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Homogeneous instabilities in nematic liquid crystals

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Abstract. The effect of a magnetic field on the instability threshold in shear flow and plane Poiseuille flow of nematics has been worked out for the case of unperturbed director orientation normal to the plane of shear. The critical shear rate for the onset of instability has been studied as a function of destabilising field. The dependence of the time constant ν on shear and field for time dependent perturbations in shear flow is found to be explained by two solutions which differ from one another to the extent that wave vectors which are real for one solution are imaginary for the other; these solutions exist for different ranges of shear and field but have one common point where they yield the same ν value making possible a continuous description of ν. In Poiseuille flow even for a destabilising field applied along the primary velocity the occurrence of an instability associated with net secondary flow (flow transverse to the primary velocity) is found to be more favourable than one having no secondary flow. This trend is also observed for low destabilising fields applied along the velocity gradient but for higher fields the occurrence of an instability without net secondary flow becomes more favourable. Simultaneous application of a stabilising electric field does not change the qualitative nature of the results.

Keywords. MHD instability; plane Poiseuille flow; shear flow; nematic liquids; MBBA.

1. Introduction

A number of theoretical and experimental investigations have been recently reported on the homogeneous instability in nematic flow. Pieranski and Guyon (1973) experimentally found that when a slab of nematic is sheared with the director aligned normal to the plane of shear (i) distortion in the director orientation occurs when the shear exceeds a critical value which varies inversely as the sample thickness, (ii) a stabilising magnetic field inhibits the onset of instability with the critical shear rate varying as the square of the field strength for strong fields. These authors also gave a theoretical treatment but did not consider time dependent perturbations nor perturbations in the velocity field. Leslie (1976) and Manneville and Dubois-Violette (1976) employed more general perturbations. Leslie (1976) showed that his exact calculations reduce to those of Pieranski and Guyon (1973) under approximation. Manneville and Dubois-Violette (1976) presented numerical calculations for a stabilizing field and showed that for large enough fields a roll instability is more favourable than the homogeneous one.

Manneville and Dubois-Violette (1976b) worked out theoretically the onset of homogeneous instability in plane Poiseuille flow for unperturbed director orientation normal to the velocity and pressure gradients and showed that above a critical pressure gradient an instability involving net secondary flow, i.e., flow transverse to the primary velocity can set in (as opposed to the case of shear flow where instability is not associated with net secondary flow). They showed that the critical pressure gradient increases with increasing stabilizing field and that an instability without net secondary flow is not favoured. Some of these results were experimentally justified by Janossy *et al* (1976).

In this paper numerical calculations on the effect of destabilizing magnetic fields on the instability thresholds in shear flow and plane Poiseuille flow of nematics have been presented. The critical shear rate for the onset of instability has been studied as a function of destabilizing field. Some of the new results obtained are the following: (i) The time constant ν of time dependent perturbations in shear flow is found to be explained by two solutions which differ from one another to the extent that wave vectors which are real for one are imaginary for the other; these solutions exist for different ranges of field and shear but have one common point where they correspond to the same ν value. (ii) In plane Poiseuille flow the occurrence of an instability associated with net secondary flow continues to be more favourable than one without net secondary flow even in the presence of a destabilising field applied along the primary flow. For low values of destabilising field applied along the velocity gradient the same trend is observed. But for higher fields the occurrence of an instability without net secondary flow becomes more favourable. Simultaneous application of a stabilising electric field does not change the qualitative nature of the results.

2. Shear flow

The nematic is assumed to be confined between two plane parallel plates occupying the planes $z=0$ and $z=d$. The director is assumed to be aligned along the x axis. The plate $z=d$ moves with a constant velocity V along the positive y direction. The notations in this paper follow closely those of Kini (1976) where the Ericksen-Leslie equations for nematics have been summarised. We seek solutions for the director and velocity fields and pressure in the form

$$n_x = \cos\theta(z,t)\cos\varphi(z,t),\quad n_y = \cos\theta(z,t)\sin\varphi(z,t),\quad n_z = \sin\theta(z,t)$$

$$v_x = u(z,t),\quad v_y = v(z,t),\quad v_z = 0$$

$$p = p(z,t)$$

subject to the boundary conditions

$$\theta(0,t) = \theta(d,t) = \varphi(0,t) = \varphi(d,t) = u(0,t) = u(d,t) = v(0,t) = 0;$$

$$v(d,t) = V. \tag{1}$$

A magnetic field $\mathbf{H}$ is applied. Inertial effects have been ignored for mathematical simplicity. The Ericksen-Leslie equations assume the following form:

$$[M(\theta)+N(\theta)C_\varphi^2]u_{,z}+N(\theta)v_{,z}S_\varphi C_\varphi+2K(\theta)C_\varphi\mathring{\theta}-2L(\theta)S_\varphi\mathring{\varphi} = 2a = 2t_{xz} \tag{2}$$

$$N(\theta)S_\varphi C_\varphi u_{,z}+[M(\theta)+N(\theta)S_\varphi^2]v_{,z}+2K(\theta)S_\varphi\overset{\circ}{\theta}+2L(\theta)C_\varphi\overset{\circ}{\varphi}$$

$$=2b=2t_{yz} \tag{3}$$

$$2F(\theta)\theta_{,zz}+\frac{dF}{d\theta}\theta^2{}_{,z}-\frac{dG}{d\theta}\varphi_{,z}{}^2+2\lambda_1\overset{\circ}{\theta}$$

$$+(\lambda_1+\lambda_2\cos 2\theta)(u_{,z}C_\varphi+v_{,z}S_\varphi)$$

$$+2\chi_a(\mathbf{H}\cdot\mathbf{n})(H_zC_\theta-H_xS_\theta C_\varphi-H_yS_\theta S_\varphi)=0 \tag{4}$$

$$2G(\theta)\varphi_{,zz}+2\frac{dG}{d\theta}\theta_{,z}\ \varphi_{,z}+2\lambda_1C_\theta^2\overset{\circ}{\varphi}+(\lambda_1-\lambda_2)S_\theta C_\theta(u_{,z}S_\varphi-v_{,z}C_\varphi)$$

$$+2\chi_a(\mathbf{H}\cdot\mathbf{n})\,(H_yC_\theta C_\varphi-H_xC_\theta S_\varphi)=0 \tag{5}$$

where

$$S_\theta=\sin\theta,\ C_\varphi=\cos\varphi,\ \overset{\circ}{\theta}=\frac{\partial\theta}{\partial t},\ \varphi_{,z}=\frac{\partial\varphi}{\partial z},\ \text{etc.},$$

$$M(\theta)=\mu_4+(\mu_5-\mu_2)S_\theta^2,\quad N(\theta)=(2\mu_1S_\theta^2+\mu_3+\mu_6)C_\theta^2.$$

$$F(\theta)=k_{11}C_\theta^2+k_{33}S_\theta^2,\ K(\theta)=\mu_3C_\theta^2-\mu_2S_\theta^2,\quad L(\theta)=\mu_2S_\theta C_\theta,$$

$$G(\theta)=(k_{22}C_\theta^2+k_{33}S_\theta^2)C_\theta^2;$$

a and b are constants which can depend on time, χ_a is the diamagnetic susceptibility anisotropy, k_{11}, k_{22}, k_{33} the elastic constants, μ_1, μ_2, μ_3, μ_4, μ_5, μ_6 the viscosity coefficients, $\lambda_1=\mu_2-\mu_3$ and $\lambda_2=\mu_5-\mu_6$.

On assuming that u, θ and φ are small and on linearising, eqs (2) and (3) reduce to

$$u_{,z}\ \mu_4(\mu_3+\mu_4+\mu_6)=2a\mu_4-2\mu_3\mu_4\overset{\circ}{\theta}-2b(\mu_3+\mu_6)\varphi \tag{6}$$

$$v_{,z}=\frac{2b}{\mu_4} \tag{7}$$

On using eqs (6) and (7), eqs (4) and (5) reduce, for different directions of the magnetic field, to the following relations:

$\mathbf{H}=(H, 0, 0)$ (*Stabilising field*)

$$\varphi_{,zz}-n_2\overset{\circ}{\varphi}+m_2\theta-h_2\varphi=0 \tag{8}$$

$$\theta_{,zz}-n_1\overset{\circ}{\theta}+m_1(\varphi+\delta)-h_1\theta=0 \tag{9}$$

$\mathbf{H}=(0, H, 0)$. *Destabilizing field*

$$\varphi_{,zz}-n_2\overset{\circ}{\varphi}+m_2\theta+h_2\varphi=0 \tag{10}$$

$$\theta_{,zz}-n_1\overset{\circ}{\theta}+m_1(\varphi+\delta)=0 \tag{11}$$

$\mathbf{H}=(0, 0, H)$. *Destabilizing field*

$$\varphi_{zz}-n_2\overset{\circ}{\varphi}+m_2\theta=0 \tag{12}$$

$$\theta_{zz}-n_1\overset{\circ}{\theta}+m_1(\varphi+\delta)+h_1\theta=0 \tag{13}$$

with $h_1=\chi_a H^2/k_{11}$, $h_2=\chi_a H^2/k_{22}$,

$m_1=(\lambda_1+\lambda_2)b/k_{11}(\mu_3+\mu_4+\mu_6)$, $m_2=(\lambda_2-\lambda_1)b/\mu_4 k_{22}$,

$\delta=a/b$, $n_2=-\lambda_1/k_{22}$,

$n_1=-[\lambda_1-\mu_3(\lambda_1+\lambda_2)/(\mu_3+\mu_4+\mu_6)]/k_{11}$.

Since a and b are functions of time δ is also a function of time. The rest of this section contains a discussion on the exact solution of the eqs (10) to (13) compatible with (1). The differential equations governing θ and φ are solved first. Using eq. (7) and the relation

$$\int_0^d u_{,z}\,dz=0 \tag{14}$$

one obtains the compatibility condition connecting b and H for stationary perturbations or b, H and the time constant ν for time dependent perturbations.

The model calculations have been carried out for MBBA for sample thicknesses of 50 μm and 100 μm. The data for viscosity coefficients and elastic constants are those reported by Gähwiller (1971) and Haller (1972) respectively. The value of χ_a has been taken from Gasparoux *et al* (1971). In every case eq. (14) reduces to a transcendental equation which has been solved by the Newton-Rhapson method coupled with iteration on an IBM 360/44 computer with double precision arithmetic. For stationary perturbations δ is treated as a constant and eq. (14) yields a relation between b and H for a given sample thickness. For a given value of H there exists a value b_c of b such that θ, φ and u cannot exist for $0<b<b_c$. In the investigation of time dependent perturbations, the time dependence of all relevant quantities is assumed to be of the form $\exp(\nu t)$ with ν being assumed to be real (since propagating modes are generally not found in nematics). A solution of eq. (14) yields ν as a function of b and H.

When a destabilizing field $H=H_y$ is applied it is found that b_c decreases with increasing field strength (figure 1) becoming zero when $H=H_c^T=\{\pi/d\}\ (k_{22}/\chi_a)^{\frac{1}{2}}$ the Freedericksz value corresponding to the twist deformation. A similar result is obtained for the field $H=H_z$; b_c goes to zero for $H=H_c^S=\{\pi/d\}(k_{11}/\chi_a)^{\frac{1}{2}}$ the critical field strength for the splay geometry (figure 1). These instabilities correspond to the mode which is such that θ and φ are symmetric about the centre of the sample

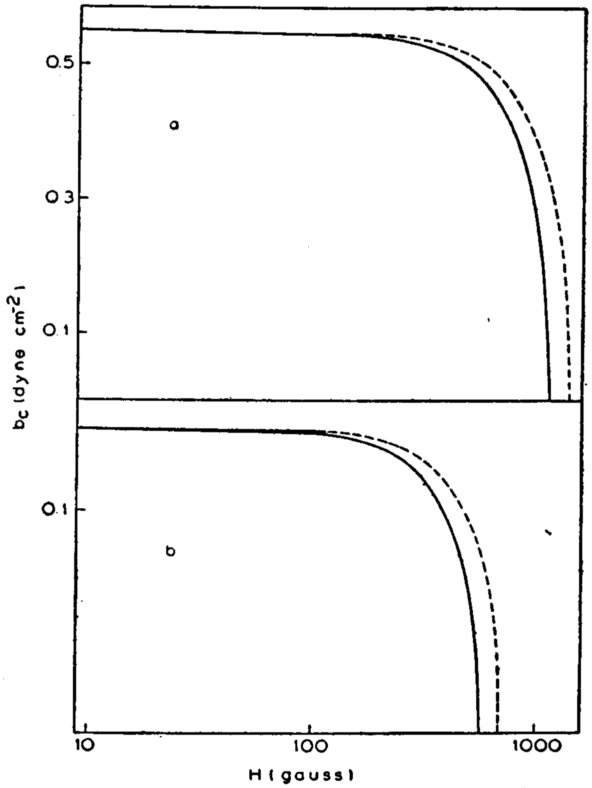

Figure 1. Variation of b_c as a function of destabilizing magnetic field for two different sample thicknesses (a) 50 μ (b) 100 μ. Dashed lines correspond to a field H_z while the solid lines correspond to a field H_y.

while u is antisymmetric and thus the net flow associated with it is zero. Since the mode corresponding to a symmetric u can occur only for special values of the sample thickness (Leslie 1976) it has not been discussed here.

Figure 2 illustrates the effect of a destabilizing magnetic field H_y on the time constant ν of a time dependent perturbation. It is found that there are two solutions—solution A which exists for $\beta = m_1 m_2 + h_2 n_1 \nu - n_1 n_2 \nu^2 > 0$ and solution B which exists for other values of β. The difference between the two solutions lies in a wave vector being real for one solution while becoming imaginary for the other. For instance θ for the two solutions has the following form:

Solution A

$$\theta = \frac{[\Delta_1(k_2^2 - n_1\nu) + m_1\Delta_2]\,[\sin k_1 z + \sin k_1(d-z)]}{(k_1^2+k_2^2)\sin k_1 d}$$

$$+ \frac{[\Delta_1(k_1^2 + n_1\nu) - m_1\Delta_2]\,[\sinh k_2 z + \sinh k_2(d-z)]}{(k_1^2+k_2^2)\sinh k_2 d} - \Delta_1$$

$$2k^2_{1,2} = [\{h_2 - \nu\,(n_1+n_2)\}^2 + 4(m_1 m_2 + h_2 n_1 \nu - n_1 n_2 \nu^2)]^{\frac{1}{2}} \pm \{h_2 - \nu(n_1+n_2)\}$$

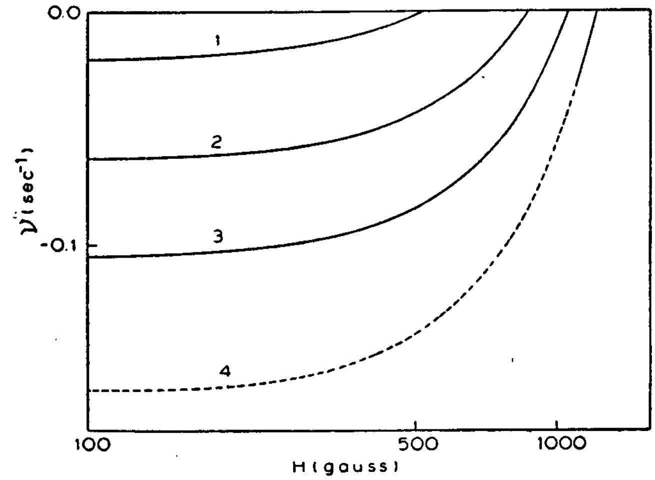

Figure 2. Variation of the time constant ν as a function of destabilizing field H_y for different values of b. (1) 0·5 (2) 0·4 (3) 0·3 (4) 0·15 dynes cm^{-2}. Solid lines pertain to solution A while the dashed line corresponds to solution B.

Solution B

$$\theta = \frac{(m_1\Delta_2 - \Delta_1(k_2^2 + n_1\nu))\,[\sin k_1 z + \sin k_1(d-z)]}{(k_1^2 - k_2^2)\sin k_1 d}$$

$$+ \frac{[\Delta_1(k_1^2 + n_1\nu) - m_1\Delta_2]\,[\sin k_2 z + \sin k_2(d-z)]}{(k_1^2 - k_2^2)\sin k_2 d} - \Delta_1$$

$$2k^2_{1,2} = h_2 - \nu(n_1+n_2) \pm [\{h_2 - \nu(n_1+n_2)\}^2 + 4(m_1 m_2 + h_2 n_1 \nu - n_1 n_2 \nu^2)]^{\frac{1}{2}}$$

$\Delta_{1,2}$ are constants depending on m_1, m_2, h_1, h_2, etc.

Solution B is found to explain the dependence of ν on b and H for low enough values of these quantities, while solution A exists for higher values of b and H where instability can set in. For a given value b' of b one finds that $\nu < 0$ for $H < H'$ (see figure 1 for b' and H'). For $H = H'$, ν becomes zero and this corresponds to the instability threshold. For $H > H'$, ν becomes positive indicating that instability can set in. In this region of field the calculation (as also the linearised equations) cease to have any significance. Similar results can be obtained for field H_z also.

3. Plane Poiseuille flow

The nematic is assumed to be confined between two plane parallel plates $z = \pm d$ and to flow along the positive y direction under the action of a constant pressure gradient $p_{,y}$. The director is initially oriented along x. We seek solutions for the director and velocity fields in the form

$$n_x = 1 \qquad n_y = \varphi(z, t) \qquad n_z = \theta(z, t)$$

$$v_x = u(z, t) \qquad v_y = v(z, t) \qquad v_z = 0$$

under the action of a magnetic field **H** and an electric field $\mathbf{E} = (0, 0, E)$. (The theory worked out here is for a *pure* nematic. But in practice a static electric field can give rise to electrohydrodynamic effects owing to ionic impurities. Hence in an experiment one has to use an alternating electric field of high enough frequency to exclude these effects and E would then correspond to the rms value). Transforming to the variable $\xi = z/d$, ignoring inertial effects and linearising the Ericksen-Leslie equations in terms of θ, φ and u one obtains

$$v = p_{,y}\, d^2 (\xi^2 - 1)/\mu_4 \tag{15}$$

$$m_3 u' = m_4 - \xi\varphi - m_5 \overset{\circ}{\theta} \tag{16}$$

Stabilizing magnetic field $H = H_x$

$$\varphi'' + m_2 d\, \xi\theta - h_2 d^2 \varphi - n_2 d^2 \overset{\circ}{\varphi} = 0 \tag{17}$$

$$\theta'' + m_1 d^3\, \xi\varphi - \theta d^2 \frac{[4\chi_a H^2 \pi - \epsilon_a E^2]}{4\pi k_1} + \frac{m_1 m_4 d^3 (\mu_3 + \mu_6)}{\mu_4} - n_1 d^2 \overset{\circ}{\theta} = 0 \tag{18}$$

Destabilizing field $H = H_y$

$$\theta'' + m_1 d^3 \left[m_4 \frac{(\mu_3 + \mu_6)}{\mu_4} + \xi\varphi \right] + e_1 d^2 \theta - n_1 d^2 \overset{\circ}{\theta} = 0 \tag{19}$$

$$\varphi'' + m_2\, \xi\, d^3 \theta + h_2 d^2 \varphi - n_2 d^2\, \overset{\circ}{\varphi} = 0 \tag{20}$$

Destabilizing field $H = H_z$

$$\varphi'' + m_2\, \xi\, d^3 \theta - d^2 n_2\, \overset{\circ}{\varphi} = 0 \tag{21}$$

$$\theta'' + m_1 d^3 \left[m_4 \frac{(\mu_3 + \mu_6)}{\mu_4} + \xi\varphi \right] + h_1 d^2 \theta - n_1 d^2\, \overset{\circ}{\theta} = 0 \tag{22}$$

where $e_1 = \epsilon_a E^2/4\pi k_{11}$, $h_2 = \chi_a H^2/k_{22}$, $h_1 = (4\pi\chi_a H^2 + \epsilon_a E^2)/4\pi k_{11}$,

$m_1 = (\lambda_1 + \lambda_2)\, p_{,y}/k_{11}\, (\mu_3 + \mu_4 + \mu_6)$, $m_2 = (\lambda_2 - \lambda_1)\, p_{,y}/\mu_4 k_{22}$,

$m_3 = (\mu_3 + \mu_4 + \mu_6)\mu_4/2p_{,y}\, d^2(\mu_3 + \mu_6)$, $m_4 = \mu_4 \delta/d(\mu_3 + \mu_6)$,

$m_5 = \mu_3\mu_4/p_{,y}\, d(\mu_3 + \mu_6)$, $n_1 = [\mu_3\{(\lambda_1 + \lambda_2)/(\mu_3 + \mu_4 + \mu_6)\} - \lambda_1]/k_{11}$,

$n_2 = -\lambda_1/k_{22}$, $\varphi' = \partial\varphi/\partial\xi$,

ϵ_a is the dielectric anisotropy and $\delta p,_y$ is the value of the stress t_{zx}. δ (and hence m_4) is a function of time alone. Since ϵ_a is negative for MBBA (see for instance table 1 of Manneville and Dubois-Violette 1976a) the electric field acts as a stabilizing force. For a given field the differential equations governing θ and φ are solved with boundary conditions

$$\theta(\pm 1, t) = \varphi(\pm 1, t) = 0 \tag{23}$$

The method of solution is exactly the same as employed by Manneville and Dubois-Violette (1976b) involving Fourier expansion of the perturbations for the case of stationary perturbations. By utilizing eq. (16) which is solved wilth the boundary conditions $u(\pm 1, t) = 0$ one obtains an infinite dimensional matrix which should have a null determinant. This condition gives the necessary relation between the Ericksen number $\mathscr{E}_r = d^3(m_1 m_2)^{\frac{1}{2}}$, the applied field, etc. For stationary perturbations Manneville and Dubois-Violette (1976b) have shown that one can get good convergence by retaining the first ten terms in the Fourier series so that the condition of null determinant has to be applied to a 10×10 matrix. In this section eqs (16)–(22) have been solved only for stationary perturbations. Since m_4 is in general a function of time for stationary perturbations it is treated as a pure number.

To get a clear picture of the homogeneous instability one can analyse eqs (16)–(22) by writing every quantity as the sum of an even function of ξ and an odd function of ξ. The system of equations breaks up into two closed sub-systems of equations corresponding to two distinct modes—(i) in which θ is even but φ and u are odd in ξ and (ii) in which φ and u are even but θ is odd in ξ. Mode (i) is called the average splay mode (or mode S in this section) and mode (ii) is called the average twist mode (or mode T). In mode S θ being an even function of ξ has a non-vanishing average over the sample and gives an average splay distortion. On the other hand, φ and u have vanishing averages. Thus mode S is characterized by the absence of net secondary flow. On the other hand, for mode T, θ has a vanishing average over the sample but φ and u have non-vanishing averages. Also, this mode, so named because φ is a twist distortion, has a net secondary flow associated with it. In any analysis these modes are treated separately. Manneville and Dubois-Violette (1976b) showed that for the field free case or for stabilizing magnetic fields mode S has an Ericksen number $\mathscr{E}_r^S$ which is always larger than the Ericksen number $\mathscr{E}_r^T$ for mode T indicating thereby that mode T is more favourable than mode S. This has been verified by Janossy *et al* (1976) who also observed net secondary flow.

The variation of $\mathscr{E}_r$ as a function of a destabilizing field $H = H_y$ for different electric fields is shown in figure 3. A sample thickness of 200 μm has been employed in the calculation. For $E=0$, $\mathscr{E}_r$ for both the modes decreases with increasing H, but $\mathscr{E}_r^S$ is always greater than $\mathscr{E}_r^T$. When $H = H_c^T = (\pi/2d)(k_{22}/\chi_a)^{\frac{1}{2}}$ $\mathscr{E}_r^T$ becomes zero. But $\mathscr{E}_r^S$ goes to zero only for $H = 2H_c^T$. This is because, for mode T, φ is an even function $\varphi_E(\xi)$ and at fields H_y close to H_c^T the occurrence of $\varphi_E(\xi)$ will be more favourable than that of its antisymmetric counterpart $\varphi_A(\xi)$ pertaining to mode S; to excite $\varphi_A(\xi)$ in the static case one would need twice the field H_c^T. In the presence of an electric field E along z the critical field for the twist configuration does not change. The presence of E simply enhances $\mathscr{E}_r$ for both the modes, due to its stabilizing influence. Thus the T mode continues to be more favourable than the S mode even for destablizing fields H_y with or without electric field along z.

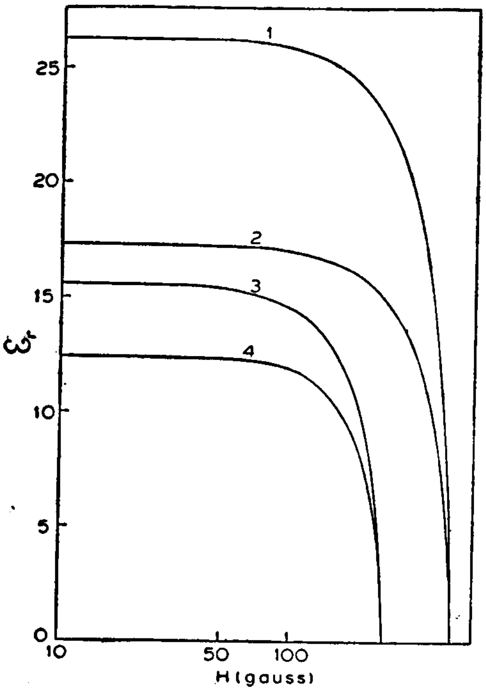

Figure 3. Variation of Ericksen number $\mathscr{E}_r$ for S and T modes as a function of destabilizing magnetic field H_y. Curves 1 and 2 are for the S mode for $E=0{\cdot}8$ cgs and $E=0$ cgs respectively; 3 and 4 are the corresponding curves for the T mode.

The effect of a destablizing field H_z is more interesting. Figure 4a illustrates the variation of $\mathscr{E}_r$ for both the modes as a function of H for $E=0$. As H increases $\mathscr{E}_r$ decreases, the decrease being more rapid for the S mode than the T mode. On further increasing H, $\mathscr{E}_r^S$ goes to zero at $H=H_c^S=(\pi/2d)\,(k_{11}/\chi_a)^{\frac{1}{2}}$ for the S mode while $\mathscr{E}_r^T$ becomes zero at $2H_c^S$. Thus for sufficiently low fields ($H<H_p$), $\mathscr{E}_r^T<\mathscr{E}_r^S$. But for larger fields ($H>H_p$) there is a reversal in the trend. This indicates that for $H<H_p$ the occurrence of the T mode is favoured while for $H>H_p$ the occurrence of the S mode is favoured. The reason for this is not far to seek. In the static case for a field H_z one gets a θ distortion $\theta_E(\xi)$ which is an even function of ξ, for $H \gtrsim H_c^S$. Since for the S mode θ is an even function it can become more favourable than the T mode in the vicinity of H_c^S.

In the presence of an electric field E along z, h_1 can be negative, zero or positive according as $H \lesseqgtr E(-\epsilon_a/4\pi\chi_a)^{\frac{1}{2}} \equiv H_L$ (say). For $H=H_L$, $h_1=0$, the effects of E and H are annulled and one has effectively the field-free case. For $h_1 \gtrless 0$ one gets two solutions for the S mode which differ from one another to the extent that the wave vector $h_1^{\frac{1}{2}}$ is real or imaginary. For instance for θ one obtains for $H < H_L$

$$\theta = \sum_{q=1}^{\infty} \theta_q(\xi)$$

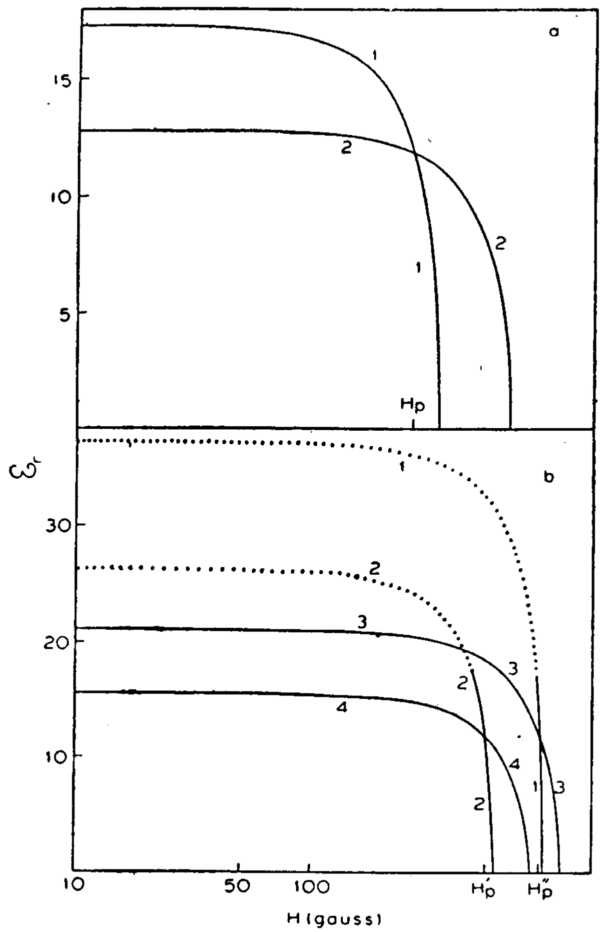

Figure 4. Variation of Ericksen number $\mathscr{E}_r$ as a function of destabilizing magnetic field H_z for different electric fields. (a) $E=0$ cgs; curve 1 for S mode and curve 2 for T mode. H_p is the magnetic field corresponding to a cross over between S and T modes. (b) Curves 1 and 2 for the S mode for $E=1{\cdot}5$ cgs and $0{\cdot}8$ cgs respectively. 3 and 4 are the corresponding curves for the T mode. H_p' and H_p'' are the cross over fields for $E=0{\cdot}8$ cgs and $E=1{\cdot}5$ cgs respectively.

where

$$\theta_q(\xi) = A_q\, m_1\, d^3 \left[\frac{2q\pi}{(k^2+q^2\pi^2)^2} \left\{ \frac{(-1)^q \cosh k\xi}{\cosh k} + \cos q\pi\xi \right\} + \frac{\xi \sin q\pi\xi}{(k^2+q^2\pi^2)} + \frac{(-1)^{q+1}(\mu_3+\mu_6)}{q\pi k^2 \mu_4} \left\{ 1 - \frac{\cosh k\xi}{\cosh k} \right\} \right]$$

with $k^2 = -h_1 d^2$ and for $H>H_L$ $\theta = \sum_{q=1}^{\infty} \theta'_q(\xi)$ where

$$\theta'_q(\xi) = A_q m_1 d^3 \left[\frac{2q\pi}{(k_1^2-q^2\pi^2)^2} \left\{ \frac{(-1)^{q+1} \cos k_1 \xi}{\cos k_1} + \cos q\pi \xi \right\} - \frac{\xi \sin q\pi\xi}{(k_1^2-q^2\pi^2)} + \frac{(-1)^{q+1}(\mu_3+\mu_6)}{q\pi\mu_4 k_1^2} \left\{ \frac{\cos k_1 \xi}{\cos k_1} - 1 \right\} \right]$$

with $k_1^2 = h_1 d^2$. A_q are the Fourier coefficients of φ. Naturally for low fields ($H < H_L$) $\mathscr{E}_r^S > \mathscr{E}_r^T$ so that for these fields the T mode is more favourable than the S mode (figure 4*d* dotted curves). On the other hand for $H > H_p > H_L$ $\mathscr{E}_r^S$ can become less than $\mathscr{E}_r^T$ so that the occurrence of the S mode is more favourable than that of the T mode. Of course for $H_L < H < H_p$ the T mode continues to be more favourable than the S mode. Also one find that the field H_p shifts to higher values with increasing E and the H values at which $\mathscr{E}_r$ becomes zero for the two modes move closer. However, since the deformation of the S mode is energetically more favourable than that of the T mode in the vicinity of small $\mathscr{E}_r$ one cannot expect any further reversal in the roles of the S and T modes with increasing E.

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