Chapter 14

RADIATION REACTION IN ELECTRODYNAMICS AND GENERAL RELATIVITY

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1. PROLOGUE

It is a privilege and pleasure to be invited to contribute an article to the JVN Fest. When I received this invitation, I tried to go back along my world-line and look for intersections with Jayant. A popular article by Jayant Narlikar entitled 'The Arrow of Time' [1] mystified and fascinated me. It roused an almost romantic longing and an urge to appreciate, if not investigate, such basic problems. Probably it was these subconscious fantasies that propelled me towards physics and eventually, general relativity. I still remember the first time I heard a public talk by Narlikar on Cosmology after his return to India. It was at the Homi Bhabha auditorium of TIFR in 1972. The hall was overflowing and I heard his (favorite?) joke on the mathematician, physicist and astronomer for the first time. I heard it again this year in his talk at the Academy and was impressed by his un-apologetic use of it to make his point! I met Jayant Narlikar at the Einstein centenary symposium in Ahmedabad in 1979 and his interests then included scale invariant cosmology (with Ajit Kembhavi) and black holes as tachyon detectors (with Sanjeev Dhurandhar). He carried his fame lightly, was unassuming and though he was not very talkative, he felt very approachable. When I finished my Ph.D. with Arvind Kumar at the Bombay University, I could not get a post doc at TIFR or work with Jayant, since he was away that particular year. Over the last sixteen years, I have had much overlap with Jayant in the organization of Relativity related activities in India. There is much to admire in Jayant and emulate. His time management, missionary zeal to the popularization of science, vision and hard work, pedagogic skills, fervor for the non-standard and ability to play devil's

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advocate in his research almost as a point of faith. In addition to the above, personally, I also admire him for his ability to take criticism and his democratic mode of functioning.

I am always impressed by Jayant's ability to start a lecture on fairly profound, subtle and technical themes like action at a distance in physics and cosmology or Mach's principle from a very elementary basic discussion. In every lecture of his that I have heard he covers a fair amount of ground starting from the very beginning and leading to what he is currently researching on. He reminds me of a capable, composed and competent guide taking a group of motley tourists up a mountain, leading everyone to the heights their capability can reach. Everyone gets a view, maybe a different glimpse, but everyone is happy to have participated in the trek and adventure that Jayant leads them on. No wonder he is a populariser par excellence and probably holds a record for such lectures and writing at least in India.

I have heard that Jayant has a soft corner for his work related to electrodynamics and action-at-a-distance [2]; he considers it to be one of the important topics in his research career. Recent progress in theoretical gravitational radiation research is very reminiscent of this research and as a tribute to Jayant, I shall try to imitate him and without getting lost in technical details compare these developments in general relativity to those in electrodynamics.

2. GRAVITATION AND ELECTROMAGNETISM

The similarity of gravitation and electromagnetism does not escape any thoughtful student of an elementary physics course [3]. Both Newton's law of gravitation and Coulomb's law of electrostatics are inverse square laws. They are proportional to their respective charges: gravitational mass and electric charge. The gravitational charge is of only one kind, while there are two kinds of electric charges, conventionally denoted as positive and negative. In electrostatics, like charges repel, while unlike charges attract. Gravitation on the other hand is always attractive and in gravitation, like charges attract! Though functionally similar, the numerical strengths of these forces is very different. The gravitational force is about 10^{39} times weaker than the electrical force and this has experimental implications, as we shall see later. Unlike electromagnetic forces, gravitation cannot be screened out. Moreover, matter in the universe is predominantly neutral. This is why, in spite of its enormous weakness, gravitation determines the large scale structure of the universe.

Both Newton's law of gravitation and Coulomb's law of electrostatics assume instantaneous action-at-a-distance. Thus they cannot be consistent with the principle of special relativity. Coulomb's law is not adequate to describe moving charges. Electromagnetic phenomena are more simply described by field equations and a moving charge produces both an electric field and a magnetic field. The laws of electromagnetism are summarized by Maxwell's equations and Lorentz equation of motion. These equations are relativistically invariant. However, in Newtonian gravitation, there is no analogue of the magnetic field; a moving mass produces the same field, as a mass at rest, if the mass distributions are identical. The situation is different in Einstein's general theory of relativity and closer to electromagnetism. Here the gravitational field produced by a body depends not only on the distribution of matter but also the state of its motion. Mathematically, the source of the gravitational field is the energy momentum tensor whose components include mass, motion and stresses. The gravitational analogue of the magnetic force is called gravimagnetism and like the Lorentz force in electrodynamics, depends on the test particle velocity. It has physical consequences like the dragging of inertial frames, Lense Thirring effect or precession of gyroscopes. Usual tests of general relativity normally involve only the gravielectric component. Like the magnetic force, the gravimagnetic component is usually smaller by a factor of v/c relative to the gravielectric part and experiments are under way to verify it directly. One can set up a detailed analogy between rotation in general relativity and magnetism. In electromagnetism, there has long been a conjecture about the possible existence of magnetic monopoles. Given the detailed similarity between rotation and magnetic fields, one can ask, if there is such a thing as the gravimagnetic monopole. The answer is in the affirmative. The famous NUT solution is the gravimagnetic monopole [4]. Of course, the Schwarzschild solution the gravielectric monopole.

3. ELECTROMAGNETIC WAVES AND GRAVITATIONAL WAVES

As mentioned earlier, the laws of electromagnetism are summarized by Maxwell's equations. Maxwell's equations admit wave like solutions and these are electromagnetic (EM) waves. EM waves are produced by accelerated electric charges. The dominant radiation is dipole radiation and is caused by the time varying dipole moment of the charge distribution. The EM field is of spin one (a vector field) and has a conserved quantity associated with it: charge. Consequently there is no monopole EM radiation. EM waves propagate at speed of light c, they are transverse and have two independent states of linear polarization corresponding to oscillations of the electric field in two perpendicular directions. The effect of an EM wave can be seen by its action on a test particle. If a sinusoidally varying EM wave is incident on a test particle, it impresses on it this sinusoidal motion. Thus, by studying the motion of a test particle, we can infer the passage of a EM wave. EM is a strong force. Consequently by the oscillation of charges and currents we can produce EM waves at one end of the laboratory and detect it at the other end: the famous Hertz experiment.

Similarly, the best description of gravitation is via Einstein's equations. These equations also admit wave like solutions. Gravitational waves are not mere artefacts of our choice of coordinates, but indeed physical, in that they carry energy. For a fascinating historical account of these debates, see Kennefick [5]. Gravitational waves are produced by accelerated motions of masses. The dominant radiation is quadrupolar and caused by the second time variation of the quadrupole moment of the mass energy distribution. The gravitational field is of spin two (a tensor field) and has conserved quantities associated with it corresponding to mass, linear momentum and angular momentum. Consequently, there is no monopole or dipole radiation. Gravitational waves also propagate with speed c, are transverse and have two independent states of linear polarization. The effect of a gravitational wave *cannot* be seen by its action on a single test particle. Gravity obeys the equivalence principle and consequently a uniform gravitational field can be transformed away by going to an accelerated frame. Tidal fields cannot be so transformed and provide a true measure of gravitational fields. Gravitational waves induce a weak time-dependent tidal field and thus, a gravitational wave can be detected by letting it impinge on a circular ring of *particles.* Due to the tidal field, the ring is squeezed in one direction and elongated along the perpendicular direction. Since the tidal field oscillates in time, the ring will go through a pattern of shapes, characteristic of the tidal field. Starting out as a ring of particles, after a quarter of a period the ring elongates into a ellipse, say along the x axis, back to a circle, then an ellipse elongated along y axis and back again to a circular shape. This pattern repeats thereafter and is characteristic of spin two. This is referred to as plus polarization. The other independent mode of polarization yields an ellipse rotated by 45^0 and is referred to as the cross polarisation. Gravitational wave detectors differ in the way they measure this minute tidal effect. Broadly we can classify them as bars (spheres), interferometers on earth and interferometers in space.

Unlike EM, gravitation is a very weak force. Consequently, the oscillation of masses in the laboratory cannot produce gravitational waves of measurable strength. The detection by any suitable method is equally difficult for the same reason. A Hertz type experiment is not possible in this case and one is forced to appeal to astronomy, to provide sources that will radiate in this bandwidth.

4. INSPIRALING COMPACT BINARIES AND GW PHASING

The Binary pulsars 1913+16 and 1534+12 establish the reality of gravitational radiation [6]. They provide proof of the validity of Einstein's general relativity in the strong field regime. More importantly, they are prototypes of inspiralling compact binaries, which are strong sources of gravitational waves for ground based laser interferometric detectors like LIGO and VIRGO [7]. The phenomenal success of the high-precision radio wave observation of the binary pulsar makes crucial use of an accurate relativistic 'Pulsar timing formula' [8, 9]. Similarly, precise gravitational-wave observation of inspiraling compact binaries would require an equivalent accurate 'Phasing formula' [7, 10] i.e. an accurate mathematical model of the continuous evolution of the gravitational wave phase. The lowest order gravitational wave radiation reaction is sufficient to treat pulsar timing. Gravitational wave phasing, on the other hand, requires higher post-Newtonian order gravitational radiation reaction, since in the final stages the systems are highly relativistic.

At this point, it is worth comparing the situation here in general relativity (GR) to that in electrodynamics (ED) to illustrate the issues. For instance, in ED we have the following categories of problems: (a) Given the charge and current distribution, compute the electromagnetic field; e.g. evaluate fields in wave-guides. (b) Given the external electromagnetic field, compute the effect on charges and currents; e.g. energy losses of charged particles moving past a nucleus. (c) Given the energy loss by say the Larmor formula, compute the reaction on the motion; e.g. Abraham-Lorentz, Planck. The corresponding situation in GR, in the inspiraling binary problem, is the following: (i) *Generation Problem:* Given the compact binary and its orbital motion, compute the gravitational field in this situation. (ii) Given the gravitational field, compute the far-zone energy and angular momentum fluxes. (iii) *Radiation Reaction problem:* Given the far zone fluxes of energy and angular momentum, compute the reaction on the near zone motion, assuming energy (angular momentum) balance. Or compute it directly, by a higher iteration of the equations of motion.

In what follows, we will discuss briefly aspects of motion, generation and radiation reaction and draw parallels to the EM case, where possible.

5. MOTION

It may be worth mentioning that unlike linear EM, non-linear GR has the feature, that its field equations contain the equations of motion. For discussions on the relation between the above feature, non-linearity and tensor nature of the field, see the review article by Havas [11]. The N-body problem as in Newtonian gravity is decomposed into an external problem and an internal problem. The former refers to the problem of defining and determining the motion of the center of mass and the latter to motion of each body around the center of mass. The effacement of internal structure in the external problem and effacement of external structure on the internal problem involves subtle issues in the problem of motion and we cannot do better than refer the reader to the beautiful review by Damour [12].

The topic of EOM for compact binary systems received careful scrutiny in the years following the discovery of the binary pulsar. There have been three different approaches to the complete kinematical description of a two body system upto the level where radiation damping first occurs (2.5PN). Damour's method explicitly discusses the external motion of two condensed bodies without ambiguities, using harmonic coordinates, in which all metric deviation components satisfy hyperbolic (wave) equations. The method employs the best techniques to treat various subproblems. (a) A Post-Minkowskian approximation to obtain the gravitational field outside the bodies incorporating a natural 'no incoming-radiation condition' whose validity is not restricted to only the near-zone. (b) A matched asymptotic expansion scheme to prove effacement and uniquely determine the gravitational field exterior to the condensed bodies. (c) An Einstein Infeld Hoffmann Kerr type approach to compute equations of orbital motion from knowledge of the external field only. The n^{th} approximate EOM is obtained from the integrability condition on the $(n+1)^{th}$ approximated vacuum field equations. (d) Use of Riesz's analytic continuation technique to evaluate surface integrals. The final EOM at 2.5PN level are expressed only in terms of instantaneous positions, velocities and spins in a given harmonic coordinate system and given explicitly in Ref.[12]. The two mass parameters in these formulas are the Schwarzschild masses of the two condensed bodies.

The conservative part of the EOM upto 2PN (excluding the secular 2.5PN terms) are not deducible from an conventional Lagrangian (function of positions and velocities) in harmonic coordinates, but only from a generalized Lagrangian (depending on accelerations). This is consistent with the result in classical field theory that in Lorentz-covariant field theories there exists no (ordinary) Lagrangian description at $O(c^{-4})$ [13]. This Lagrangian is invariant under the Poincare group and thus allows one to construct ten Noetherian quantities that would be conserved during the motion. These include the 'Energy', 'Angular Momentum', 'Center of Mass' and thus a solution to the problem of 'motion' provides the Energy that enters into the phasing formula. The EOM for the general case is given in [12] and crucially used in the following studies of generation and radiation reaction. All the above has detailed parallels in the electromagnetic case and the relevant Lagrangian and associated subtleties are discussed in the Les Houches lecture by Damour [9].

Schafer's [14] approach, on the other hand, is based on the Hamiltonian approach to the interaction of spinless point particles with the gravitational wave field. The Hamiltonian formulation is best done in the Arnowitt-Deser-Misner (ADM) coordinates, in which two metric coefficients satisfy hyperbolic equations (evolution) while the remaining eight are of elliptic type (constraints). It uses a different gauge that allows an elegant separation of conservative and damping effects. One recovers the damping force acting on the Hamiltonian subsystem of instantaneously interacting particles coming from its interaction with the dynamical degrees of freedom of the gravitational field. In this approach, point masses are used as sources and regularisation uses Hadamard's 'partie finie' based on Laurent's series expansion regularisation.

The last approach due to Grischuk and Kopejkin [15] on the other hand is based on (a) Post-Newtonian approximation scheme (b) assumption that bodies are non-rotating 'spherically-symmetric' fluid balls. The symmetry is in the coordinate sense. The EOM of the center of mass of each body are obtained by integration of the local PN EOM. These are explicitly calculated retaining all higher derivatives that appear. One then reduces the higher derivatives by EOM and obtains the final results. Formally collecting the various relativistic corrections into a 'effective mass', one can have a PN proof of effacement of internal structure and provide a plausibility argument for validity of 'weak field formulas' for compact objects.

The fact that three independent methods give formally identical equations of motion at 2PN order is a strong confirmation of the validity of the numerical coefficients in the EOM. This work provides the basis for the timing formula mentioned earlier. The damping terms can be considered as perturbation to a Lagrangian system which is multi-periodic – a radial period and a angular period corresponding to periastron precession – and leads to the observed secular acceleration effect in the binary pulsar. No balance argument is involved at any stage.

The situation is now under investigation at the 3PN level. The work on 3PN generation crucially requires the EOM at 3PN accuracy and work is in progress to obtain the 3PN contributions by different techniques. These include the MPM method supplemented by Hadamard 'partie-finie' [16], the Epstein Wagoner Will Wiseman method [17] as also the Hamiltonian formalism [18]. As mentioned above, upto 2.5PN, three distinct computational techniques led to a unique EOM. Preliminary investigations have even raised questions about whether this sort of uniqueness will persist at 3PN.

It is interesting to note that both the Riesz regularisation and the Hadamard finite part averaged over all directions of approach to the singularity are techniques employed in the discussions of EOM in EM [19]. Both continuous source distributions and point sources (delta functions) have also been used in these computations. However, the situation in EM is much better than in gravitation because all the divergent terms can be renormalized into the mass after regularization. In gravitation, these offensive terms have a more complicated structure and we do not renormalize and simply throw away these divergent terms. The procedure in EM is also different since it is Lorentz invariant. In gravitation on the other hand we work in a particular frame and *hope* that in the end the EOM is nevertheless Lorentz invariant. Of course, if they are, it is a very powerful check that all is well with the computation [20]!

6. GENERATION

There are two approaches to calculate gravitational wave generation to higher orders, philosophically following the approaches of Fock and Landau-Lifshitz; the Blanchet-Damour-Iyer (BDI) [21] approach and the Epstein-Wagoner-Thorne-Will-Wiseman (EWTWW) [22, 23] approach respectively. Blanchet, Damour and Iyer build on a Fock type derivation using the double-expansion method of Bonnor. This approach makes a clean separation of the near-zone and the wave zone effects. It is mathematically well defined, algorithmic and provides corrections to the quadrupolar formalism in the form of compact support integrals or more generally well defined analytically continued integrals. The BDI scheme has a modular structure: the final results are obtained by combining an 'external zone module' with a 'radiative zone module' and a 'near zone module'. For dealing with strongly self-gravitating material sources like neutron stars or black holes one needs to use a 'compact body module' together with an 'equation of motion module'. It correctly takes into account all the nonlinear effects. It should be noted that, in generation problems, as one goes to higher orders of approximation, two independent complications arise. Though algebraically involved in principle, the first is simpler: contributions from higher multipoles. The second complication is not only algebraically tedious but technically more involved: contributions from higher nonlinearities e.g for 2PN generation cubic nonlinearities need to be handled.

The Epstein and Wagoner (EW) approach, also starts by rewriting the Einstein equations in a "relaxed" form. As in electromagnetism, one can write down a *single* formal solution valid everywhere in spacetime based on the flat-spacetime retarded Green function. The retarded integral equation for $h^{\alpha\beta}$, can then be iterated in a slow-motion ($v/c < 1$), weak-field ($||h^{\alpha\beta}|| < 1$) approximation as shown by Thorne [22]. Unlike in the electromagnetic case, however, the non-linear field contributions make the integrand of this retarded integral non-compact. The EW formalism leads to integrals that are not well defined, or worse, are divergent. Though at the first few PN orders, different arguments were given to ignore these issues, they provide no justification that the divergences do not become fatal at higher orders. Consequently, the EW formalism did not appear to be a reliable route to discuss higher PN approximations. Recently, Will and Wiseman have critically examined the EW formalism and provided a solution to the problem of its divergences by taking literally the statement that the solution is a *retarded* integral, *i.e.* an integral over the *entire* past null cone of the field point. The new EW method proposed by Will and Wiseman can be carried to higher orders in a straightforward, albeit very tedious manner and the result is a manifestly finite, well-defined procedure for calculating gravitational radiation to high PN orders.

The end result of the computations are expressions for the radiative mass and current multipole moments characterizing the source distribution. Once they are on hand, one can proceed to compute the associated gravitational waveform. From the waveform, the far zone energy flux may be computed by time differentiation (this is why one needs the EOM) and integration over all directions. The energy flux can also be computed directly from the moments and this provides a simple check on the algebraic correctness of the long computations. The angular momentum flux can also be computed for non-circular orbits. At the 2PN level this program is complete not only for circular, but also general orbits [24]. The extension to spinning bodies is also available [25]. The extension of these results to 3PN accuracy is an algebraically heavy and conceptually involved exercise, under investigation since 1996, using the multipolar post-Minkowskian approach [26]. The Hadamard regularization, based on the Hadamard partie finie, used in the computation of motion is also used in generation and provides consistent results. Though the known test particle limits are recovered, the finite mass correction introduces a plethora of new contributions. Hopefully in the near future the EW and ADM formalisms [17, 18] should provide a check on these results.

The solution to the generation problem thus provides the second input for phasing once we make the assumption of energy balance.

7. RADIATION REACTION IN ELECTRODYNAMICS

The idea of a damping force associated with an interaction that propagates with a finite velocity was first discussed in the context of electromagnetism by Lorentz. He obtained it by a direct calculation of the total force acting on a small extended particle due to its 'self-field'. The answer was incorrect by a numerical factor and the correct result was first obtained by Planck using a 'heuristic' argument based on energy balance which prompted Lorentz to re-examine his calculations and confirm Planck's result, $F^i = \frac{2}{3}\frac{e^2}{c^3}\ddot{v}^i$, where v_i is the velocity of the particle. The relativistic generalization of the radiation reaction by Abraham based on arguments of energy and linear momentum balance preceded by a few years the direct relativistic self-field calculation by Schott and illustrates the utility of this heuristic, albeit less rigorous, approach [9]. The argument based on energy balance proceeds thus: A non-accelerated particle does not radiate and satisfies Newton's (conservative) equation of motion. If it is accelerated, it radiates, loses energy and this implies damping terms in the equation of motion. Equating the work done by the reactive force on the particle in a unit time interval, to the negative of the energy radiated by the accelerated particle in that interval (Larmor's formula) the reactive acceleration is determined and one is led to the Abraham-Lorentz equation of motion for the charged particle. Lorentz's direct method of obtaining radiation damping, on the other hand, is based on the evaluation of the retarded action of each piece of the charge on the other parts. Starting with the momentum conservation law for the electromagnetic fields, one rewrites this as Newton's equation of motion, by decomposing the electromagnetic fields into an 'external field' and a 'self-field'. Expanding the self-field in terms of potentials, solving for them in terms of retarded fields and finally making a retardation expansion, one obtains the required equation of motion, when one goes to the point particle limit. For a historical summary of classical theories of radiation reaction see Erber's account [27].

There have been two broad approaches to radiation reaction later: The **field theory** one originally due to Dirac [28], that considers the *total* field at all points in space to be a fundamental physical quantity and point charges as singularities of the field; **the action-at-a-distance** one originally due to Wheeler and Feynman [29], that considers only forces exerted on the charge by *other* charges as physically meaningful. Each approach strictly goes beyond Maxwell's equations and uses an additional assumption: the conservation law for the EM energy momentum tensor in field theory and the relation between Lorentz force and momentum of the particle in action-at-a-distance theory. Though the plausibility of the physical idea of reducing everything to interaction of particles is the fascinating advantage of action-at-a-distance theories, none of the viewpoints appears preferable to the other from considerations of simplicity. Hoyle and Narlikar [3] have assessed the status of action-at-a-distance theories both in classical and the quantum electrodynamics. As there are no fields, the usual problems of divergences are absent in this treatment. When considered within cosmological models, these theories place stringent requirements on the future and past null cones of the universe. The theories will not work in Friedman cosmologies but do in steady state or quasi-steady state models. Issues related to the use of advanced fields in the Dirac derivation, were clarified later [30] and an approach to radiation reaction without advanced fields was presented by properly taking into account the retarded self-field of the point charge as required by the idea of energy-momentum localization. Since the retarded field diverges on the world line of the particle and the 'limit' depends on the direction of approach, one defines the field at the singularity as the average value over all possible directions [19]. A recent novel approach to radiation reaction is due to Gupta and Padmanabhan [31]. They show that fields of charged particles moving on arbitrary trajectories in an inertial frame can be related in a simple manner to the fields of a uniformly accelerated charged particle in its proper rest frame. Since the latter field is static and easily calculable, the former field is obtained by a coordinate transformation. It also allows them to compute the self force on the charged particle and recover the Dirac result.

8. RADIATION REACTION IN GR

As in electromagnetism, radiation reaction forces arise in gravitation from the use of retarded potentials satisfying time asymmetric boundary conditions like no-incoming boundary condition at past null infinity.

The problem is more complicated because of the nonlinearity of general relativity.

The approach to gravitational radiation damping has been based on the balance methods, the reaction potential or a full iteration of Einstein's equation. The first computation in general relativity was by Einstein who derived the loss in energy of a spinning rod by a far-zone energy flux computation. The same was derived by Eddington by a direct near-zone radiation damping approach. He also pointed out that the physical mechanism causing damping was the effect discussed by Laplace, that if gravity was not propagated instantaneously, reactive forces could result. An useful development was the introduction of the radiation reaction potential by Burke and Thorne [32] using the method of matched asymptotic expansions. In this approach, one derives the equation of motion by constructing an outgoing wave solution of Einstein's equation in some convenient gauge and then matching it to the near-zone solution. Restricting attention only to lowest order Newtonian terms and terms sensitive to the outgoing (in-going) boundary conditions and neglecting all other terms, one obtains the required result. The first complete direct calculation à la Lorentz of the gravitational radiation reaction force was by Chandrasekhar and Esposito. Chandrasekhar and collaborators [33] developed a systematic post-Newtonian expansion for extended perfect fluid systems and put together correctly the necessary elements like the Landau-Lifshitz pseudo-tensor, the retarded potentials and the near-zone expansion. These works established the balance equations to Newtonian order, albeit for weakly self-gravitating fluid systems. The revival of interest in these issues following the discovery of the binary pulsar and the applicability of these very equations to binary systems of compact objects follows from the works of Damour [9] and Damour and Deruelle [8] discussed earlier.

Many other approaches to radiation reaction problems have emerged in the last five years. For instance, given the formulas for the far-zone energy and angular momentum fluxes to a particular PN accuracy, to what extent can one infer the radiation reaction acceleration in the (local) EOM? Given the algebraic complexity of various computations and subtle evaluations of various small coefficients, it is worthwhile to check the obvious consistency requirement on the far-zone fluxes. To this end, Iyer and Will (IW) [34] proposed a refinement of the text-book treatment of the energy balance method used to discuss radiation damping. This generalization uses both energy and angular momentum balance to deduce the radiation reaction force for a binary system made of non-spinning structureless particles moving on general orbits. Starting from the 1PN conserved dynamics of the two-body system, and the radiated energy and angular momentum in the gravitational waves, and taking into account the arbitrariness of the 'balance' upto total time derivatives, they determined the 2.5PN and 3.5PN terms in the equations of motion of the binary system. The part not fixed by the balance equations was identified with the freedom still residing in the choice of the coordinate system at that order. The explicit gauge transformations they correspond to has also been constructed. Blanchet [35], on the other hand, obtained the post-Newtonian corrections to the radiation reaction force from first principles using a combination of post-Minkowskian, multipolar and post-Newtonian schemes together with techniques of analytic continuation and asymptotic matching. By looking at "antisymmetric" waves – a solution of the d'Alembertian equation composed of retarded wave minus advanced wave, regular all over the source, including the origin – and matching, one obtains a radiation reaction tensor potential that generalizes the Burke-Thorne reaction potential, in terms of explicit integrals over matter fields in the source. The *validity* of the balance equations upto 1.5PN is also proved. By specializing this potential to two-body sytems, Iyer and Will [34] checked that this solution indeed corresponds to a unique and consistent choice of coordinate system. This provides a delicate and non-trivial check on the validity of the 1PN reaction potentials and the overall consistency of the direct methods based on iteration of the near-field equations and indirect methods based on energy and angular momentum balance. It should be noted that the 'balance method' by itself cannot fix the particular expression for the reactive force in a given coordinate system. In order to solve a practical problem (in which we erect a particular coordinate system), the method is in principle insufficient by itself, but it provides an extremely powerful check of other methods based on first principles. Gopakumar, Iyer and Iyer [36] have applied the refined balance method to obtain the 2PN radiation reaction – 4.5PN terms in the equation of motion. Different facets of the IW choice like the functional form of the reactive acceleration have been systematically and critically explored and a better understanding of the origin of redundant equations is provided by studying variants obtained by modifying the functional forms of the ambiguities in energy and angular momentum. These reactive solutions are general enough to treat as particular cases any reactive acceleration obtained from first principles in the future.

Within the ADM approach, the radiative 3.5PN terms in the ADM Hamiltonian has been obtained by Jaranowski and Schafer [37]. Work is in progress to check that this leads to expressions for 3.5PN acceleration that is a particular case of the general IW solution. In the test particle case, work on radiation reaction has focussed on understanding the evolution of Carter constant in Kerr geometry by a variety of methods. Issues related to radiative versus retarded fields, adaptation of Dewitt-Brehme and asymptotic matching methods, axiomatic treatments as well as extension to spinning particles have also been investigated in the last three years [38].

9. CONCLUSION

It is amazing that in the macroscopic world, the computations of small higher order corrections so reminiscent of Lamb shift corrections in quantum electrodynamics (microscopic world) are in-expendable to extract the best from the LIGO and VIRGO facilities that will be able to look for gravitational wave signals by 2001. General relativity, far from being an esoteric and abstruse theory driven by aesthetic considerations is in a situation where experiments are driving the theory. We are on the threshold of opening another window to this marvelous universe and gravitational wave astronomy could well be the new astronomy of the 21st century. With the inauguration of the Gravitational Wave Astronomy, more than ever before, General Relativity will have found its true home.

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