Ground Roll Time for Aircrafts: A Quick Estimate*

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The time taken by typical aircrafts to take off from runway, or more precisely, its ground roll time, is estimated, starting from simple estimates and then with details added.

1. Introduction

Over the years, I have developed a habit, whenever I get on an aeroplane, of checking the time an aircraft takes to lift off the ground. I once saw a co-passenger checking the time with his stopwatch, and I have been doing it ever since, almost as a reflex, perhaps because it gives me something to do instead of being anxious about leaving the assurances of Mother Earth. After a few flights, however, I began to see a correlation between the type of aircraft and the take off time (the bigger the plane, the longer the time) and began to wonder about the basis for the relation and the factors that determine the time. I present a summary of what I have learnt and how I have tried to reconcile the estimates with my own observations.

Keywords
Aerodynamics, ground roll time, take off time, aerodynamic lift,drag, thrust force, drag polar.

I have used the phrases ‘ground roll time’ and ‘take off time’ interchangeably in the abstract, but they are not the same. The take off time includes an additional time, when the aircraft leaves the ground and reaches a height of 15 m ($\sim$ 50 ft) (sometimes a different height, say, of 35 ft, is used). But I will ignore this additional segment and call the ground roll time also the ‘take off time’ since the difference is small, and we are interested in a rough estimate anyway.

First, let us see if we can make a rough order-of-magnitude estimate. Let us assume that the thrust of the engine provides acceleration during the ground roll, and the take off time is the time

*Vol.30, No.6, DOI: https://doi.org/10.1007/s12045-025-1817-y

Figure 1. Coefficient of lift for different configurations of the wing and flap (From Sun *et al*., 2018).

Flap types	$C_{L,max}$	Illustration
airfoil only	1.5	
leading-edge slat	2.4	
plain flap	2.5	
split flap	2.6	
Fowler single-slotted flap	2.9	
Fowler multi-slotted flap	3.0	
with leading-edge slat	3.3	
with boundary layer suction	3.9	

required to attain the take off speed. For this, we need to know (i) the engine thrust and (ii) the take off speed. The latter depends on the wing area (S), lift coefficient $C_{L,max}$ (which incidentally varies with speed, angle of attack, etc., so we can take its maximum value here), and the weight of the aircraft (W), as well as the density of air (ρ). One defines the take off speed υ_{TO} as 20% larger than the stalling speed, where

$$\frac{1}{2}\rho\upsilon_{stall}^2 S C_{L,max} = W \,. \tag{1}$$

In other words, the υ_{stall} is the speed which just about supports the weight of the aircraft by producing the necessary lift. The formula derives from the fact that pressure due to airflow is $(1/2)\rho\upsilon^2$, and it is multiplied by the relevant area and the lift coefficient.

Let us take a specific model of aircraft, say, A320 (perhaps the most common aircraft plying in India). We have the empty carriage weight m = 42,600 Kg and a maximum landing weight of 62,500 Kg. We can take an intermediate value of 50,000 Kg as an example. The wing area is given as S = 122.6 m^2. The maximum value[1] of lift coefficient $C_{L,max}$ depends on the flap condition. Let us take a value of 2.5 (corresponding to A320 with plain flap). The standard atmospheric condition gives a density (at sea level) of ρ = 1.1225 Kgm^{-3}.

[1] The specifications can be found in https://wiki.fsairlines.net/index.php/Airbus_A320_Family

We can then estimate the stalling speed as,

$$v_{stall} = \left[2mg/(\rho S C_{L,max})\right]^{1/2} = 53.4\,\text{m/s}\,. \quad (2)$$

Therefore, our take off speed should be 20% higher than this[2] or, 64 m/s.

Next, the maximum thrust of CFM56-5B engine that is often used in A320 is 120 kN each. This gives an acceleration (force/mass) of $2 \times 1.2 \times 10^5/5 \times 10^4 = 4.8$ ms^{-2}. With this acceleration, the time to attain the take off speed of 64 m/s is v_{TO}/acceleration≈ 13.3 seconds. As an order-of-magnitude estimate, this is not too bad since this is somewhat less than half the actual value. It is an underestimate because of many oversimplifications, e.g., neglect of friction on the runway, neglect of air drag, and the assumption that the aircraft maintains a large acceleration that is implied by the maximum thrust of the engine. However, it does give an idea of the important parameters involved in the problem. In terms of the thrust (T), the acceleration can be written as $g\,(T/W)$, which is the product of the acceleration due to gravity and the thrust-to-load ratio (T/W), which is a characteristic of any aircraft model.

For a Boeing 747-100 aircraft, the wing area is 510 m^2 (5500 sq ft), and the maximum take off weight is 33, 3390 Kg, which gives a take off speed of (with the same value of $C_{L,max}$)[3] , 81 m/s. There are four engines, each with 206.8 kN thrust; in total, a thrust of 827 kN. In other words, the thrust to load ratio (T/W) is 0.25 (roughly half of A320). The acceleration is 2.45 ms^{-2}. Therefore, the run time is roughly 33 s (in reality, it is more than a minute).

[2]Incidentally, at the Leh airport, at an altitude of 3.5 km, the atmospheric density is 0.863 Kgm^{-3} (using https://www.pdas.com/atmosTable2SI.html (this table), which makes the stalling speed roughly 20% higher than at sea level (64 m/s would become 76.3 m/s). This would work against the use of large (and heavy) aircrafts at such high altitude airports.

[3]These values can be found in https://www.boeing-747.com/boeing_747_family/747-100.php

2. Importance of Drag and Friction

Let us now move beyond rough estimates and write the full equation of motion of the aircraft as it moves on the runway. The force diagram is shown in *Figure* 2. The forces acting on the aircraft are the aerodynamic lift (L) and drag (D), the thrust force (T), weight ($W = mg$), the ground normal force (R), and the ground friction force (μR), where μ is the coefficient of (rolling) friction.

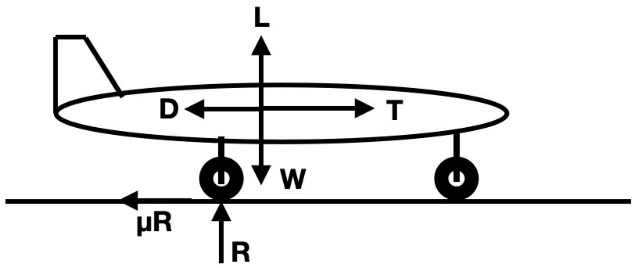

Figure 2. Forces acting on an aircraft as it rolls on the runway: T is the forward thrust, D is the drag, L is the lift, R is the normal force, while μ is the coefficient of friction.

The equations of motion in two directions (vertical and horizontal) are given by:

$$L + R - W = 0, \quad \Rightarrow \quad R = W - L, \quad (3)$$
$$T - D - \mu R = m\frac{\mathrm{d}v}{\mathrm{d}t}, \Rightarrow T - D - \mu(W - L) = \frac{W}{g}\frac{\mathrm{d}v}{\mathrm{d}t}. \quad (4)$$

The most important parameter that determines the take off time is the thurst to load (T/W) ratio of an aircraft.

The second equality in (4) follows the result of the previous equation. We now write $D = \frac{C_{Dg}}{2}\rho S v^2$, $L = \frac{C_{Lg}}{2}\rho S v^2$, where C_{Lg} is the lift coefficient during the ground run, C_{Dg} is the drag coefficient, S is the wing area, ρ is the density of the surrounding air. Incidentally, these constants differ in their values during the ground roll and cruising in the air due to aircraft configurations (e.g., flaps in the wing) and due to the proximity of the ground, which is why the subscript g has been added to them.

Rewriting and re-arranging, we have:

$$g(\frac{T}{W} - \mu) - \frac{g}{W}(D - \mu L) = \frac{\mathrm{d}v}{\mathrm{d}t}$$
$$\Rightarrow g(\frac{T}{W} - \mu) - \frac{g}{W}\frac{1}{2}S v^2(C_{D_g} - \mu C_{L_g}) = \frac{\mathrm{d}v}{\mathrm{d}t}. \quad (5)$$

Now, we can re-write the force equation as, (gathering the terms depending on v^2 together)

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g\Big(\frac{T_0}{W} - \mu\Big) - \frac{g}{W}\Big[\frac{1}{2}\rho S\Big(C_{D_g} - \mu C_{L_g}\Big)\Big]v^2$$
$$= A - Bv^2, \quad (6)$$

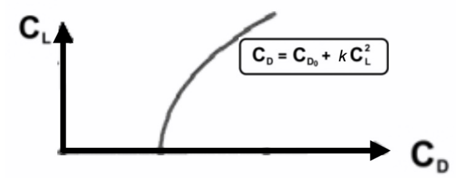

Figure 3. The relation between drag coefficient and lift coefficient has a parabolic shape.

where the terms

$$A = g\Big(\frac{T_0}{W} - \mu\Big), \quad B = \frac{g}{W}\Big[\frac{1}{2}\rho S\Big(C_{D_g} - \mu C_{L_g}\Big)\Big]. \qquad (7)$$

The time for ground roll can be obtained by integrating (6) for dt, starting from rest, to reach a take off speed of v_{OT}, (for $A, B > 0$)

$$\mathrm{d}t = \frac{\mathrm{d}v}{A - Bv^2}, \Rightarrow t_{OT} = \frac{1}{2\sqrt{AB}} \ln \frac{\sqrt{A} + v_{OT}\sqrt{B}}{\sqrt{A} - v_{OT}\sqrt{B}}. \qquad (8)$$

The relation between drag coefficient and lift coefficient can be figured out from wind tunnel experiments. Historically, it was first done by Otto Lilienthal in 1880 and in 1910 by Gustav Eiffel. They used polar coordinates, so the relation is referred to as 'drag polar'. It depends on the angle of attack, the Reynolds number and the Mach number. Typically, $C_D = C_{D0} + kC_L^2$, where k is fixed for an aircraft, as shown in *Figure* 3.

The relation between the drag coefficient and lift coefficient was first determined by Otto Lilienthal in 1880 and Gustav Eiffel in 1910.

The ground drag coefficient C_{Dg} can be estimated from this by requiring that the acceleration is maximum during the ground roll. Going back to (5), assuming that the take off speed v_{TO} is independent of C_{Lg}, we can find the maximum acceleration by taking the derivative with respect to C_{Lg} and equating the result to zero. We have

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}C_L}\Big(\frac{\mathrm{d}v}{\mathrm{d}t}\Big) &= \frac{\mathrm{d}}{\mathrm{d}C_L}(C_{Dg} - \mu C_{Lg}) = \frac{\mathrm{d}}{\mathrm{d}C_L}\Big(C_{D0} + kC_{Lg}^2 - \mu C_{Lg}\Big) = 0\,, \\ \Rightarrow C_{Lg} &= \frac{\mu}{2k}\,. \end{aligned} \qquad (9)$$

Let us now gather other specifications for the A320 model. For the runway, the friction $\mu = 0.03$ (somewhere between 0.02 and

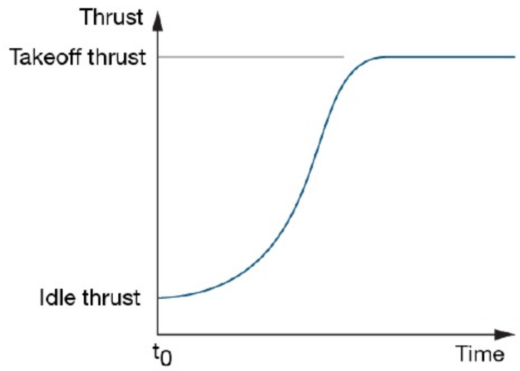

Figure 4. A schematic diagram of how thrust increases from a low to its maximum value.

0.05). The values of $C_{D0} = 0.032$, and $k = 0.0334$, as given in the specifications of the aircraft. We therefore have, $C_{Lg} = \frac{\mu}{2k} = 0.449$. This gives us $C_{Dg} = C_{D0} + kC_{Lg}^2 = 0.0387$.

The maximum thrust of A320 engines is given as 120 kN. There are two engines. (Sometimes, the engine power is given instead of the thrust. This value can be used to estimate the thrust at take off. There is an efficiency factor η, which, when multiplied to the power, equals thrust times the take off speed, from which the thrust can be estimated.)

It is interesting to find that the rough estimates presented in the beginning are quite robust and are not significantly changed after taking into account the air drag and runaway friction.

We can calculate the two terms A, B that we need,

$$\begin{aligned} A &= g\left(\frac{T_0}{W} - \mu\right) = 9.8\left(\frac{2 \times 1.2 \times 10^5}{5 \times 10^4 \times 9.8} - 0.03\right) = 4.5, \\ B &= \frac{g}{W}\left[\frac{1}{2}\rho S\left(C_{D_g} - \mu C_{L_g}\right)\right] = 3.4 \times 10^{-5}\,. \end{aligned} \tag{10}$$

Equation (8) then gives us the ground roll time as $t_{TO} = 14.4$ s. Notice that this takes care of ground friction and air drag. The term A is similar to the acceleration we began with, but now it takes into friction, and $g\,(T/W)$ has now become $g\,(T/W) - \mu$. Also, the air drag is included in the term B, which further reduces the acceleration.

3. Summary

As these corrections due to drag and friction show, the rough estimates we had presented to begin with were quite good indeed. The drag and friction slow down the aircraft during the ground roll, but not by a large amount.

However, an obvious factor we have not discussed here is that the pilots slowly increase the thrust by pushing the thrust lever, the manner of which cannot be captured by any equation. In other words, the thrust is much smaller, to begin with, and slowly attains the maximum value. This is what makes the take off time longer than we have estimated here. The time we have calculated is the time of take off using the maximum thrust, and is a fraction of the total ground roll time.

We can use a function that mimics the slow increase of the thrust, and use its average. For example, consider the function

$$g(t) \equiv \frac{1}{2}(\tan h\,(t-2)+1)\,. \tag{11}$$

We can use $g(t=0)=0$, $g(t=2)=0.5$, and $g(t=4)=1$. Taking its average between $t=0$ and $t=4$, we have $(1/4)[\int_0^4 g(t)\,\mathrm{d}t] = 0.5$. Therefore, we need a correction factor of 0.5 for the average thrust applied to the aircraft, which would increase the take off time by a factor of 2. This will make the estimate consistent with an observed time scale of roughly 30 seconds.

One also needs to take into account the fact that pilots slowly increase the thrust, and that the thrust does not remain constant at its maximum during the ground roll.

Conclusions

We have derived the time taken by aircrafts to take off from the runway, starting from a quick estimate and then with details for drag and frictional forces added to it, and compared with what one observes.

Suggested Reading

[1] J Sun, J Hoekstra and J Ellerbroek, Aircraft drag polar estimation based on a stochastic hierarchical model, in '*Eighth SESAR Innovation Days*, 3–7 December 2018, Salzburg, Austria', 2018.