CELEBRATING 75 YEARS OF SCIENTIFIC EXCELLENCE

LOOKING BACK

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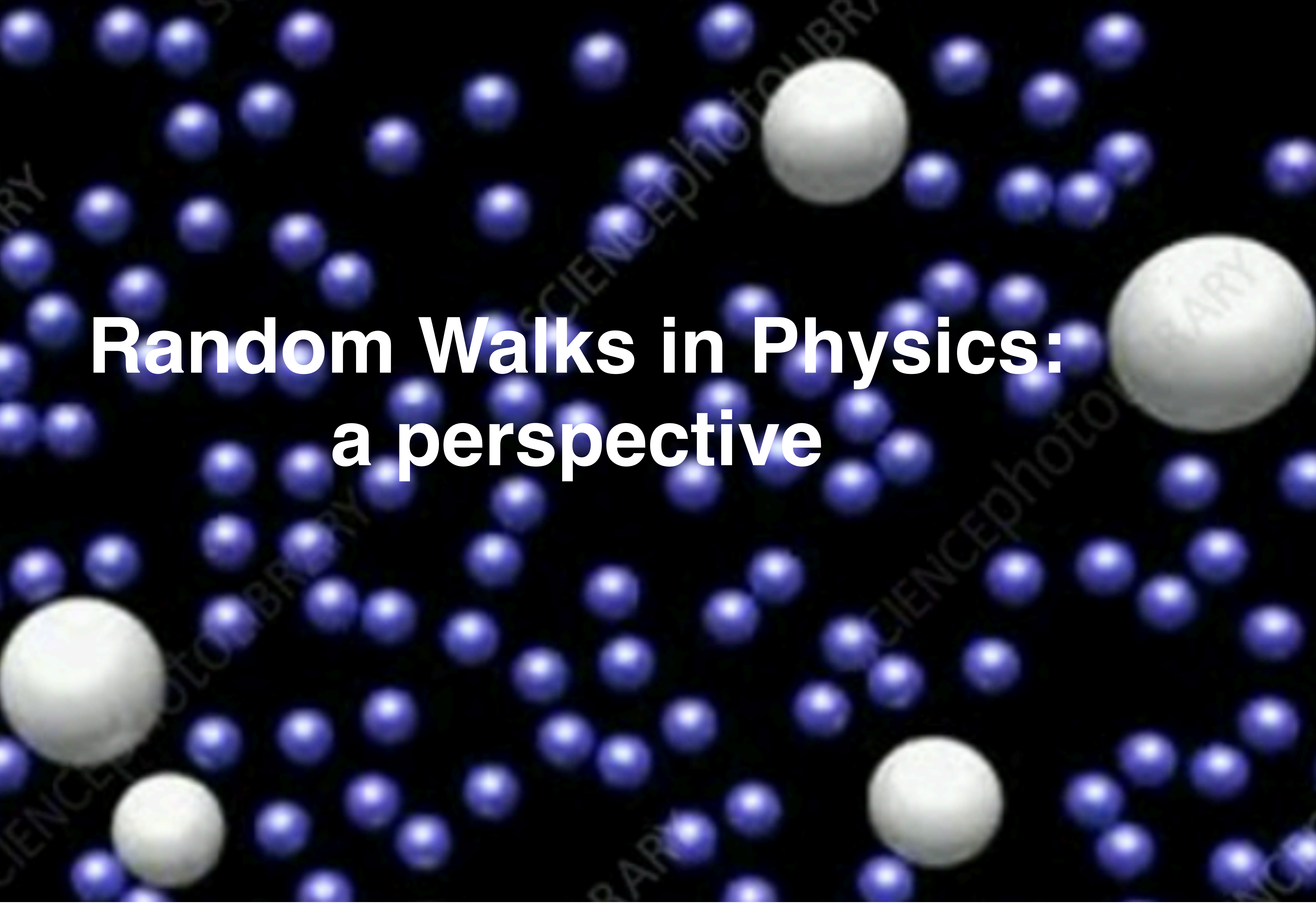
Over the years at RRI, my research has touched upon a range of different topics : classical and quantum statistical mechanics (equilibrium and non-equilibrium), soft matter physics, classical and quantum optics, quantum information, soft matter analogue of a quantum gravity prediction.

Today, I will restrict to my research involving a random walk or its continuum limit, Brownian motion and its various applications (Biological Physics, Decoherence and so on)

Random Walks in Physics: a perspective



RANDOM WALKS

A random walk in one dimension is generated by coin flips of a fair coin. The walker takes steps to the left or right depending on the result of the coin flip (head or tail).

The continuum limit of a RANDOM WALK is BROWNIAN MOTION - for instance, movement of milk globules kicked around by surrounding water molecules.

I will mention some of my work where these notions have appeared.

Topics

- Brownian Motion and Magnetism.
- The Worm Like Chain (WLC) model of biopolymers : DNA elasticity.
- A `Gaussian’ for diffusion on a sphere.
- Active Brownian Dynamics.
- Surface Tension and the Cosmological Constant.
- Decoherence at absolute zero.
- Brownian Motion at absolute zero.

Brownian Motion and Magnetism

- The question of interest is the following:
- Given a Brownian particle moving on a plane which returns to its starting point, what is the distribution of areas.
- This led to a natural connection between the area A and its conjugate variable, the magnetic field B and led to an interesting observation on diamagnetism of Bosonic systems.
- . $\tilde{P}(B) \equiv \overline{e^{ieBA}} = \int P(A) e^{ieBA} dA,$ $\tilde{P}(B) = Z(B)/Z(0)$
- Thus an insight into magnetism appeared as a byproduct of studying the distribution of areas of closed Brownian paths.

Ref: (PRB (1994) Joseph Samuel and Supurna Sinha)

Brownian Motion and Magnetism contd …

- One central question of interest:`Distribution $P(\Omega)$ of solid angles described by a closed Brownian path on a sphere’ [Depolarised light scattering]

- Solved via the mapping

to magnetism due to a

monopole located at the centre of

the sphere :

$$Z_g = \sum_{j=|g|}^{\infty} (2j+1) e^{-\frac{\beta\{j(j+1)-g^2\}}{2}}.$$

$$P(\Omega) = \mathrm{Re}\, \frac{1}{2\pi Z_0} \sum_{l=0}^{\infty} \left\{ (2l+1) \frac{1+\zeta}{2(1-\zeta)} + \left[\frac{2\zeta}{(1-\zeta)^2} \right] \right\} e^{\frac{-\beta}{2} l(l+1)},$$

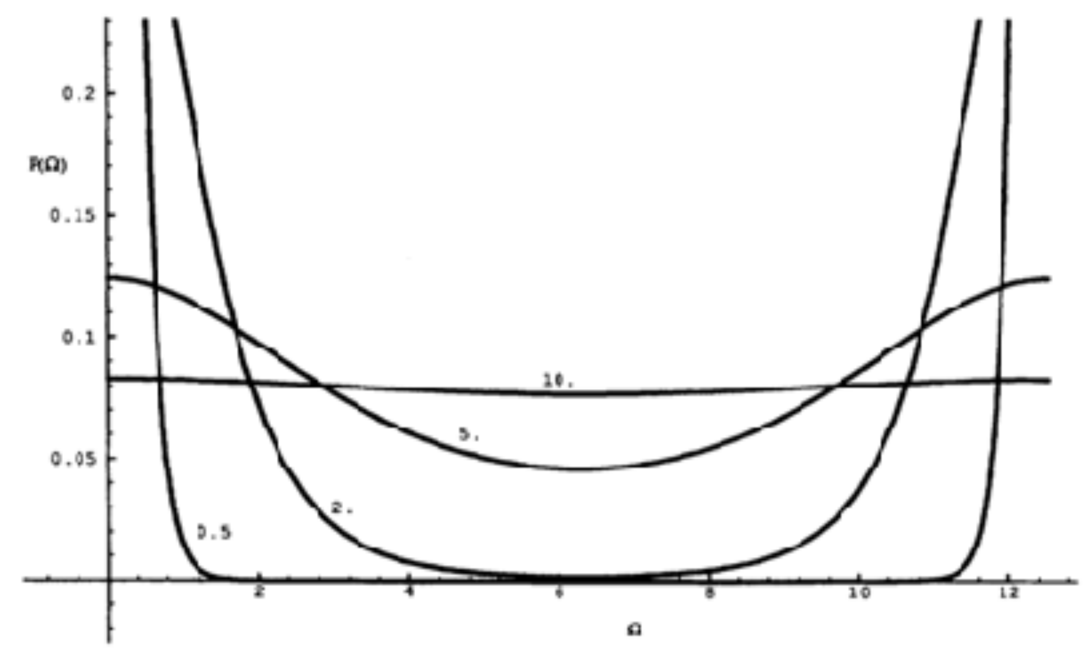


FIG. 1. The probability distribution $P(\Omega)$ of solid angles for closed random walks lasting a time β. $P(\Omega)$ is plotted above for four values of β: 0.5, 2, 5, and 10.

Diamagnetism

- We consider N spinless Bosons and consider the area enclosed by their trajectories (we allow for exchanges of particles). We consider the area enclosed $\mathscr{A}[q(\tau)]$

The probability distribution $P(A) \equiv \langle \delta(\mathscr{A}[\vec{x}(\tau)] - A) \rangle$ is related to the conjugate distribution $\tilde{P}(B)$ via a Fourier transform.

- Since $\tilde{P}(B)$ is the Fourier transform of a positive function , it follows that $Z(B) \leqslant Z(0)$

Or equivalently, $F(B) \geqslant F(0)$. (since $F = -\frac{1}{\beta} log[Z(B)]$)
(Diamagnetism)

Diamagnetism (contd..)

- We also conclude that: $\tilde{P}(B) \geqslant \cos(\pi B/2B_c)$ for $|B| \leqslant B_c$

  where $B_c = \pi/[2\sqrt{-\beta\chi(0)}]$. (FT of a +ve fn)

- This gives a lower bound on the partition function or an upper bound on the free energy and puts constraint on the extent of diamagnetism.

- Plot of $\tilde{P}(B) = Z(B)/Z(0)$ versus B.

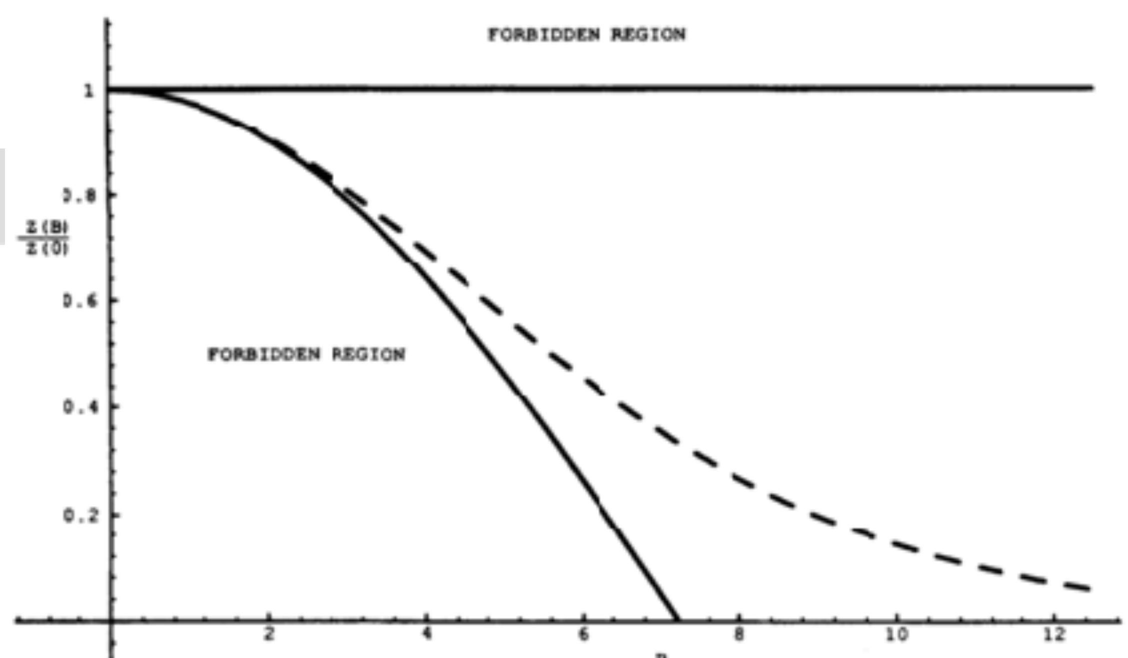


FIG. 2. The region forbidden by the bounds [inequalities (14) and (18)] on the partition function. These bounds are shown as solid lines. The dotted curve is the partition function for a charged simple harmonic oscillator in an external magnetic field. Notice that the dotted curve lies outside the forbidden region.

A `Gaussian' for diffusion on a sphere

- The probability distribution for diffusion

on a plane: $$P(r,t) = \left(\frac{r}{2Dt}\right) \exp - \frac{r^2}{4Dt}$$

- Question of interest: Counterpart of this on a sphere.

- We used a saddle point method to arrive at a short time propagator which is non-perturbative in the spatial domain:

$$Q(\theta,\tau) = \frac{\mathcal{N}(\tau)}{\tau}\sqrt{\theta \sin\theta}\, e^{-\frac{\theta^2}{2\tau}}$$

[A. Ghosh, J. Samuel and S. Sinha, EPL (2012)]

A `Gaussian' for diffusion on a sphere (contd..)

We thus obtained:

• An analytically elegant `Gaussian'

probability distribution formula for diffusion on a sphere.

• This formula also leads to an efficient algorithm

for simulating diffusion on a sphere which improves

on the standard tangent space approximation based

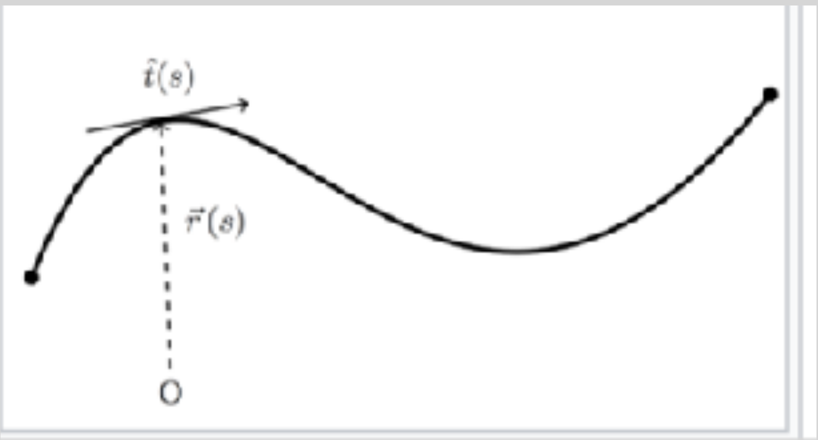
algorithms since it takes into account the Curvature of the sphere.

- The formula allows for generalisation to saddles and other curved manifolds.

which have many applications in soft matter and biological physics.

The Worm Like Chain Model of semiflexible polymers

- In the WLC model,



a semiflexible biopolymer

molecule like DNA is viewed as a curve in space.

- It is convenient to solve the statistical mechanics

and elastic properties of such a polymer by mapping it on the problem of BROWNIAN MOTION ON A SPHERE (the space of tangent vectors $\hat{t}(s)$).

  - Excellent quantitative agreement with single molecule expts. on DNA and other biopolymers.

WLC model : a few highlights

- `Elasticity of semiflexible polymers' Joseph Samuel and Supurna Sinha - Phys. Rev. E 66, 050801(R) (2002).
- `Inequivalence of statistical ensembles in single molecule measurements' Supurna Sinha and Joseph Samuel, Phys. Rev. E 71, 021104 (2005).
- 'DNA Elasticity : Topology of Self-Avoidance', Joseph Samuel, Supurna Sinha, Abhijit Ghosh, J. Phys. : Condens. Matter 18 , S253-S268 (2006).
- `Writhe distribution of stretched polymers' Supurna Sinha, Phys. Rev. E 70, 011801 (2004).
- `Free energy of twisted semiflexible polymers' Supurna Sinha, Phys. Rev. E 77, 061903 (2008).
- `Ring closure in actin polymers' Supurna Sinha and Sebanti Chattopadhyay, Physics Letters A, 381, (2017).

Active Brownian Dynamics

- Active Brownian particles - self-propelled particles which generate dissipative directed motion by consuming energy from the environment - for instance, Bacterial run and tumble motion.
- The speed of an ABP is fixed but its direction is a vector diffusing on the unit sphere. (BROWNIAN MOTION ON A SPHERE)

Orientational Probability distribution of an ABP(Active Brownian Particle)

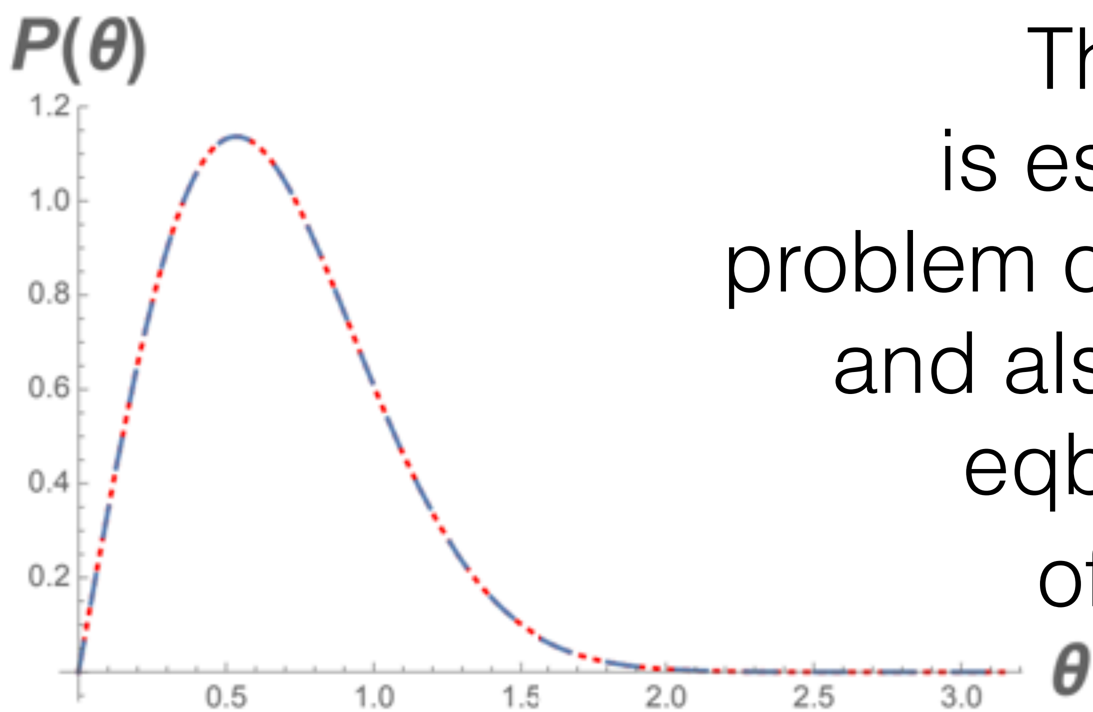


FIG. 1: Comparison of the short time approximate Probability Distribution (Eq.(16))(red dotted line) and the exact Probability Distribution (Eq. (4))(blue dashed line) at time $t = 0.3$.

The dynamics of an ABP is essentially the same as the problem of Brownian motion on a sphere and also maps onto the problem of eqbm. statistical mechanics of semiflexible polymers.

[Supurna Sinha J. Stat. Mech. (2020) 083201)]

[Predictions testable in expts. on Janus particles (ABP)]

Surface tension and the cosmological constant

- A small nonzero value of the cosmological constant stemming from the discreteness of spacetime on quantum gravity scales (Sorkin, Int. J. Th. Phys. 36 2759 (1997)).
- We proposed a soft matter analog of this effect - a small nonzero surface tension of a fluid membrane stemming from a finite number N of amphiphilic molecules (Joseph Samuel and Supurna Sinha (PRL, 2006), Rohit Katti, Joseph Samuel, Supurna Sinha (Classical and Quantum Gravity (2009)) (exptally testable)
- Fluctuation effects (Brownian motion) .

Decoherence at absolute zero

- Decoherence plays a central role in understanding the quantum to classical transition. [MODEL is the same as the system+environment model that is used to study QUANTUM BROWNIAN MOTION].

- The central question : Is there decoherence even at absolute zero (where only quantum fluctuations are present)

- Noticed that the law is distinct from the usual high temp. exponential decay of coherences.

[Supurna Sinha, Physics Letters A (1997).]

- Obtained a power law decay: $\tilde{\rho}(x, x', t) \sim t^{-\alpha}$,

  with $\alpha = (2/\pi\hbar) M\gamma(x - x')^2$.

- Such a slow decay of coherence is testable in cold atomic systems (PRL(2017)-S. Sarkar et al)

Brownian Motion at absolute zero:

- Brownian motion is the random motion of particles suspended in a fluid resulting from their collision with fast-moving molecules in the fluid.
- At room temperature, the law of diffusion is given by the well-known Einstein's law of diffusion: $\langle \Delta x^2 \rangle = 2Dt$
- What happens to this law of diffusion as we lower the temperature to close to absolute zero and scale down the size of the Brownian particle?
  - In the quantum domain we find that $< \Delta x^2 > \sim ln[t]$

Refs: 1. S. Sinha and R. D. Sorkin, PRB [1992];
2. U. Satpathi, S. Sinha and R. D. Sorkin, JSTAT[2018];

BM at absolute zero contd…

- Our derivation uses the Fluctuation-Dissipation theorem - independent of the details of the system.
- Recently there has been a theory-experiment collaboration with the LAMP group analysing the Response Function of a cloud of cold atom- sets the groundwork for testing the predicted logarithmic law of diffusion.
- [Ref: `Optics Continuum'(2022)

$S.Bhar, M.Swar, U.Satpahi, S.Sinha, R.D.Sorkin[$ $S.Chaudhuri, S.Roy$]

A few recent papers on QBM:

`Interplay of dissipation and memory in the quantum Langevin dynamics of a spin in a magnetic field'
[Suraka Bhattacharjee, Koushik Mandal, Supurna Sinha(IJMPB (2023)]
Quantum Brownian Motion of a charged oscillator in a magnetic field coupled to a heat bath through momentum variables
[Suraka Bhattacharjee, Urbashi Satpathi, Supurna Sinha Physica A (2022).]
Long Time Tails in Quantum Brownian Motion of a charged particle in a magnetic field
[Suraka Bhattacharjee, Urbashi Satpathi, Supurna Sinha (Physica A, 2022)]
Quantum Langevin dynamics of a charged particle in a magnetic field : Response function, position-velocity and velocity autocorrelation functions
[Suraka Bhattacharjee, Urbashi Satpathi, Supurna Sinha Pramana - J Phys 96, 53 (2022)]
Quantum Brownian Motion: Drude and Ohmic Baths as Continuum Limits of the Rubin Model
[Avijit Das, Abhishek Dhar, Ion Santra, Urbashi Satpathi, Supurna Sinha [PRE(2020)]
Decoherence and the ultraviolet cutoff: non-Markovian dynamics of a charged particle in a magnetic field
Suraka Bhattacharjee, Koushik Mandal, Supurna Sinha (arXiv (2023) arXiv: 2301.06365).

Conclusion:

- A common thread in many of my research projects at RRI has been Brownian Motion.
- Brownian Motion and Magnetism : a connection.
- Brownian Motion on a Sphere - WLC biopolymer model, A `Gaussian' for diffusion on a sphere, ABP dynamics.
- Surface tension and the cosmological constant - $\frac{1}{\sqrt{N}}$ fluctuation effects.
- Decoherence at absolute zero.
- Brownian Motion at absolute zero [continued interest (theory-expt collaboration)— more

recently applied to a charged Brownian particle in a magnetic field and a

spin in a magnetic field.].

THANK YOU!