

Pancharatnam phase as a purely geometric phase

Any optical system designed to demonstrate the Pancharatnam phase¹ as a purely geometric phase must generate each new polarization state by projecting the previous state on it². Retarders cannot be used to cycle the polarization, since they may introduce additional dynamic phases³. The only optical elements that can be used to cycle the state of polarization of the beams are analysers, where we define an analyser as an optical element that transmits the desired polarization and rejects (absorbs or reflects) the orthogonal polarization⁴. We describe here an interferometric arrangement for such a demonstration.

The optical system that we have used

(see Figure 1) is based on Young's interference experiment. An expanded, linearly polarized beam from a He-Ne laser ($\lambda = 0.633 \mu\text{m}$) illuminates a pair of slits through two circular analysers RCA and LCA to yield, respectively, right- and left-circularly polarized states. Both these circularly polarized states are then projected by a lens on to a linear analyser LA to obtain the final linearly polarized states. A magnified image of the interference pattern produced by these two states is formed on a screen by a second lens.

While sheet polarizers can be used as linear analysers, circular analysers, which transmit one circular polarization and

reject the other, are not readily available. We have used $50 \mu\text{m}$ thick films of two cholesteric liquid-crystal materials^{5,6} (76% by weight of cholesteryl oleyl carbonate in cholesteryl chloride, at 24°C , and 80.7% by weight of cholesteryl chloride, 17.6% of cholesteryl oleyl carbonate and 1.7% of cholesterol, at a temperature of 38°C) as left- and right-circular analysers, respectively.

The operation of this system can be

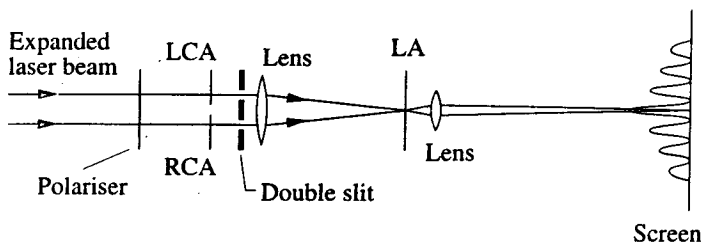


Figure 1. Optical system of the interferometer used to demonstrate the Pancharatnam phase as a purely geometric phase.

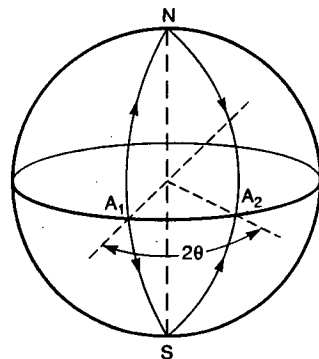


Figure 2. Poincaré sphere representation of the operation of the interferometer.

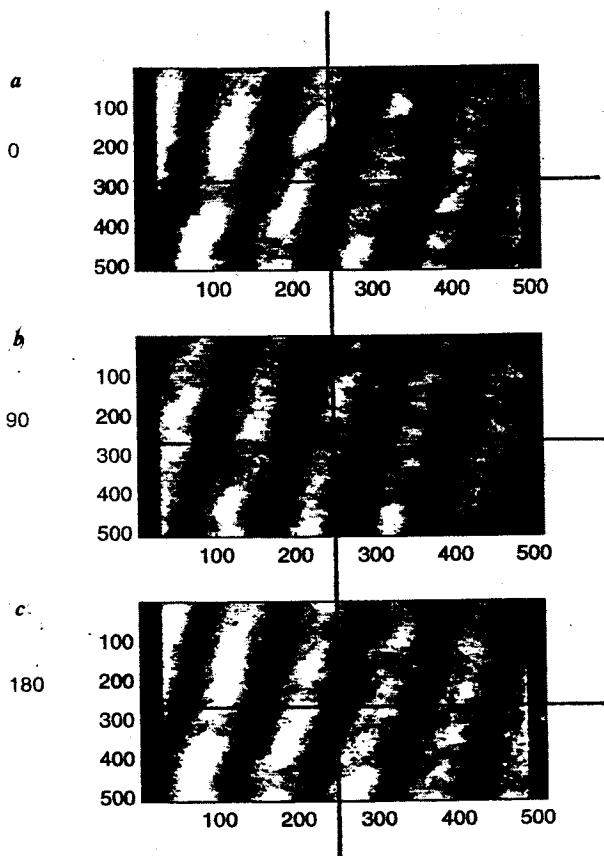


Figure 3. Intensity distribution in the interference fringes with the linear analyser set at angles of (a) 0° , (b) 90° and (c) 180° .

followed by means of the Poincaré sphere⁷. We assume that, as shown in Figure 2, the input is a vertically polarized state which is represented by the point A_1 on the equator of the Poincaré sphere. This state is projected onto the left- and right-circularly polarized states represented, respectively, by N , the North pole of the sphere, and S , the South pole of the sphere. Finally, these two circularly polarized states are projected onto the linearly polarized state represented by the

point A_2 on the equator. If the linear analyser is set at an azimuth angle θ to the vertical, A_2 is located at an angular distance 2θ from A_1 , and the phase difference $\Delta\phi$ that is introduced between the two beams, which is equal to half the solid angle subtended at the centre of the sphere by the area enclosed between the two geodesics A_1NA_2 and A_1SA_2 , is

$$\Delta\phi = 2\theta. \quad (1)$$

To verify the operation of the system, the fringe pattern was recorded with a CCD camera interfaced to a computer. Figure 3 shows the interference fringes with the linear analyser LA set at angles of 0° , 90° and 180° . As can be seen, a rotation of the linear analyser through an angle of 90° resulted in a shift of the fringes corresponding to the introduction of a phase difference $\Delta\phi = 180^\circ$ between the two beams. Rotation of the analyser through 180° brought the fringes back to their original position.

In this interferometer, each state is produced by projecting the previous state on it. The two beams do not traverse any birefringent elements and do not undergo any reflections. Accordingly, the additional phase difference introduced between the two beams by a rotation of the linear analyser is a purely geometric phase.

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