J. Astrophys. Astr. (1982) 3, 351–361

A New Look at the Birthrate of Supernova Remnants

G. Srinivasan and K. S. Dwarakanath *Raman Research Institute, Bangalore 560080*

Received 1981 August 12; accepted 1982 July 29

Abstract. We have reanalysed a homogeneous catalogue of shell-type supernova remnants and we find that the radio data are consistent with a birthrate of one in 22 ± 3 yr. Our approach is based on the secular decrease of surface brightness of the historical remnants whose ages are precisely known. The above-mentioned birthrate is significantly higher than most previous estimates which range from one in 50–150 yr, and is consistent with the supernova rate in our galaxy derived from historical observations, as well as with recent estimates of the pulsar birthrate.

Key words: supernova remnants, birthrates

1. Introduction

The rate of supernovae (SN) in our galaxy has been variously estimated to be one in every 11 yr to one in 30 yr (Clark and Stephenson 1977a; Katgert and Oort 1967; Tammann 1977). These numbers were arrived at in two different ways. One way was to determine the observed SN rate in external galaxies of similar morphology to our own. The other method was to estimate the rate of SN in our galaxy from historical records. Katgert and Oort, using the latter approach have suggested a rate of one in 25 yr, and Clark and Stephenson suggested one in 30 yr or less. It may be recalled that only 7 SN were recorded in the last 2 millennia. The failure to detect the other 70 or so SN which must have gone off in the last 2000 yr (as suggested by the above rates) is explained by the above authors as due to obscuration, horizon limitations *etc.*

It is generally accepted that SN explosions are associated with the formation of neutron stars as originally suggested by Baade and Zwicky (1934). According to this picture, the birthrate of neutron stars must match the SN rate. It is also generally believed that all neutron stars function as pulsars. Therefore it is natural to try to relate the birthrate of pulsars to the SN rate. The pulsar birthrate estimates in the literature vary from one every 7–40 yr (Taylor and Manchester 1977; Phinney and

Blandford 1981). Recently, Vivekanand and Narayan (1981) have arrived at a rate of one in 21^{+6}_{-4} yr; since this is the first calculation to incorporate luminosity selection effects, it is probably the most reliable number to date. Thus it would appear that the pulsar birthrate and SN rate are reasonably consistent, though there are inherent errors in each of these estimates.

The third number which must agree with the above two rates is the birthrate of supernova remnants (SNRs). There have been numerous attempts in the literature to estimate this in various ways. We mention a few of the methods and the results obtained.

1. One assumes that all SNRs with surface brightness Σ greater than that of a given age calibrator are younger. Using this method Caswell (1970) arrived at $\tau=55^{+25}_{-15}$ yr (where τ is the mean interval between births of SNRs).

2. A variant of the above method is to assume that all SNRs with a linear diameter D less than that of an SNR with known age are younger. Ilovaisky and Lequeux (1972) obtained $\tau=50 \pm 25$ yr, using this approach.

3. Clark and Caswell (1976) have argued that the cumulative counts of SNRs [$N(<D)$ and $N(>\Sigma)$ plots] suggest that all SNRs with diameter $\leqslant$ 32 pc evolve according to the self-similar solution given by Sedov. Using consistency arguments, they relate τ to E_0/n, where E_0 is the energy of the explosion and n is the density of the interstellar medium. Using a value of $E_0/n=5\times 10^{51}$ erg cm^3, as suggested by X-ray observations of four remnants, they arrive at $\tau\sim150$ yr.

The above methods have some inherent drawbacks. For example, it need not be true that a remnant with Σ greater than a given one is necessarily younger. The brightness of the remnant may be intimately connected with the initial parameters of the explosion, as well as the density of the ISM (Gull 1973). The same criticism would apply to the $N(<D)$ approach. Again, it may not be true that all remnants with $D<32$ pc are evolving according to the Sedov solution. It is well known that the exploding shell will evolve in a self-similar manner only when the mass swept up far exceeds the mass ejected. This may not be true for a remnant of a given diameter, if it is expanding in a region of very low density or the mass ejected is very high.

In this paper, we wish to re-examine this question. Our method, a variant of the one used by Caswell (1970), is based on the secular decrease of the surface brightness of historical shell SNRs. In a recent paper, Higdon and Lingenfelter (1980) have derived a birthrate for SNRs using a similar approach, though there are important differences in detail. We shall return to this in later sections. In Section II, we shall outline our procedure and derive a birthrate for shell SNRs. In Section III, we attempt to incorporate the possible effect of the density of the interstellar medium on the radio brightness of the remnants.

2. Surface brightness – age relation

The radio surface brightness of a remnant is a function of its age. Basically, two kinds of mechanisms for radio emission have been proposed in the literature for shell remnants. In the first it is assumed that the magnetic field in the shell is the compressed interstellar field, and that the particles responsible for the synchrotron radiation are either the interstellar cosmic rays or the particles produced in the original explosion (van der Laan 1962). The alternative is to say that the magnetic field and/or the relativistic particles are generated within the shell itself. Gull (1973) has advanced a model where both the magnetic field and the particle energy are amplified by turbulent motions in the convective interface between the ejecta and the interstellar gas. He has argued that this will continue till equipartition is reached between the turbulent energy, the particle energy and the field energy. In this paper we shall assume that this is the operative mechanism for young remnants. According to this model, the radio emission will reach a maximum at a time $t=t_0$ when the mass swept up is of the order of 0·1–0·3 times the mass ejected, and then will decay with time. Thus, for $t \geqslant t_0$ one may write

$$\Sigma = \Sigma_{\max} (t_0/t)^{\alpha} \tag{1}$$

where $\Sigma_{\max}$ is the brightness at time t_0. Gull's theory does not give any specific value for α. In what follows we shall write

$$\Sigma = At^{-\alpha} \tag{2}$$

and treat α as a parameter. It should be emphasized here that this equation is general in character and is not necessarily tied to Gull's theory, though consistent with it. For any given value of α, one can determine the constant A for a specific age calibrator. This combination of A and α can then be used to calculate the 'ages' of all the known remnants, and thus a birthrate can be obtained. This assumes that α is constant in time. We shall impose the following criteria for the choice of α.

1. If one restricts oneself to a time interval which is small compared to the mean lifetime of remnants, then one must have $N(<t) \propto t$, where t is the age of the remnant. This will be true as long as 'deaths' of remnants have not yet set in.

2. More than one age calibrator is available (see below) for determining the constant A, for a given α. The most appropriate value of α must satisfy the criterion that it give reasonably good ages for all the other age calibrators.

As was mentioned in the Introduction, there were seven definite recordings of SN in historical times. They were recorded in AD 185, AD 393, AD 1006, AD 1054, AD 1181, AD 1572 (Tycho), and AD 1604 (Kepler). Clark and Stephenson (1977b) have discussed in detail the association of radio SNRs with these historically-observed explosions. They come to the conclusion that—with the exception of the supernova of AD 393—a reasonably certain identification can be made in every other case. Recently, it has been suggested (Ashworth 1980; Brecher and Wasserman 1980) that the SN that produced the radio remnant Cas A may have been seen by the astronomer Flamstead in AD 1680, but we shall not use this remnant as an age calibrator. The Crab nebula and 3C 58 (probably associated with the SN of AD 1181) have a morphology quite different from the other historical remnants, which are of the shell type. There are reasons to believe (see for example Weiler and Panagia 1980; Radhakrishnan and Srinivasan 1980a) that the evolution of these objects is different from those of shell SNRs. In what follows, we shall therefore restrict ourselves to shell SNRs. One is thus left with four shell remnants whose ages are accurately known, *viz.* SNR 185, SNR 1006, SNR 1572 and SNR 1604, with which one can calculate the ages of all the known SNRs. We have used the catalogue of SNRs compiled by Clark and Caswell (1976). They list 120 SNRs out of which Σ (the surface brightness at 408 MHz) is quoted for all but four. From this list of 116, we delete the Crab, 3C 58, MSH 15–56 and Vela X because of their filled-in appearance, and apply our analysis to the remaining 112.

In Figs 1–3, we show $N(<t)$ *vs.* t plots for $a = 1{\cdot}2$, $2{\cdot}5$, $3{\cdot}5$ respectively. $a = 1{\cdot}2$ corresponds to evolution according to the Sedov solution if $\beta = 3$ (where, $\Sigma \propto D^{-\beta}$). For a given a, each one of the curves corresponds to the constant A in Equation (2) determined through one of the age calibrators.

There are three striking features in these plots:

1. Beyond 300 years or so, the $N(<t)$ curves are remarkably linear (till about 2000 years), indicating a uniform birthrate. Since, as we have already mentioned, it takes a time t_0 (which depends on the density of the medium *etc.*) for the radio brightness to build up, the behaviour of the curves for the first few hundred years can be understood.
2. As a increases, the $N(<t)$ curves calculated from SNR 185, SNR 1572 and SNR 1604 approach each other; but the curves are insensitive to an increase in a beyond 2·5. It may be noted that when SNR 1006 is used as a calibrator, one obtains a much higher birthrate, irrespective of a. We shall discuss the possible reasons for this in some detail in the next section.
3. Most of the SNRs appear to be less than 2000 years old.

An examination of Figs 1–3 shows that it is difficult to choose the optimum value of a on this basis. In order to do this we have computed the residual R defined by

$$R = \left[\frac{1}{4}\sum_{i=1}^{4} \ln^2\left(\frac{t_{\mathrm{cal},\,i}}{t_{\mathrm{true},\,i}}\right)\right]^{1/2} \tag{3}$$

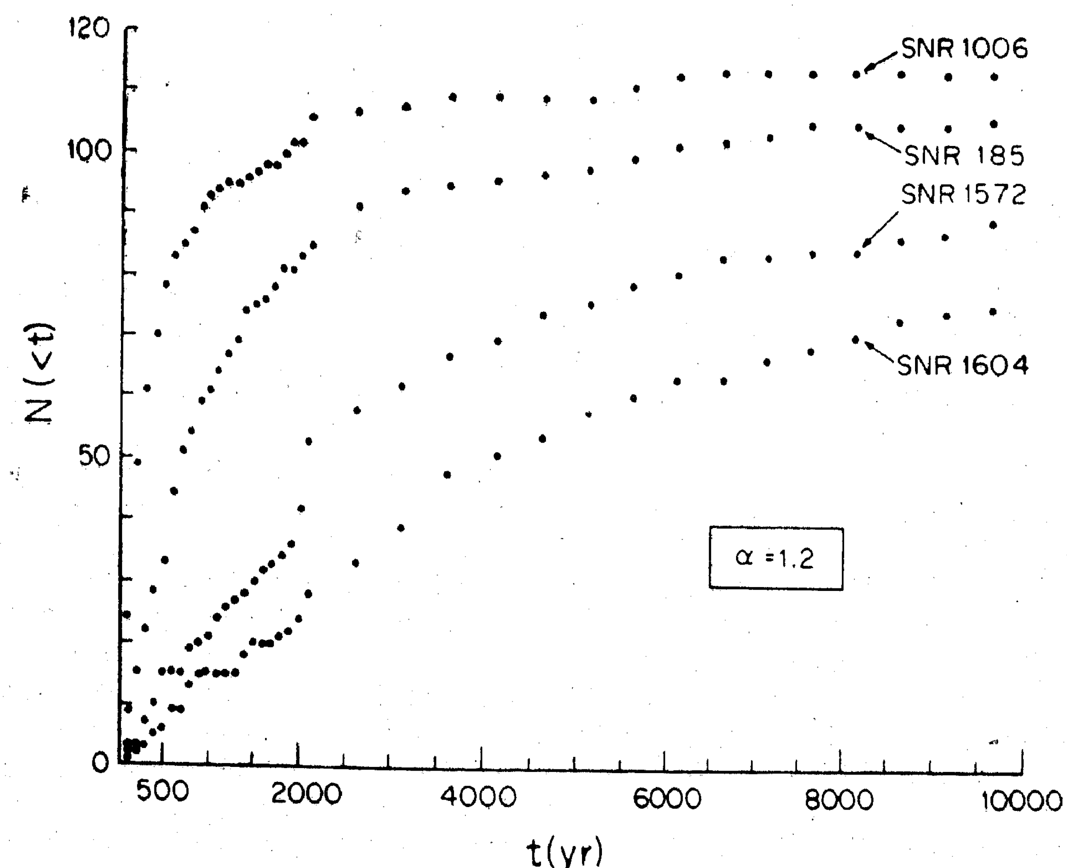

Figure 1. The number of shell SNRs with ages less than a given age, for $a = 1{\cdot}2$. The labels refer to the calibrator used to determine the constant A in Equation (2).

where $t_{cal,\,i}$ is the age of one of the above-mentioned calibrators derived from Equation (2) and $t_{true,\,i}$ is its actual age. It was found that R is minimum for $\alpha = 3{\cdot}5$ and $A = 4{\cdot}4 \times 10^{-10}$. Since the number of calibrators are only four, one cannot obtain a meaningful statistical error for R. So we have adopted the following approach to estimate the scatter in α and A. There are reasons to suspect that SNR 1006 may be subluminous for its age (possible reasons are discussed in the next section). If so, it will strongly bias the above procedure towards a larger value of α. To get a feeling for this we left out SNR 1006 as a calibrator and repeated the above procedure. The minimum value of R now corresponded to $\alpha = 2{\cdot}5$ and $A = 1{\cdot}1 \times 10^{-12}$. In what follows we shall assume that α lies in the range 2·5–3·5.

In Fig. 4 we show the number of SNRs whose ages are less than 2000 years, as a function of α. The curves are labelled by the calibrator used. The flatness of the curve labelled SNR 185 is an artifact because SNR 185 itself is $\sim$ 2000 years old. The dashed curve is a ' least-squares fit ' using all the four calibrators. For $\alpha = 3{\cdot}5$, this gives for the mean interval τ between SN that produce shell remnants, a value of 19 yr.

A similar procedure, but leaving out SNR 1006 this time, gives $\tau = 25$ yr (for the appropriate value of $\alpha = 2{\cdot}5$). Since the true value of α can lie anywhere in the range 2·5–3·5, we obtain $\tau = 22 \pm 3$ yr. In this context it is worth mentioning that the *measured* value of α for Tycho is $2{\cdot}0 \pm 0{\cdot}8$ (Strom, Goss and Shaver 1982), consistent with our range for α. There is one other remnant, *viz.* Cas A, for which α has been measured. In this case $\alpha \sim 3$ (Stankevich 1979).

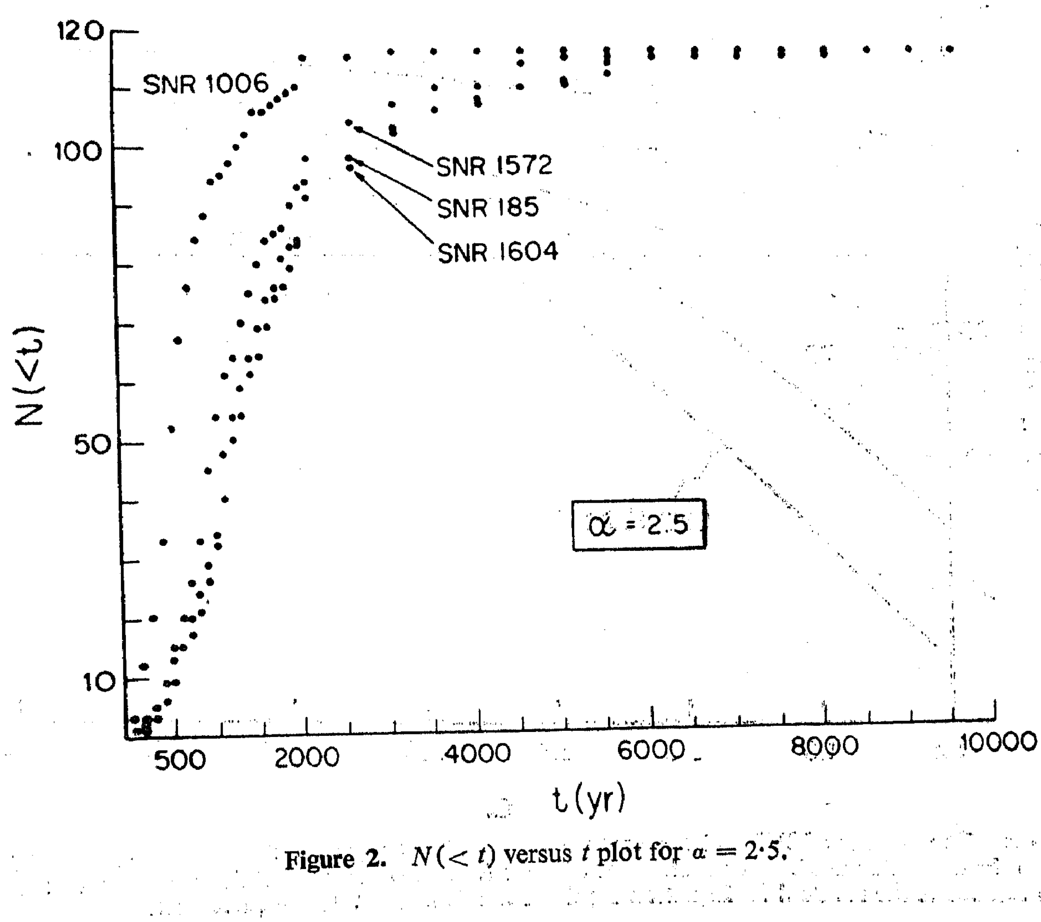

Figure 2. $N(<t)$ versus t plot for $\alpha = 2{\cdot}5$.

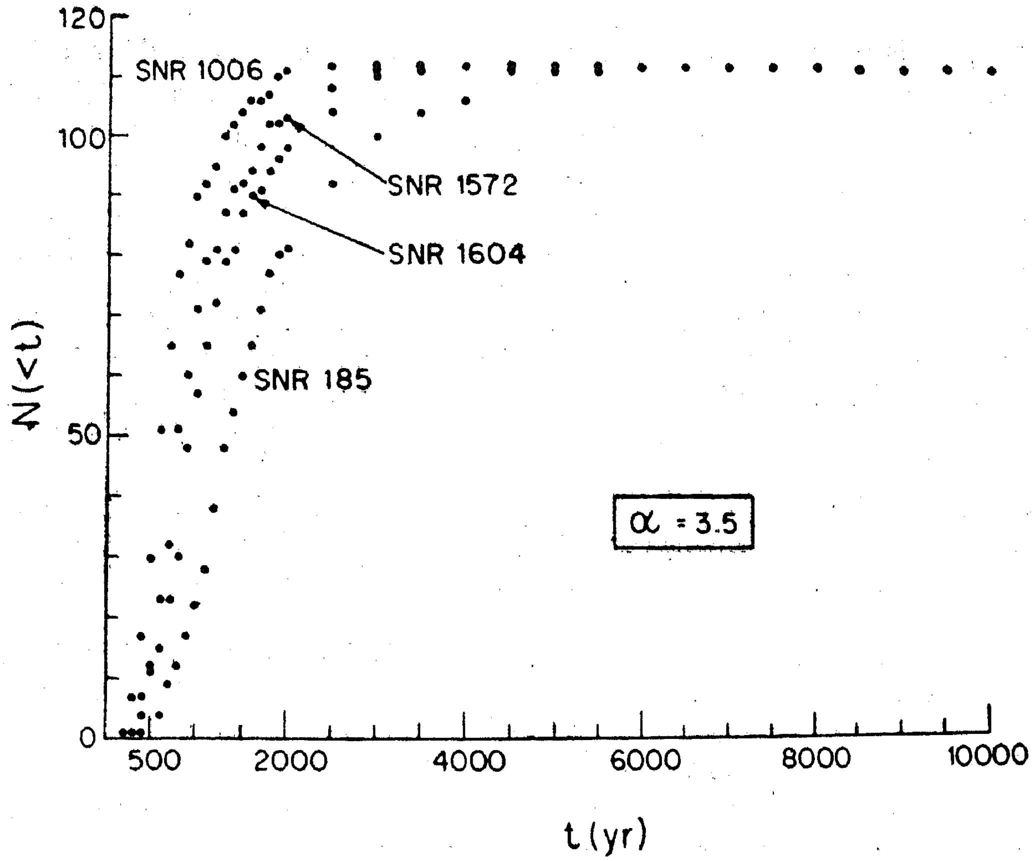

Figure 3. $N(<t)$ versus t plot for $\alpha = 3{\cdot}5$.

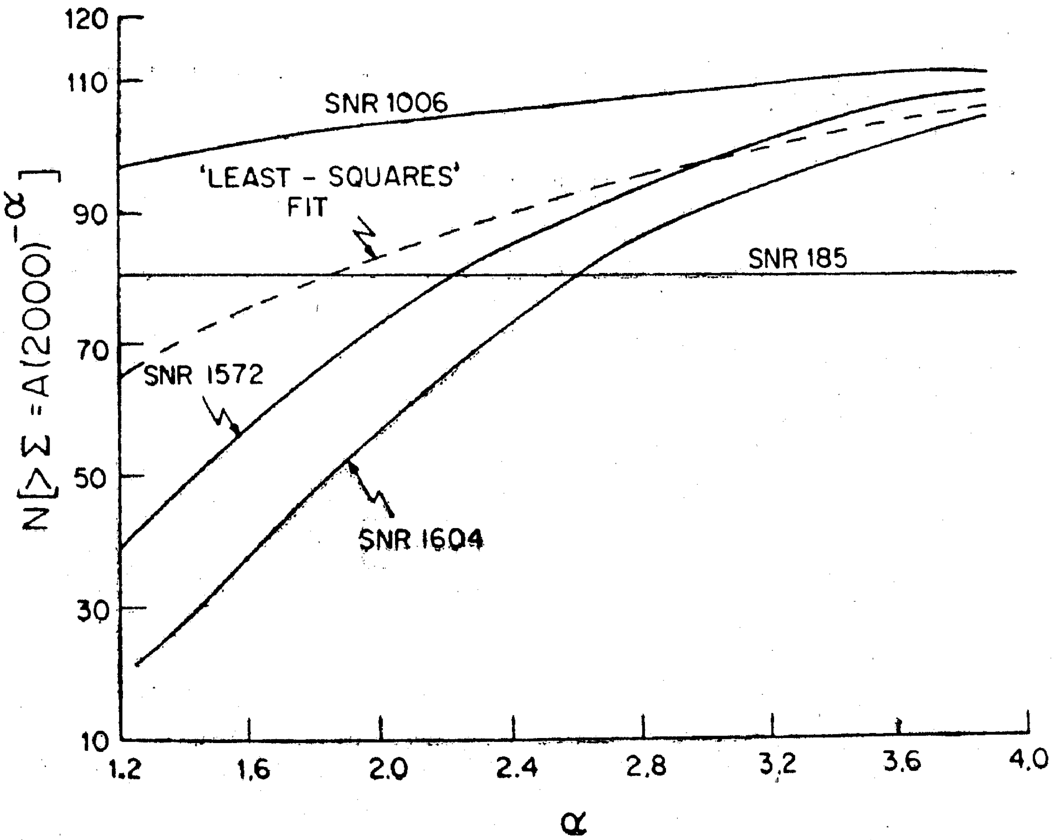

Figure 4. The number of SNRs whose ages are less than 2000 years as a function of α. The labels refer to the calibrator used. For each value of α we have also determined this number with an A which minimises the residual R (Equation 3); this curve is labelled 'least-squares' fit.

At this stage it is perhaps worth emphasizing the salient differences between the method outlined above and that adopted by Higdon and Lingenfelter (1980). The starting point is the secular decrease in the surface brightness (Equations 1 and 2). However, we determine both A and α by 'fitting' the historical remnants; Higdon and Lingenfelter *assume* $\alpha = 2{\cdot}3$, which they say is given by Gull's theory (though we cannot find such an explicit statement in Gull 1973). They then normalise Equation (1) by assuming that Kepler is at its maximum brightness and that Kepler, Tycho, SNR 1006 and SNR 185 are all expanding in regions of similar density.

3. Dependence of the surface brightness on the density of the interstellar medium

The method outlined above assumes that all remnants brighter than a given one are younger. This need not be true. For example, the remnant SNR 1006 may be very subluminous since its surface brightness is somewhat smaller than that of Vela XYZ. If the Vela pulsar is associated with the latter remnant its age would be $\sim 10^4$ yr (Clark and Caswell 1976). Thus if one uses SNR 1006 as an age calibrator one would grossly underestimate the ages of other remnants. This is precisely what we found in Figs 1–3. One possible reason why SNR 1006 is subluminous is that it is at a height of 325 pc above the galactic plane, where the hydrogen density is expected to be smaller than in the plane. For a given initial energy of the explosion (E_0), a remnant expanding in a region of low ambient density will attain a larger diameter in a given time than one expanding in a higher-density medium and will, therefore, have a lower surface brightness. In addition to this, two remnants of the same diameter may have different Σs if the density of the medium determines the radio flux in some way. Combining these two effects Caswell and Lerche (1979) have argued that the Σs of two SNRs of the same age but at different z (distance from the galactic plane) may be related as follows:

$$\Sigma(z, t) = \Sigma(z = 0, t) \exp(-|z|/z_\Sigma) \tag{4}$$

where z_Σ is the 'scale height for the surface brightness'. They have suggested that $z_\Sigma \sim 110$ pc. If one pursued similar arguments put forward by Milne (1979), one would get $z_\Sigma = 50$ pc. It must be remembered that Equation (4) merely takes into account the variation of density as a function of z and does not take into account local fluctuations of density in the interstellar medium (ISM). We shall come back to this point later.

Let us now return to the SNR 1006 and its anomalously low surface brightness. If one ignores the fluctuations in density in the plane, one can use Equation (4) to normalize the Σs of all the remnants and thus convert a catalogue complete down to a limiting surface brightness (Clark and Caswell 1976) to one complete to a limiting age (Caswell and Lerche 1979; Milne 1979). One can then repeat the procedure adopted in Section 2 and obtain a new birthrate for SNRs. The results of the calculations using $z_\Sigma = 100$ pc are shown in Fig. 5; the labels have the same meaning as in Fig 4. We have not included the results obtained with Kepler as a calibrator; the reasons for this will be discussed below. Again, it will be seen that the mean birthrate is fairly insensitive to α. Using the same criteria mentioned in the previous

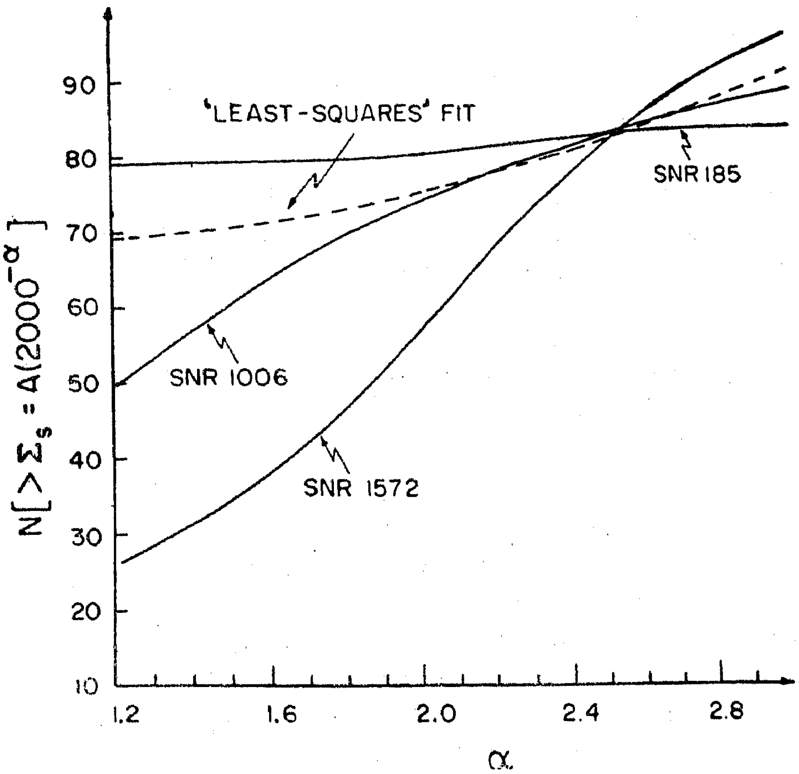

Figure 5. The curves have the same meaning as in Fig. 4, but the surface brightness of all the remnants were scaled up using Equation (4) and $z_\Sigma = 100$ pc; this is denoted by Σ_s.

section, we would like to suggest that for $z_\Sigma = 100$ pc, the evolution of the surface brightness is best described by $\alpha = 2{\cdot}5 \pm 0{\cdot}1$ yielding $\tau = 25 \pm 1$ yr. For $z_\Sigma = 150$ pc, we find $\tau = 22 \pm 3$ yr for the same range of α.

An examination of Figs 4 and 5 reveals that the birthrate obtained through SNR 185 and SNR 1572 are not affected much by the z_Σ used to normalize the Σs (Fig. 4 corresponds to $z_\Sigma = \infty$, *i.e.* no scaling). This is because most of the SNRs are at fairly small z ($z_{185} = 100$ pc, $z_{1572} = 150$ pc *etc.*); and hence their Σs are not scaled up very much (Clark and Caswell 1976, give a scale height of 60 pc for the distribution of SNRs).

SNR 1604 (Kepler) poses a problem, however. There are no reliable measurements of the distance to Kepler and therefore its height above the galactic plane is uncertain. Assuming that it was a typical Type I SN one can place it at a distance of between 6·5 to 10 kpc (Clark and Stephenson 1977b); this would correspond to a z-distance of 770 to 1180 pc above the plane. Yet, its diameter (9 pc at $d = 10$ kpc) and surface brightness (twice that of Tycho) suggest that it is expanding into a region of density similar to that in the plane. Clark and Stephenson (1977b) have suggested that if one adopted a distance of 10 kpc, it would place the remnant in the nuclear bulge of the galaxy; perhaps the scale height of the gas in the bulge is much higher than in the disk. At any rate, it would be meaningless to scale up the surface brightness of Kepler with a $z_\Sigma \sim 100$ pc; it would make it more than 100 times brighter than Tycho! Recently, Danziger and Goss (1980) have suggested a distance of 3·2 kpc to Kepler. This would place the remnant at 380 pc above the plane, but would make its linear diameter a mere 3 pc. Given its age, such a small diameter would imply a very low value for $E_0 / n \sim 10^{49}$ erg cm^3, where E_0 is the energy of the explosion and n the density of the medium. It is because of these gross uncertainties that we have not used Kepler as a calibrator in the analysis given in this section.

As was mentioned above, we have so far ignored possible variations in the density of the ISM at any given z. This may be at least as important as the variation in density with z. According to the emerging picture of the large-scale structure of the ISM there are two kinds of intercloud medium; one with $n_{\mathrm{W}} \sim 0{\cdot}2$-$0{\cdot}3$ cm^{-3}, $T \sim 8000$ K and the other with $n_{\mathrm{H}} \sim 10^{-2{\cdot}5}$ cm^{-3}, $T \sim 10^6$ K (McKee and Ostriker 1977; Radhakrishnan and Srinivasan 1980b). The precise filling factors of these two components of the ISM are not known, but it is clear that if the above picture is true then some of the remnants must be expanding in the tenuous medium and the rest in the dense medium.

According to Gull (1973) the maximum surface brightness and the time t_0 when this is reached will scale as $\Sigma_m \propto n^{1{\cdot}42}$, $t_0 \propto n^{-1/3}$. These scaling laws when combined with Equation (1) tell us how the ages of two remnants, with a given Σ, scale with the density of the ISM into which they are expanding. One can formally write

$$N(>\Sigma) = \frac{t_{\mathrm{H}}}{\tau} f_{\mathrm{H}} + \frac{t_{\mathrm{W}}}{\tau} f_{\mathrm{W}} \tag{5}$$

where f_{H} and f_{W} are the filling factors of the hot and warm medium respectively. If all the calibrators were expanding in the same component of the ISM, and if we knew the densities and filling factors of both the components, one could in principle use Equation (5) to correct the birthrate obtained in Section 2. Since these are not known, we shall not attempt to do this in this paper. Higdon and Lingenfelter (1980) have assumed that Kepler, Tycho and SNR 185 are expanding in the hot, low-density medium and deduced an effective birthrate for SNRs by treating the filling factor as a parameter. However, recent observation of the expansion of Tycho suggests that it was decelerated significantly implying that it may not be expanding in the coronal gas (Strom, Goss and Shaver 1982).

4. Discussion and summary

There has been a long-standing discrepancy between estimates of the SN rate in our galaxy and the SNR birthrate which ranged from one in 50 yr to one in 150 yr (Ilovaisky and Lequeux 1972; Clark and Caswell 1976; Caswell and Lerche 1979). Clark and Stephenson (1977a) suggested that the above discrepancy could be reconciled if only ~ 20 per cent of SNRs are ‘long lived’. This is acceptable only if SNRs do not turn on till they are ‘middle aged’. Bright young remnants such as Cas A, Kepler and Tycho militate against this. Alternatively, one can say that not all supernovae leave behind radio remnants. This possibility has to be rejected because all the recorded SN in historical times have produced a remnant of one kind or another. It has also been suggested (Lozinskaya 1979; Tomisaka, Habe and Ikeuchi 1980) that most of interstellar space is filled with a hot, very low-density gas (as suggested by McKee and Ostriker 1977) and that remnants expanding in this gas will not be detected. The difficulty with this argument is that it would require all the historical SN to have occurred in the dense medium with a very low filling factor. All these force one to the conclusion that the SNR birthrate must match the SN rate.

This was our motivation to have a fresh look at the SNR birthrate using the secular decrease in the surface brightness of the historical remnants as the starting point. The distinct advantage of this method over many of the previous ones is that it uses only the surface brightness which is independent of the distance to the source. Age calculations in the 'standard model' explicitly or implicitly involve the linear diameter, to determine which a distance estimate is needed. This is possible only for a fraction of SNRs. The disadvantage of our method is that there are very few historical remnants and one has to assume they are typical of the whole sample.

We now summarise our important conclusions and their implications:

1. The secular decrease of the surface brightness of the calibrators used is best described by $\Sigma \propto t^{-\alpha}$, where α is between 2·5 and 3·5. This is consistent with the measured slope of $2{\cdot}0 \pm 0{\cdot}8$ for Tycho and $\sim$3 for Cas A.

2. This rate of decrease, if taken as typical of all SNRs, implies a birthrate for shell remnants of 1 in 22 ± 3 yr. This is in reasonable agreement with the supernova rate deduced from historical observations.

3. A majority of SNRs are relatively young with ages of a few thousand years. This is to be contrasted with the typical SNR lifetime of $\gtrsim 10^4$ yr as derived from the 'standard' model.*

4. Since only $\sim$6 out of 120 SNRs resemble the Crab nebula and the mean lifetime of a shell remnant as derived by us is comparable to the expected lifetime of the Crab nebula (Weiler 1978; Weiler and Panagia 1980) it becomes unimportant to correct the birthrate of shell SNRs to obtain the total birthrate of all SNRs.

Acknowledgements

We wish to thank R. Nityananda for suggesting the method outlined in Section 2 and for many illuminating comments. Critical comments on an earlier version of the manuscript by V. Radhakrishnan, Ramesh Narayan, W. M. Goss, C. J. Salter, P. A. Shaver, K. W. Weiler and two anonymous referees are also gratefully acknowledged.

References

Ashworth, W. B. 1980, *J. Hist. Astr.*, **11**, 1.
Baade, W., Zwicky, F. 1934, *Phys. Rev.*, **45**, 138.
Brecher, K., Wasserman, I. 1980, *Astrophys. J.*, **240**, L 105.
Caswell, J. L. 1970, *Astr. Astrophys.*, **7**, 59.
Caswell, J. L., Lerche, I. 1979, *Mon. Not. R. astr. Soc.*, **187**, 201.
Clark, D. H., Caswell, J. L. 1976, *Mon. Not. R. astr. Soc.*, **174**, 267.
Clark, D. H., Stephenson, F. R. 1977a, *Mon. Not. R. astr. Soc.*, **179**, 87p.
Clark, D. H., Stephenson, F. R. 1977b, *The Historical Supernovae*, Pergamon Press, Oxford.
Danziger, I. J., Goss, W. M. 1980, *Mon. Not. R. astr. Soc.*, **190**, 47p.

*The recent discovery of a pulsar with a characteristic age of 1677 yr, in the shell remnant MSH 15–52, clearly shows that the SNR cannot be older than $\sim$ 1600 yr. The age calculated from the 'standard model' is, however, $\sim 10^4$ yr. This lends support to our viewpoint that the standard ages are often gross overestimates. Our age for MSH 15–52 is $\sim$ 1200 yr, consistent with the maximum age of 1677 yr as implied by the pulsar (Srinivasan, Dwarakanath and Radhakrishnan 1982).

Gull, S. F. 1973, *Mon. Not. R. astr. Soc.*, **161**, 47.
Higdon, J. C., Lingenfelter, R. E. 1980, *Astrophys. J.*, **239**, 867.
Ilovaisky, S. A., Lequeux, J. 1972, *Astr. Astrophys.*, **20**, 347.
Katgert, P., Oort, J. H. 1967, *Bull. astr. Inst. Netherl.*, **19**, 239.
Lozinskaya, T. A. 1979, *Soviet Astr.*, **23**, 506.
McKee, C. F., Ostriker, J. P. 1977, *Astrophys. J.*, **218**, 148.
Milne, D. K. 1979, *Austr. J. Phys.*, **32**, 83.
Phinney, E. S., Blandford, R. D. 1981, *Mon. Not. R. astr. Soc.*, **194**, 137.
Radhakrishnan, V., Srinivasan, G. 1980a, *J. Astrophys. Astr.*, **1**, 25.
Radhakrishnan, V., Srinivasan, G. 1980b, *J. Astrophys. Astr.*, **1**, 47.
Srinivasan, G., Dwarakanath, K. S., Radhakrishnan, V. 1982, *Curr. Sci.*, **51**, 596.
Stankevich, K. S. 1979, *Austr. J. Phys.*, **32**, 95.
Strom, R. G., Goss, W. M., Shaver, P. A. 1982, *Mon. Not. R. astr. Soc.*, **200**, 473.
Tammann, G. A. 1977, *Ann. N. Y. Acad. Sci.*, **302**, 61.
Taylor, J. H., Manchester, R. N. 1977, *Astrophys. J.*, **215**, 885.
Tomisaka, K., Habe, A., Ikeuchi, S. 1980, *Progress theor. Phys.*, **64**, 1587.
van der Laan, H. 1962, *Mon. Not. R. astr. Soc.*, **124**, 125.
Vivekanand, M., Narayan, R. 1981, *J. Astrophys. Astr.*, **2**, 315.
Weiler, K. W. 1978, *Mem. Soc. astr. Ital.*, **49**, 545.
Weiler, K. W., Panagia, N. 1980, *Astr. Astrophys.*, **90**, 269.