
The Fine Structure Constant

Biman Nath

The article discusses the importance of the fine structure constant in quantum mechanics, along with the brief history of how it emerged. Although Sommerfeld's idea of elliptical orbits has been replaced by wave mechanics, the fine structure constant he introduced has remained as an important parameter in the field of atomic structure.

The values of the constants of Nature, such as Newton's gravitational constant 'G', determine the nature of our Universe. Among these constants, there are a few which are pure numbers and have no units. For example, there is the 'fine structure constant', denoted by α , which has a value roughly $1/137$. The value of α is related to the electromagnetic force between subatomic charged particles, and essentially determines how an atom holds together its electrons.

It is however not obvious why this constant has this particular value. Why $1/137$ and not some other value? One might think the question is meaningless, but it is not. If the value of this constant had been slightly smaller or larger, even by as little as 4%, then stars would not have been able to sustain the nuclear reactions in their core that produce carbon. As a result, there would not have been any carbon-based lifeforms in our Universe. Therefore, the question why $\alpha \approx 1/137$ is not completely irrelevant.

Scientists have even wondered if its value remains a constant over time. Astronomers have looked for signatures of tiny variation in its value in the spectral lines of distant quasars (which existed a long time ago), after com-



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Keywords

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¹ *Resonance*, Vol.16, No.9, 2011.

² *Resonance*, Vol.13, No.2, 2008.

³ *Resonance*, Vol.5, No.3, 2008.

⁴ In 1913, two physicists Gilbert Lewis and Elliot Adams conjectured that

$$e^2/hc = 16 \pi^2 (8\pi^5/15)^{1/3},$$

which agrees with the numerical value within 0.2%! In 1922, Arthur Lunn offered a number of algebraic expressions for e^2/hc : $\pi/2^4 3^3$, $7/\pi^6$, $32/45\pi^4$ and so on. These attempts are probably a measure of how desperate physicists were in their pursuit of a 'fundamental' reason for the value of the fine structure constant.

⁵ *Resonance*, Vol.18, No.10, 2013.

paring with laboratory reference measurements. If it does vary, then the standard scenario of the evolution of matter in the universe might need revision. It is perhaps no wonder that Richard Feynmann¹ called it a “magic number” and its value “one of the greatest damn mysteries of physics”.

Although it was Arnold Sommerfeld who formally introduced the fine structure constant in 1916 (as discussed later in this article), its history can be traced back to Max Planck². Planck had noticed that the combination of e^2/c (where e is the charge of electrons and c is the speed of light) has the same dimensions as the quantum of action, h (the Planck constant). (This is in CGS units. In MKS, one would have to consider $e^2/4\pi\epsilon_0$ instead of e^2 .) He wondered if h was identical to e^2/c and if this could somehow ‘explain’ the value of the elementary charge.

In 1909, while reviewing the current status of the theory of blackbody radiation, Einstein³ noted that one could take $h = e^2/c$ as an order of magnitude estimate, but ‘three decimals are missing’. Lorentz reacted to Einstein’s remarks saying that although one could think of a missing factor of 4π , three missing decimals were too much and concluded that h had nothing to do with e . In 1913, James Jeans speculated that perhaps $\frac{e^2}{c} = \frac{h}{32\pi^3}$, by supplying the missing three decimals with an ad-hoc factor $32\pi^3$. This sort of numerological experiments continued for some time⁴.

The initial attempts to consider a simple model of a hydrogen atom, with an electron orbiting around a nucleus, led to the first glimpse of the importance of the two parameters, e^2/c and h . Consider the Bohr⁵ model of the hydrogen atom, with the electron in the ground level, so that $mvr = h/2\pi \equiv \hbar$. Equating the Coulomb force between the positive nucleus and the electron ($\frac{e^2}{4\pi\epsilon_0 r^2}$) with the centripetal force ($\frac{mv^2}{r} = \frac{\hbar^2}{mr^3}$, one has for the



Bohr radius (denoted as a_0),

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2}. \quad (1)$$

Compare this to the classical radius of electron. Although the size of the electron is beyond the scope of ordinary quantum mechanics, one can think of its size as something the electron would need to have if its rest energy were only due to its electrostatic potential energy:

$$\frac{e^2}{4\pi\epsilon_0 r_e} = mc^2 \Rightarrow r_e = \frac{e^2}{4\pi\epsilon_0 mc^2}. \quad (2)$$

The ratio of the classical radius of electron to the Bohr radius is then,

$$\frac{r_e}{a_0} = \left(\frac{e^2}{4\pi\epsilon_0 \hbar c} \right)^2 \equiv \alpha^2, \quad (3)$$

where α contains the ratio $e^2/\hbar c$. This was first noted by an Austrian physicist Arthur Erich Haas in 1910, even before Bohr formally announced his model of hydrogen atom in 1913. In other words, the size of a hydrogen atom is a factor $\alpha^{-2} \approx 20000$ times the size of an electron.

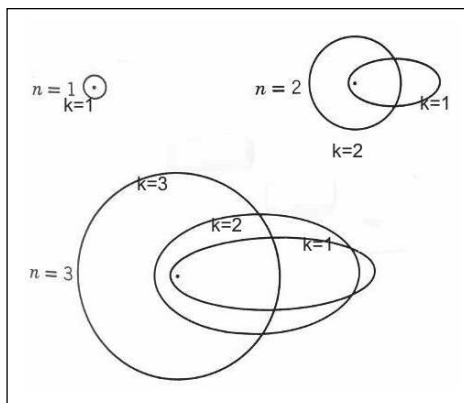
Another way of looking at α is to consider the ratio of the orbital speed of electron to the speed of light: $\alpha = v/c$, since $v = \hbar/m a_0$. Quantizing the angular momentum as $mvr = n\hbar$, where $n = 1, 2, \dots$, Bohr was able to determine the energy difference between different n levels, and therefore explain the different spectral lines of hydrogen atom. For example, the red $H\alpha$ line at 6563 \AA was interpreted as due to a jump of an electron from $n = 3$ to $n = 2$. However, with the advent of high resolution experiments, it was soon discovered that the red line was actually a doublet, which was termed the 'fine structure' of lines.

Arnold Sommerfeld thought he could improve upon the Bohr model by assuming that the orbits can be elliptical (*Figure 1*). For each principal quantum number

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Figure 1. Elliptical orbitals in the Bohr–Sommerfeld model. For each value of n (the total quantum number), k can take the values $1, 2, \dots, n$.



n , there would be a number of possible elliptical orbits, which were described by another quantum number, k . For a given value of n , k could take the values $1, 2, \dots, n$. For example, for the level $n = 3$, there could be three elliptical orbits of different shapes. Bohr had quantised the principal quantum number stating that $mvr = n\hbar$. Consider the de Broglie wavelength $\lambda = h/p$, where $p = mv$ is the momentum. Bohr's quantisation meant that orbits of each principal quantum number n would contain an integral multiple of the de Broglie wavelength. In other words, the $n = 3$ level orbit would be as large as to fit 3 de Broglie wavelengths. In Sommerfeld's model, electrons in an elliptical orbit would have both radial and azimuthal motions. He assumed that the total radial length traversed by the electron would contain n_r de Broglie wavelengths, while the total azimuthal length covered by the electron would contain k such wavelengths. The total quantum number would be the sum of these two quantum numbers, $n = n_r + k$. For example, for $n = 2$, there would be two possible orbits, one circular orbit, for which $k = 2$ and $n_r = 0$, because there would be no radial motion. The other orbit would be elliptical, with $k = 1$ and $n_r = 1$, because it would involve both radial and azimuthal motions.

In addition, he considered the effect of variation of mass with speed. All these made the calculation of total energy somewhat lengthy, but Sommerfeld came up with



an expression for the total energy of electron in the n, k level of hydrogen, as

$$W(n, k) = -\frac{Rhc}{n^2} \left[1 + \frac{\alpha^2}{n^2} \left(\frac{n}{k} - \frac{3}{4} \right) \right], \quad (4)$$

where R is the Rydberg constant. Sommerfeld found that the energy difference between the levels $(n, k) = (2, 2)$ and $(2, 1)$ could explain the difference in the frequencies of the doublet of the red $H\alpha$ fine structure lines,

$$\Delta\nu = R\alpha^2/16 = 0.365 \text{ cm}^{-1}. \quad (5)$$

He presented his calculations at the Bavarian Academy of Sciences in December 1915 and January 1916. The spectroscopist Friedrich Paschen soon set to work on comparing this prediction with observations. By May 1916, he reported to Sommerfeld that “my measurements are now finished, and they agree everywhere most beautifully with your fine structures”. One month later, Paschen determined the value of α^{-1} as 137.9.

This was when the parameter α got its name ‘fine structure constant’. Sommerfeld’s model was praised as a great progress. Einstein wrote to him a year later that, “Your investigation of the spectra belongs among my most beautiful experiences in physics. Only through it do Bohr’s ideas become completely convincing.” Planck went to the extent of comparing this work with that of the prediction of Neptune’s orbit in astronomy.

However, all this work was superseded by the advent of wave mechanics of Schrödinger⁶, when the classical picture of fixed orbits of electrons was abandoned in favour of a probabilistic wave function. The uncertainly principle pointed out that the classical way of calculating the electron orbit was wrong because the position and velocity could not be determined at any given time. These models could explain the fine structure and much more, without referring to elliptical orbits. For the fine

⁶ *Resonance*, Vol.10, No.12, 2005.



Suggested Reading

- [1] Helge Kragh, *Magic number: a partial history of the fine-structure constant*, *Archive for History of Exact Sciences*, Vol.57, pp.395–431, 2003.
- [2] Arthur Beiser, *Concepts of Modern Physics*, McGraw-Hill, 1973.

⁷ *Resonance*, Vol.15, No.10, 2010.

⁸ *Resonance*, Vol.14, No.4, 1999.

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structure of spectral lines, a new quantum number was invoked, that of the electron ‘spin’, which took the place of Sommerfeld’s ‘ k ’ quantum number. But the role of the fine structure constant in the scheme of the sub-atomic world was already secured, and it keeps appearing in all expressions of energy levels in atoms. It is now viewed as one of the ‘coupling constants’ of Nature. The force of gravity couples all particles with the Newton’s gravitational constant G . Similarly, one can think of the fine structure constant being a parameter that couples all charged particles. This is how the electromagnetic force manifests itself. Similarly one can consider two more coupling constants for the weak and strong forces which tell us the relative strength of these forces.

Since the value of α is important for the electronic energy levels in atoms, scientists have wondered what would have happened if its value had been different. In the 1950s, astronomers Fred Hoyle⁷ and others worked out the detailed process with which stars produce heavy elements such as carbon, oxygen and so on. They found that the abundance of carbon in the Universe could be explained only if the fine structure constant had a value that made the nuclei of helium atoms far more likely to fuse to produce carbon nuclei than they otherwise would be. This enhanced probability of fusion reaction, called a ‘resonance’, made it possible for carbons to be produced, and then other heavier atoms like oxygen, etc. It appears that this resonance is a key to the production of all heavy elements in the universe, and consequently, of the emergence of carbon-based life form.

What then determines the value of α ? Are there hidden dimensions in Nature that somehow fix its value? Some scientists think so. But the enigma of α remains. As one of the students of Sommerfeld, Wolfgang Pauli⁸ wrote about α in 1948: “The theoretical interpretation of its numerical value is one of the most important unsolved problems of atomic physics.”

