NON-STEADY-STATE AERODYNAMICS OF THE FLIGHT OF ENCARSIA FORMOSA

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ABSTRACT

Weis-Fogh (1973) demonstrated that the high lift coefficients necessary for flight in certain insects are incompatible with steady-state aerodynamics and proposed two novel mechanisms, the fling and the flip, whereby the wing movements generate circulation around the wings prior to and independently of translation. This premise is investigated for the flight of the small chalcid wasp Encarsia formosa by re-estimating the minimum steady-state lift coefficient by another method and including some possible refinements. The steady-state lift coefficient obtained in this manner is still too high for the relevant Reynolds number. An average circulation of about 3.8 $cm^2 s^{-1}$ may be created around each wing in the fling and flip, however, and this value is higher than the circulation during translation which satisfies the Kutta-Zhukovski condition. Trailing edge vortices, with the same sense as stopping vortices, should then be shed as the wings translate. A rough calculation shows that the corresponding decrease in bound vorticity during each halfstroke is small enough to allow an average circulation remaining around the wings sufficient for flight.

INTRODUCTION

In his analysis of the flight of the small chalcid wasp Encarsia formosa Gahan, Weis-Fogh (1973) demonstrated that the minimum average coefficient of lift, $\bar{C}_L$, necessary for sustained flight was higher than can be expected from steady-state aerofoil theory. He calculated $\overline{C}_L$ to be 3.2, which is 3 to 4 times that which can be expected at the relevant Reynolds number (approximately 20) based on conventional aerofoil principles. The increased viscous and diffusive effects in this range of Reynolds numbers prevent a circulation of such high strength from being established around the wing. To explain the discrepancy he proposed two novel mechanisms, the fling and the flip, which occur at the end of the upstroke and downstroke respectively, whereby circulation is created around each wing prior to and independently of translation. The circulation thus created is independent of normal aerofoil limitations, and an analysis by Lighthill (1973) indicates that the circulation generated by the fling can reach a sufficiently high value to permit sustained flight.

The periods at the ends of the upstroke and downstroke are aptly described aerodynamically as non-steady-state, and occur in all modes of flapping flight. For insects flying in the range of small Reynolds numbers the importance of these non-steady-state periods may increase as the effectiveness of conventional aerofoils decreases.

This paper attempts to estimate the non-steady-state effects for the flight of _Encarsia_ and compare them to what would be expected from steady-state principles. If circulation can be established around the wing independently of a translational velocity, what happens to this circulation during the wing stroke? It is possible that the circulation generated during non-steady periods satisfies the Kutta-Zhukovski condition for the translating wings, so that no trailing edge vortices are shed and the wing acts as a conventional aerofoil during translation. Perhaps the fact that the wings are shaped like aerofoils is somewhat incidental, and they simply translate the bound vorticity created in non-steady periods to produce lift by the Magnus effect. Finally, a combination of the two effects may occur as in unsteady aerofoil theory, so that the wing continually sheds trailing vortices while obtaining lift from the bound vortex.

THE WINGBEAT OF _ENCARSIA_

The morphology and average dimensions of _Encarsia_ are shown in Figure 1, redrawn from Weis-Fogh (1973). The wing span is approximately 0.15 cm. The brim of marginal hairs should be noted in particular since their significance will be discussed shortly. The mass of the insect is 25 μg. The angular movement of the long axes of the wings with time, $\gamma(t)$, was obtained from high-speed cine films (Figure 2, redrawn from Weis-Fogh (1973)). The upstroke and downstroke differ little from a harmonic motion, although the upstroke is slightly faster. The average wing beat frequency is 400 per second, and the long axes

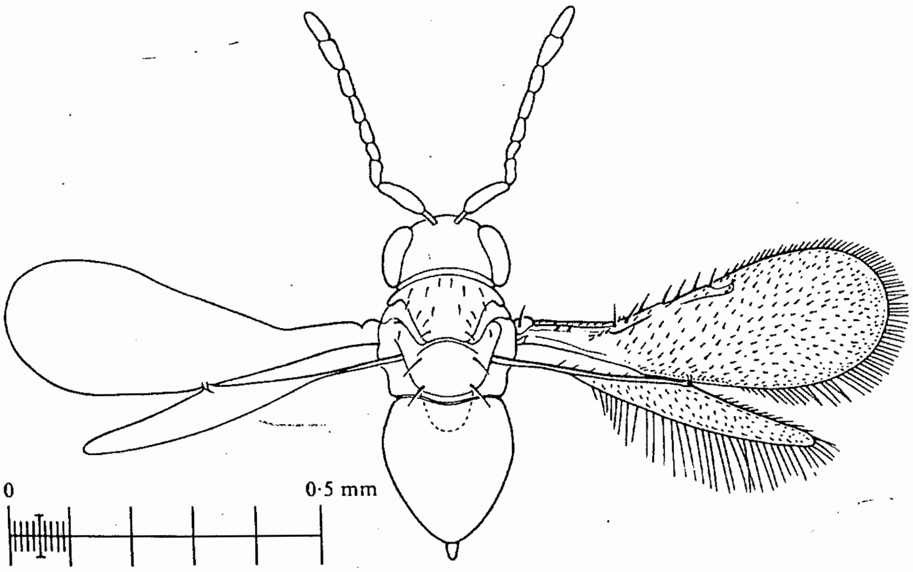

Figure 1. The morphology of Encarsia formosa Gahan, re-drawn from Weis-Fogh (1973). On the right-hand side the entire wing is drawn, including the brim of marginal hairs. On the left-hand side only the membranous wing is drawn.

of the wings move through about 2.09 radians (120°) in the stroke plane. The average angular velocity, $d\gamma/dt$, of the wings during the wing beat is 2.68×10^3 rad s^{-1}.

The clap, fling, and flip periods are indicated in Figure 2. During the clap, the wing surfaces touch dorsally and γ is 3.32 rad (190°). The clap lasts about one-fourth to one-fifth of the stroke period and cannot produce any useful aerodynamic forces. At the end of the clap, the wings fling open and generate a bound vortex around each wing. The fling ends at γ approximately equal to 2.71 rad (155°), and the wings begin translating with an angle of attack, α, of about 0.52 rad (30°). Towards the end of the downstroke, α increases to about 1.22 rad (70°), but an increased tilt of the stroke plane makes the effective α about 0.87 rad (50°). When the harmonic translational velocity equals 0 at the end of the downstroke the flip occurs and the wings begin the upstroke. During the upstroke, α is 0.52 rad (30°) but the tilt of the stroke plane again gives an effective α of 0.87 rad (50°). This α is relatively constant during the upstroke. As the wings clap together to end the wing beat a jet motion is produced directed downwards. The force of this jet is several orders of magnitude smaller than the weight of the insect and is therefore insignificant in terms of lift. The long duration of the clap is puzzling. It may allow shed vortices from the upstroke to be

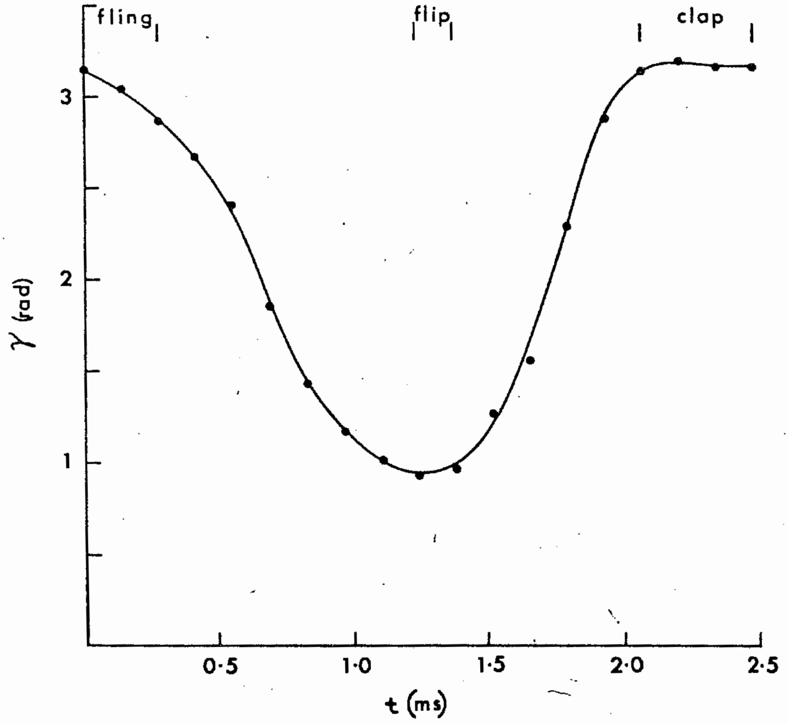

Figure 2. Variation of the angular position of the wings with time during the wing beat of Encarsia, re-drawn from Weis-Fogh (1973). The fling, flip, and clap periods are indicated.

carried sufficiently far away before the fling begins, and it could possibly indicate elastic storage of the high mechanical energy needed for the fling motion.

ESTIMATING $\overline{C}_L$

The method used in estimating the minimum steady-state $\overline{C}_L$ necessary for flight involves calculating the circulation as a function of r, the distance along the long axis of the wing from the fulcrum. The lift per unit length is then calculated as a function of r, and an average figure obtained with a corresponding value of r where this average figure occurs. This value of r is used to calculate an average velocity during the wing beat, and $\overline{C}_L$ is then estimated.

The chord of the wing as a function of r, c(r), was determined by measurements made from Figure 1. Two wing outlines were used in this measurement; an outline enclosing only the membranous portions of the wing, and one which included the additional area occupied by the brim hairs. The chordwise variation with r is presented in Figure 3 for both outlines.

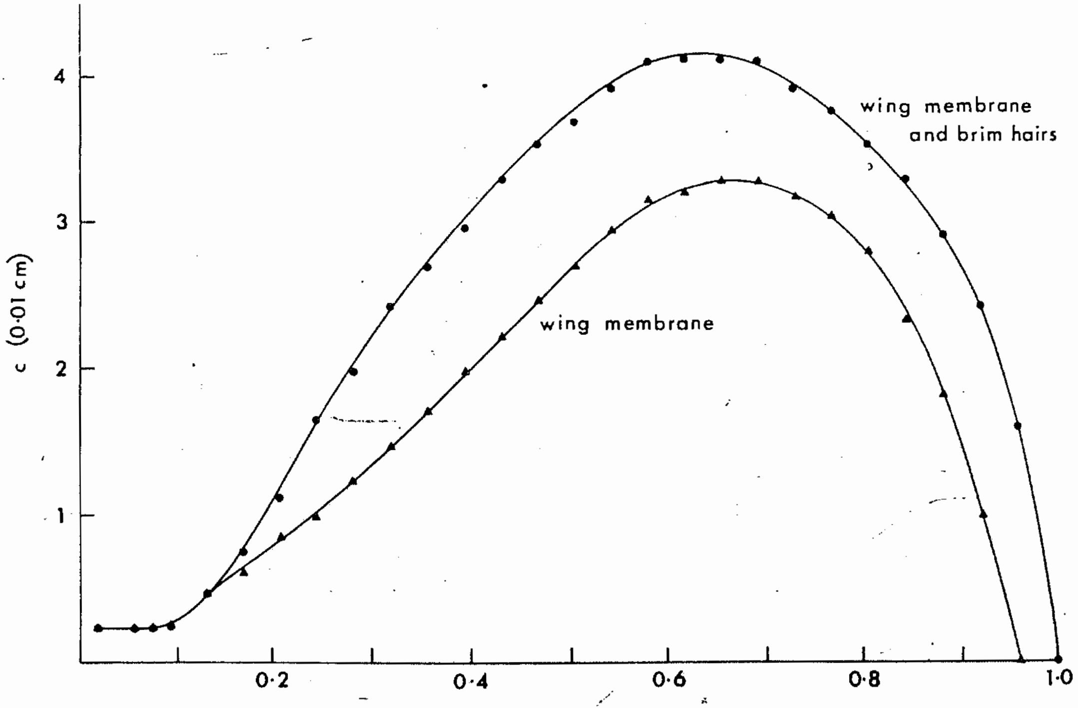

Figure 3. Relation between the wing chord for Encarsia and distance from the fulcrum, for the membranous wing and the membranous wing plus brim hairs.

To determine which outline should be used to estimate $\overline{C}_L$ it is necessary to know if the brim hairs effectively act as an extension of the membranous wing under flight conditions. The order of magnitude of the boundary layer around the brim hairs during flight is given by

$$0(\frac{\delta^2}{d^2}\ \mathrm{Re}) = 1 \qquad (1)$$

where δ is the boundary layer thickness, d is the diameter of the brim hairs (about 2×10^{-4} cm), and the Reynolds number, Re, is dU_0/ν, where ν is the kinematic viscosity of air and U_0 is the free-stream velocity normal to the wing chord. Taking a value of 50 cm s^{-1} for U_0, the order of magnitude for the boundary layer around each hair is thus 8×10^{-4} cm. The spacing of the brim hairs leaves a gap of about 1.1×10^{-3} cm between adjacent hairs, so the boundary layer of each hair occupies at least half the gap. This indicates that little flow through the hairs may be expected, and that the hairs possibly extend the wing area close to their distal ends. In further calculations c(r) will be taken corresponding to the outline including the brim hairs.

The circulation, Γ, around the wings is a function of r and t.

$$\Gamma(r,t) = \frac{1}{2} C_L(r,t)c(r)U(r,t) \tag{2}$$

It is assumed that variation of C_L with r and t is small compared to the variation of c and U, so that an average value $\bar{C}_L$ may be used. U(r, t) is the variation of velocity with r and t and is given by $r\,d\gamma(t)/dt$. Thus,

$$\Gamma(r,t) = \frac{1}{2} \bar{C}_L c(r) r \frac{d\gamma}{dt}(t) \tag{3}$$

At any given instant $\Gamma(r)$ is proportional to c(r)r. Using c(r) measured for the wing outline including brim hairs, $\Gamma(r)$ is calculated and presented in Figure 4. The distribution of Γ over r is independent of the value of $d\gamma/dt$. This calculation does not correct for effects from the trailing vortices.

An interesting sideline is the speculation that the circulation distribution calculated above may correspond closely to that which would give minimum induced drag. This speculation is based on the idea that the most efficient design is selected through evolution when possible. We have no analytical treatment of the circulation profile for a flapping wing as far as I know, and it would be interesting to compare this calculation with the results of such a treatment. With the circulation distribution as calculated, the system of trailing vortices would appear as in Figure 5.

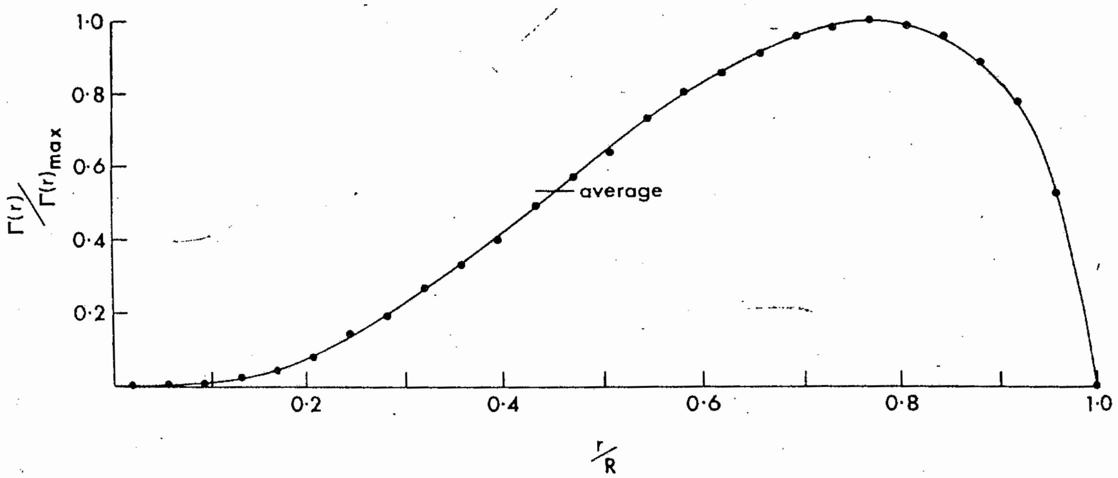

Figure 4. Distribution of circulation along the wing of Encarsia, based on distance from the fulcrum and the corresponding chord for a wing area including the brim hairs.

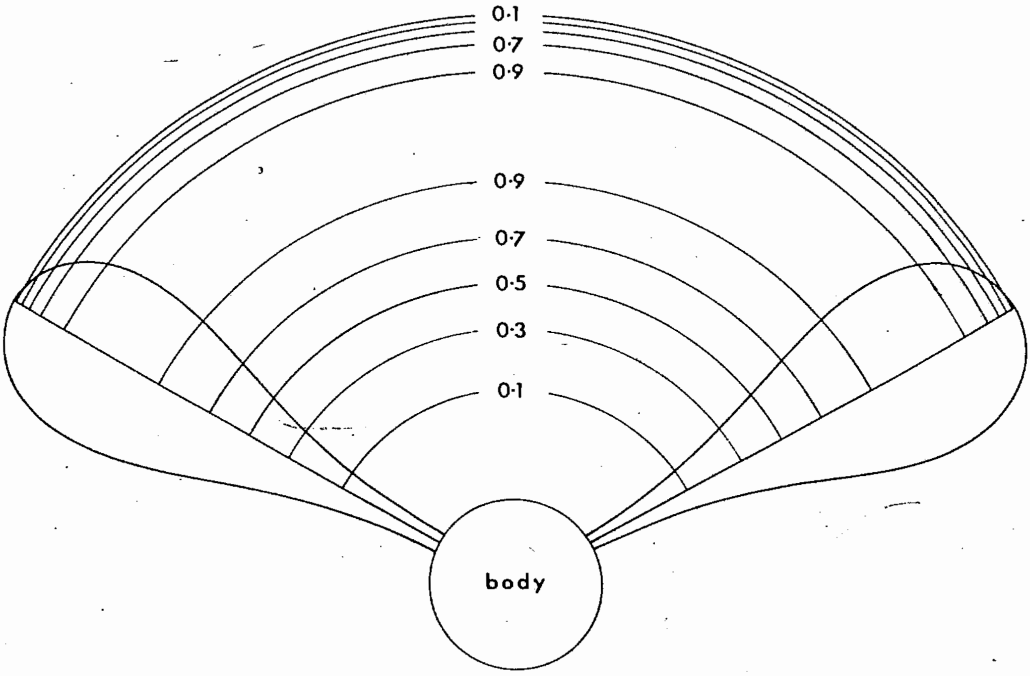

Figure 5. Pattern of trailing vortices obtained from Figure 4. The circular arcs are lines of equal circulation, expressed as a fraction of the maximum circulation around the wing.

The lift per unit length along r at any time is:

$$\frac{L}{\ell}(r) = \rho\, U(r)\Gamma(r) \tag{4}$$

and is proportional to $c(r)r^2$. ρ is the density of air. This distribution is shown in Figure 6, and the average value occurs at r equal to 0.034 cm with a corresponding chord of 0.037 cm.

The average lift per unit length for the wing is

$$\frac{\overline{L}}{\ell} = \frac{1}{2}\rho\, cU^2\overline{C}_L \tag{5}$$

when c and U take values corresponding to r equal to 0.034 cm. So,

$$\frac{\overline{L}}{\ell} = \frac{1}{2}\rho\, cr^2\overline{C}_L\left(\frac{d\gamma}{dt}\right)^2 \tag{6}$$

and $d\gamma/dt$ is the average value during the wing beat. The total lift of the wings is equal to $2R\,\overline{L}/\ell$, where R is the maximum

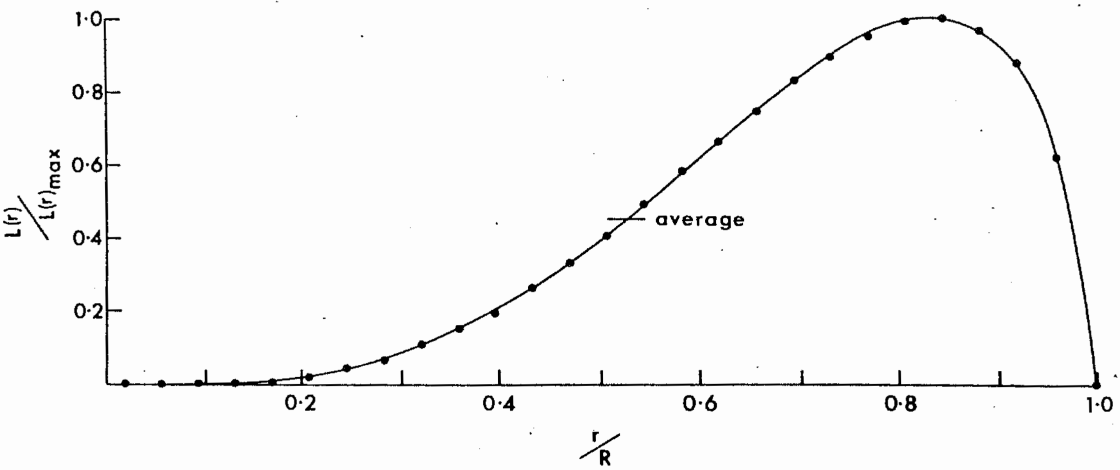

Figure 6. Distribution of lift along the wing of Encarsia as calculated from equation (4). The brim hairs are included in the wing area.

value of r - the length of one wing. This lift must balance the weight, G, of the animal. Since the wings generate lift for only 62% of the stroke period due to the clap, fling, and flip periods,

$$2R\frac{\overline{L}}{\ell} = 1.6\ G \qquad (7)$$

Thus

$$\overline{C}_L = \frac{1.6G}{\rho\, Rcr^2 (\frac{d\gamma}{dt})^2} \qquad (8)$$

and $\overline{C}_L = 1.6$.

Although $\overline{C}_L$ calculated by this method is considerably lower than that calculated by Weis-Fogh (1973), it is still too high to be compatible with steady-state aerofoil theory. The experimental work on aerofoils in this Reynolds range is unfortunately, inadequate, but from Thom and Swart (1940) a steady-state C_L for a flat plate at this Reynolds number could be expected to be about 1.1 or 1.2. The $\overline{C}_L$ calculated above may also be higher during flight due to the small amount of flow through the brim hairs.

CIRCULATION AROUND THE WINGS

Lighthill (1973) investigated the circulation that would be created during the fling for two-dimensional flow in an inviscid fluid and concluded that it would have a value of 0.69 ωc^2 around one wing and -0.69 ωc^2 around the other, where ω is the angular velocity of each wing during the fling and c is the chord. From analysis of films ω has an average value of 4000 rad s^{-1}, and at c = 0.037 cm as above this would create circulations of strengths ±3.8 $cm^2 s^{-1}$. Lighthill pointed out that viscous modifications may increase the value of circulation, but for this analysis 3.8 $cm^2 s^{-1}$ will be used.

The flip has not been theoretically analysed as yet, but for this investigation it will be assumed that it can create a bound vortex of approximately the same strength as in the fling. During the flip, ω for the forewing and hindwing is about 4000 rad s^{-1}. The vortex around the hindwing is shed as a starting vortex, and the vortex around the forewing remains bound to the wing. Lacking a more sophisticated analysis, it will be assumed that the circulations remaining around the wings at the end of the fling and flip are roughly equal.

The next step is to compare this circulation to that which could be expected around the wings by steady-state theory as they translate. Taking $\overline{C}_L = 1.0$ as a conservative estimate, the circulation possible around the wing by steady-state theory is $\Gamma = \frac{1}{2}cUC_L = 1.7$ $cm^2 s^{-1}$. This is the steady-state value of Γ satisfying the Kutta-Zhukovski condition for $\overline{C}_L = 1.0$. However, a circulation of 3.8 $cm^2 s^{-1}$ already exists around the wing as it begins to translate. The Kutta-Zhukovski condition is not satisfied, therefore, and trailing edge vortices will be shed as the wings moves. The shed vortices decrease the strength of the bound vortex, which will approach the steady-state value of 1.7 $cm^2 s^{-1}$ with time. If the amount of vorticity shed is sufficiently small during the half-stroke, however, the strength of the bound vortex will remain high enough to produce a lift balancing the weight of the animal.

To estimate how the circulations generated in the flip and fling vary with time is extremely difficult at this stage. If may prove helpful to consider the work of Wagner (1925) and Walker (1932) on the build-up of circulation around an aerofoil moved with an impulsive start. Figure 7 shows Wagner's theoretical data as calculated by Walker, for the flat plate, for the ratio of the existing circulation to the final steady-state value, Γ/Γ_{ss}, as a function of the distance (in chord lengths) travelled by the aerofoil. Walker's experimental data are in good agreement. A plot of the function

$$\frac{\Gamma}{\Gamma_{ss}} = 1 - e^{-0.7L} \qquad (9)$$

where L is the distance travelled in chords by the aerofoil is also shown in Figure 7. This function resembles the actual curve,

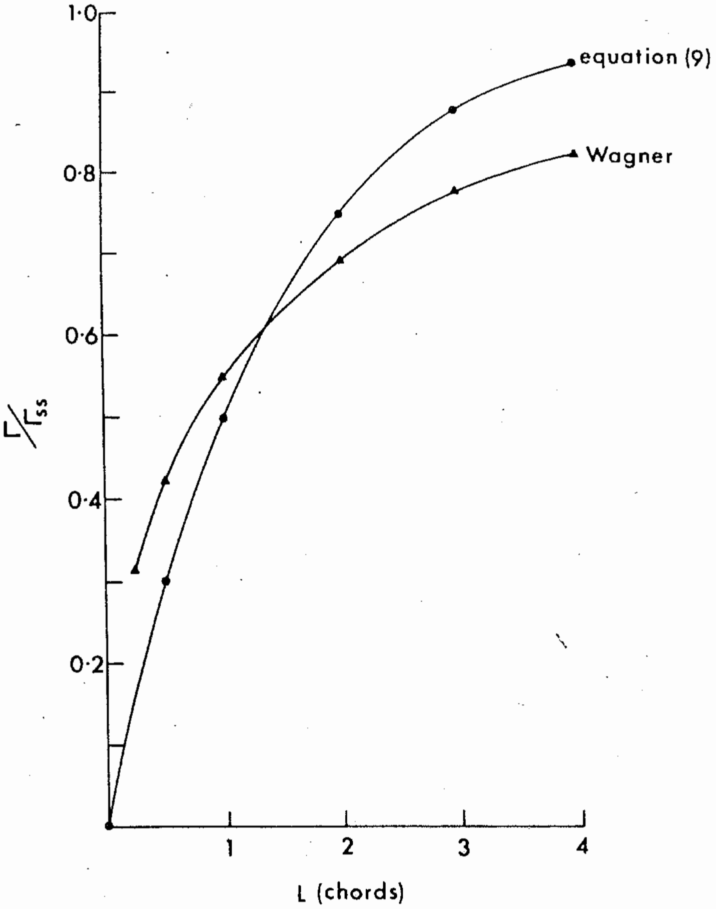

Figure 7. Change in the ratio of transient circulation to the final steady-state circulation with distance travelled, in chords, for a flat plate moved with an impulsive start. Wagner's theoretical curve is shown, along with a plot of equation (9).

but overestimates the change of circulation with L, $d\Gamma/dL$. In both cases $d\Gamma/dL$ decreases as Γ/Γ_{ss} approaches the steady-state value of 1. After the fling and flip, Γ/Γ_{ss} is about 2.2, and presumably will decrease until it reaches 1. $d\Gamma/dL$ is negative as Γ approaches Γ_{ss}, and should increase to 0 as the steady-state is obtained. For this approximation a function for Γ/Γ_{ss} can be used similar to equation (9):

$$\frac{\Gamma}{\Gamma_{ss}} = 1.2\, e^{-0.7\, L} + 1 \qquad (10)$$

and is plotted in Figure 8. Since the time constant is the same in equations (9) and (10), we might expect $d\Gamma/dL$ to be somewhat overestimated in equation (10) also. During the half-stroke the

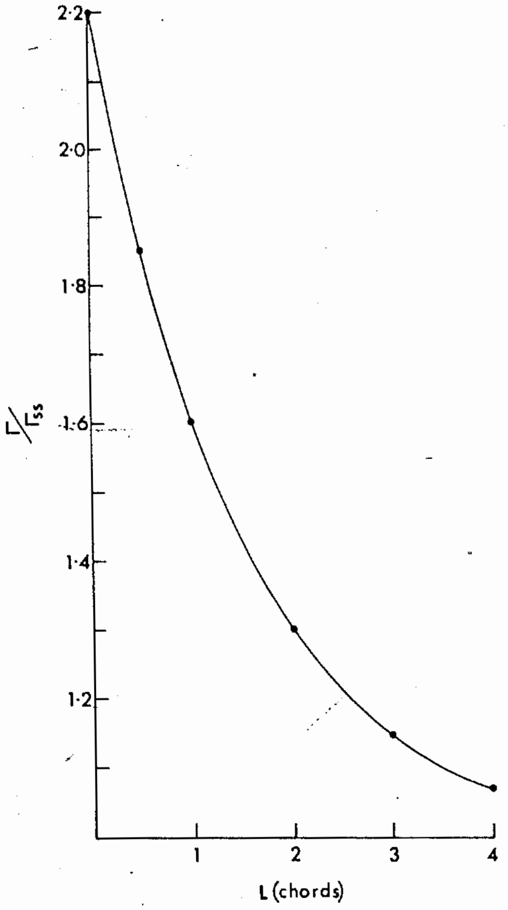

Figure 8. Change in the ratio of transient circulation to the final steady-state circulation with distance travelled, in chords, for the wings moving with a bound vortex generated in the fling and flip phases.

wings travel about twice their chord at $r = 0.034$ cm. For a conservative estimate of $d\Gamma/dL$, $L = 3$ will then be used. With $\Gamma/\Gamma_{ss} = 2.2$ at $L = 0$, equation (10) gives $\Gamma/\Gamma_{ss} = 1.2$ at $L = 3$. The average value of Γ/Γ_{ss} during the half-stroke is then 1.5, and the average value of Γ is 2.6 $cm^2 s^{-1}$. This circulation would correspond to a lift of 0.96 G during the entire wing beat.

CONCLUSIONS

Weis-Fogh (1973) demonstrated that consideration of non-steady-state aerodynamics was necessary to explain the flight of the small wasp, Encarsia formosa, and in particular that circulation around the wings is created independently of conventional aerofoil action. This analysis has been extended to show that even during periods when the wings are moving like aerofoils non-steady-state aerodynamics must be considered. The assumptions used in this analysis are sometimes crude, especially concerning the rate of change of the established circulation with time. Until more experimental evidence is available it is pointless to refine them. A rough estimate of lift is achieved, though, and it is close to the necessary value. It seems probable that non-steady-state effects must be invoked during the entire wing beat. Before each downstroke and upstroke, wing movements generate a circulation around each wing which is greater than the maximum steady-state value obtainable by aerofoil theory. During the downstroke and upstroke, trailing vortices are continually shed, reducing the strength of the circulation. The short duration of the half-stroke may enable the average circulation remaining around each wing to be large enough to produce the necessary lift.

The idea that non-steady-state aerodynamics can be advantageous during flight is an important addition to past investigations. Before Weis-Fogh proposed the fling and flip, all known non-steady-state effects, except the lift hysteresis near the stall point, reduced the possible lift of an aerofoil. Using the steady-state $\overline{C}_L$ calculated by equation (8), maximum lift would be obtained when the circulation around each wing was always at the steady-state value, and would only be 0.6 G. Conventional non-steady-state effects would delay the build-up of circulation during each half-stroke and reduce the lift further. The fling and flip are non-steady-state effects which increase the possible lift, however. The finite rate of decrease of the circulation established by the fling and flip can be significantly advantageous in a transient situation also. The wing beat period of Encarsia is short enough for this transient response to be useful, if not essential.

There are several possible refinements to this analysis which would be beneficial to the insect. The rate of vortex shedding depends on the difference between the circulation generated before translation and the steady-state circulation expected throughout the stroke. The wing velocity decreases towards the end of each half-stroke, and this would decrease the steady-state circulation. As the wing decelerates during the downstroke, however, the angle of attack increases so that the steady-state circulation remains relatively constant and at a high value. When the wings approach the clap position at the end of the upstroke they are decelerating but also becoming increasingly influenced by the upwash of the other wing, which effectively increases the angle of attack. As the wings separate after the fling and the angle between their long axes is about 1.31 rad (75°), so that the downwash of each wing does not significantly reduce the angle of attack, and therefore steady-state circulation, of the other. As Lighthill (1973) noted, the circulation generated by the fling is characterized by an approximately equal pressure on both sides of the trailing edge when the wings separate, so that trailing edge vortex shedding would not be expected immediately after separation.

SUMMARY

1. An estimation of the boundary layer around the wing brim hairs indicates that little flow through the hairs is to be expected during flight. The effective wing area may then be extended from the membranous wing to near the distal ends of the hairs, an increase of about 40%.

2. The distribution of circulation and lift along the long axis of the wing is calculated neglecting the effects of the trailing vortices.

3. The minimum steady-state lift coefficient necessary for flight is calculated to be 1.6. This value is lower than Weis-Fogh's estimate but still too high for steady-state aerofoil principles at this Reynolds number.

4. The circulation generated by the fling and flip may be about 3.8 $cm^2 s^{-1}$. Creation of circulation according to aerofoil theory would permit a steady-state value of about 1.7 $cm^2 s^{-1}$. Trailing vortices would then be shed as the wing translates with the circulation pre-established by the fling and flip. After estimating the vorticity shed under these conditions, it is seen that the average circulation remaining about the wing during each half-stroke is sufficient for sustained flight.

ACKNOWLEDGMENTS

I am most grateful to Professor T. Weis-Fogh for his advice and encouragement during this work, and for the data and films he generously allowed me to use. I also thank Professor Sir James Lighthill, F.R.S., for his helpful discussions, Mr. B.J. Fuller for information on wing dimensions for Encarsia, and Mr. G.G. Runnalls for assistance with films.

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