if the neutron star happens to be in a binary system. During the mass transfer phase the angular momentum of the accreted matter could 'spin up' the neutron star (see, *e.g.*, review by Henrichs (1983) and references therein). Such a decrease in period will reduce

* This corresponds to an initial magnetic moment of the neutron star less than that of a single neutron (by about 10^{-645}!).

the characteristic age of the pulsar. However, it cannot go on indefinitely and the minimum period the neutron star can attain depends on its surface magnetic field and corresponds to the Eddington accretion rate (Srinivasan & van den Heuvel 1982). As a consequence, the final characteristic age will be $\sim 10^7$ yr (Radhakrishnan 1984). It is easily seen therefore that such a scenario cannot explain the 1200 yr characteristic age of the Crab pulsar, no matter how the field varies with time.

7. Conclusions

If the Crab pulsar magnetic field has grown significantly in the past 930 years since its birth, the Crab nebula could not have been powered by it unless the field growth occurred in a very short time ($\lesssim$ 200 yr). However, this is then equivalent to the pulsar having its present field essentially at birth. Alternatively, if the pulsar, being a product of the first supernova explosion in a binary system, is older than the nebula, then the field growth picture cannot accommodate the values of the present magnetic field and the characteristic age of the pulsar simultaneously.

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