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Perturbative Growth of Cosmological Correlations

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Abstract. We briefly describe the results of our recent detailed analysis, using the BBGKY hierarchy, of the perturbative evolution of correlation functions in cosmological models. The equations for the evolutions of the two and three point correlation functions that are discussed here can be used to study the growth of clustering in the weakly nonlinear regime for various kinds of initial conditions. They can also be used to test approximation schemes which are currently used to describe the nonlinear growth of density perturbations. As an example, we present the form of the three point correlation function that arises from Gaussian initial conditions in an $\Omega = 1$ universe. This does not have the 'hierarchical' form often assumed.

Key words: Cosmology:large scale structure of the universe—Galaxies: clustering.

Here we shall discuss in brief some results, a detailed derivation and discussion of which are presented elsewhere (Bharadwaj 1993). The reader is also referred to Peebles (1980) for a comprehensive discussion of this subject.

We consider deviations of the matter distribution in the universe from the homogeneous and isotropic state. Correlation functions describe the statistical properties of these disturbances. We wish to study the evolution of the correlation functions due to gravitational instabilities in an expanding universe.

The correlation functions only contain information about the spatial distribution of the matter. They have no velocity information. To study the dynamics one has to use distribution functions on phase space. The evolution of these quantities is governed by the BBGKY hierarchy.

We choose initial conditions where the deviation of the matter distribution from the homogeneous and isotropic state is small, and is characterized by a small parameter ε. All peculiar motions are also of this order. Then, by taking velocity moments of the first three equations of the BBGKY hierarchy we derive equations for perturbatively evolving the irreducible two and three point correlation functions.

We display below the equation for perturbatively evolving the two point correlation function ξ. The parameter λ is defined as

$$d\lambda = \frac{dt}{a(t)^2},$$

where $a(t)$ is the scale factor and t the cosmic time. Using this simplifies the equations

$$\frac{\partial^3}{\partial\lambda^3}\xi - 8\pi G\rho\left[a\frac{\partial}{\partial\lambda}\xi + \frac{\partial}{\partial\lambda}(a\xi)\right] = f \tag{1}$$

In the equation above f is a function (which is rather lengthy and not shown here) of order ε^3 and higher which depends on the three point correlation function and the third velocity moment of the two particle distribution function. To the lowest order i.e. ε^2, 'f' is zero and the equation for ξ gives the results we expect from the linear theory of density perturbations. To go further, the evolution of the function f has to be considered.

We have a similar equation for the three point correlation function also. To order ε^3 this reproduces the results of linear theory.

These equations can be used to perturbatively study the growth of cosmological clustering beyond the linear regime. They may also be used to test various approximation schemes used to describe the nonlinear growth of density perturbations. The perturbative approach will only work in the weakly nonlinear regime. It is hoped that a better understanding of the growth of clustering in the weakly nonlinear regime may motivate schemes to close the BBGKY hierarchy and get some understanding of the highly nonlinear regime.

Next, we restrict ourselves to Gaussian initial conditions in a universe with $\Omega = 1$. Many scenarios for the generation of the initial perturbations predict Gaussian initial conditions. In this case there is no initial three point correlation function. We use the equation for the three point correlation function to calculate the induced three point correlation function that arises from the two point correlation function due to gravity. In the calculation we have kept only the growing mode. The irreducible three point correlation function that arises takes the form

$$\zeta(1,2,3,t) = \sum_{a,b,c}\left[\frac{1}{7}(5+2\cos^2\theta)\xi(x)\xi(y) + \cos\theta\frac{d}{dx}\xi(x)y\bar{\xi}(y) + \frac{4}{7}(1-3\cos^2\theta)\xi(x)\bar{\xi}(y) + \frac{2}{7}(9\cos^2\theta - 3)\bar{\xi}(y)\bar{\xi}(x)\right], \tag{2}$$

where

$$\bar{\xi}(x) = \frac{1}{x^3}\int_0^x \xi(x')x'^2 dx' \tag{3}$$

In the equation above

$$x = |x^a - x^b|$$
$$y = |x^a - x^c|,$$

and

$$\cos\theta = \frac{x_\mu y_\mu}{xy}$$

where the subscript μ refers to the cartesian components. The superscripts a, b and c refer to the points between which the correlation is being evaluated and are summed over the values shown in the table below.

a	1	1	2	2	3	3
b	2	3	3	1	1	2
c	3	2	1	3	2	1

The three point correlation is of order ε^4.

In the linear theory the evolution is local. Here we see the first signs of nonlocality developing. The three point correlation function ζ does not depend only on the values of the two point correlation function ξ at the separations occurring in ζ. It depends on the two point correlation at all scales smaller than the scales where the three point correlation function is being evaluated.

Fry (1984) has calculated the three point correlation function for the special case of power-law initial two point correlation function

$$\xi(x) = Ax^{-n}. \tag{4}$$

The result shown above agrees with Fry's result when 'n' is less than three. For higher values of 'n' the integral of the two point correlation function diverges and deviations from the power law behaviour are required at small separations to obtain meaningful results.

The expression for the three point correlation function presented here can in principle be used to test the assumption of Gaussian initial conditions against observations. This can only be done at large scales where the perturbations are still weakly nonlinear.

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References

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