Fermi Transport

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Spinning particles, geometric transport laws.

This article introduces Fermi Transport, starting with elementary examples and slowly rising in level. A number of exercises are suggested for the reader's active participation.

We learn in high school that when linearly polarised light passes through a sugar solution, its plane of polarisation is rotated. What is perhaps less well known is that the same thing happens when light is passed through a helically coiled optical fibre. This is due to a geometric effect. The polarisation vector $\mathbf{E}$ is perpendicular to the direction of propagation $\mathbf{k}$ of the light beam. If the beam changes direction, and $\hat{\mathbf{k}}$ swings towards the $\mathbf{E}$ vector, the polarisation vector rotates with it in the $\mathbf{E}-\mathbf{k}$ plane, maintaining orthogonality $\mathbf{E}\cdot\mathbf{k} = 0$. The three directions $(\hat{\mathbf{k}}, \hat{\mathbf{E}}, \hat{\mathbf{k}}\times\mathbf{E})$ form a right-handed frame. If $\hat{\mathbf{k}}(s)$ describes a closed curve on the sphere of directions, (*Figure* 1), the electric vector is found rotated by an angle equal to the signed area enclosed by the curve $\hat{\mathbf{k}}(s)$. This is due to the curvature of the sphere of directions. In

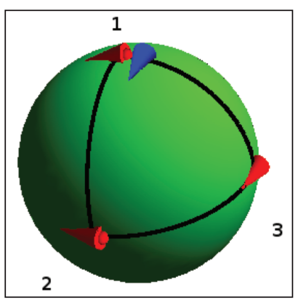

Figure 1. It shows the sphere of directions in green, with three radial directions labeled 1, 2, 3. The polarisation vectors are shown in red perpendicular to the radius vector and therefore tangential to the sphere. If the direction of a light ray changes from 1 to 2 to 3 to 1, the polarisation vector is rotated as shown. On its return to the 1 direction, the polarisation vector is as shown in blue, rotated relative to the initial polarisation shown in red. The angle of rotation is equal to the area of the geodesic triangle 1-2-3-1, which in this case is an octant of the sphere $4\pi/8$, i.e., $\pi/2$, which is a right angle. This is also equal to the angle excess – the sum of the angles of the geodesic triangle 1231 minus π.

the special case of the helix mentioned above, the propagation direction describes a small circle on the sphere (a circle of latitude), which encloses an area per turn of $\alpha = 2\pi(1 - \cos\theta)$ (where θ is the polar angle, or co-latitude) and gives rise to a rotation of the plane of polarisation by an angle α.

Geometric effects rotate the plane of polarisation just as a sugar solution does.

Light is a travelling electromagnetic wave described by Maxwell's equations. These equations tell us that light is a transverse wave: the direction of the electric vector is perpendicular to the direction of propagation $\hat{\mathbf{k}}$. If light travels in a straight line, the electric vector remains parallel to itself and orthogonal to the direction of propagation. If the light changes direction slowly (over many wave lengths) the electric vector must also change direction in order to maintain orthogonality. Suppose that light, initially travelling along $\hat{\mathbf{k}}_1$ with polarisation $\hat{\mathbf{E}}_1$ ($\hat{\mathbf{E}}_1 \cdot \hat{\mathbf{k}}_1 = 0$), changes direction infinitesimally to a neighboring direction $\hat{\mathbf{k}}_2$. What would the polarisation at $\hat{\mathbf{k}}_2$ be? Our first guess, $\hat{\mathbf{E}}_1$ is definitely wrong because it is not in general orthogonal to $\hat{\mathbf{k}}_2$. A geometrically natural guess is to pick *the* vector *nearest* to $\hat{\mathbf{E}}_1$ from all the vectors orthogonal to $\hat{\mathbf{k}}_2$. In other words, minimise $(\mathbf{E}_2 - \hat{\mathbf{E}}_1)^2$ subject to the constraint $\mathbf{E}_2 \cdot \hat{\mathbf{k}}_2 = 0$. Using Lagrange multipliers, we find

$$\mathbf{E}_2 - \hat{\mathbf{E}}_1 = \lambda\hat{\mathbf{k}}_2, \tag{1}$$

where λ is determined from the condition $\mathbf{E}_2 \cdot \hat{\mathbf{k}}_2 = 0$ to be $-\hat{\mathbf{k}}_2 \cdot \hat{\mathbf{E}}_1$. So,

$$\mathbf{E}_2 = \hat{\mathbf{E}}_1 - (\hat{\mathbf{k}}_2 \cdot \hat{\mathbf{E}}_1)\hat{\mathbf{k}}_2 = \mathcal{P}_{k_2}\hat{\mathbf{E}}_1\,, \tag{2}$$

where $\mathcal{P}_{k_2}$ is the projector orthogonal to $\hat{\mathbf{k}}_2$:

$$\mathcal{P}_{k_2} = \mathrm{I} - \hat{\mathbf{k}}_2 \otimes \hat{\mathbf{k}}_2\,. \tag{3}$$

Maxwell's equations tell us that light is a transverse wave.

We see from (2) that $\mathbf{E}_2$ lies in the $\hat{\mathbf{E}}_1 - \hat{\mathbf{k}}_2$ plane. A small computation assures us that since $\mathbf{E}_1$ is nearly orthogonal to $\hat{\mathbf{k}}_2$, projection does not alter its length (to

Fermi's transport rule is parallel transport followed by projection.

first order in infinitesimals) and $\mathbf{E}_2$ is also a unit vector. Motivated by this geometric argument, let us define the Fermi derivative along a space curve $\mathbf{x}(s)$ as the ordinary derivative followed by projection orthogonal to the tangent vector $\hat{\mathbf{k}}(s) = \frac{d\mathbf{x}(s)}{ds}$:

$$\frac{D_F \mathbf{E}}{ds} = \mathcal{P}_{\hat{k}(s)} \frac{d\mathbf{E}}{ds}\,. \tag{4}$$

Using $\hat{\mathbf{k}}(s) \cdot \hat{\mathbf{E}}(s) = 0$, we can write

$$\frac{D_F \mathbf{E}}{ds} = \frac{d\mathbf{E}}{ds} + \hat{\mathbf{k}} \left(\frac{d\hat{\mathbf{k}}}{ds} \cdot \hat{\mathbf{E}} \right)\,. \tag{5}$$

We arrive at Fermi transport by requiring that the Fermi derivative of the polarisation vector vanishes along a curve. This leads to the Fermi transport rule

$$\frac{d\hat{\mathbf{E}}}{ds} = -\hat{\mathbf{k}} \left(\frac{d\hat{\mathbf{k}}}{ds} \cdot \hat{\mathbf{E}} \right), \tag{6}$$

which ensures that $\hat{\mathbf{k}} \cdot \hat{\mathbf{E}} = 0$ is maintained if it was true initially. Along straight lines, polarisation is transported parallel to itself, but along curved lines, it changes according to (6).

Adding a term which vanishes and switching to index notation, we can rewrite Fermi's rule as

$$\frac{d\hat{E}^a}{ds} = \left(\frac{d\hat{k}^a}{ds}\, \hat{k}^b - \hat{k}^a\, \frac{d\hat{k}^b}{ds} \right) \hat{E}_b \tag{7}$$

or

$$\frac{dE^a}{ds} = A^{ab}\, E_b, \tag{8}$$

where $A^{ab} = \left(\frac{d\hat{k}^a}{ds}\, \hat{k}^b \, - \hat{k}^a\, \frac{d\hat{k}^b}{ds} \right)$ is an antisymmetric tensor describing the rotation of our right-handed frame $(\hat{\mathbf{k}}, \hat{\mathbf{E}}, \hat{\mathbf{k}} \times \hat{\mathbf{E}})$. This form brings out the fact that the transport rule is a rotation and preserves inner products. This transport rule (equivalently written as (6) or (7)) is an elementary example of Fermi transport.

The transport law for the electric field (8) was written down by the geometrical consideration that we wish to maintain **k** and **E** orthogonal, since light is a transverse wave. This law of transport can also be derived from Maxwell's equations. The gentle reader is encouraged to perform the derivation using the method described in *Box* 1.

The Fermi transport rule can be derived from Maxwell's equations.

Box 1.

For the reader's entertainment, we suggest that she derives the transport law[1] starting from Maxwell's equations. First introduce the vector potential $\vec{A}$ to solve the homogeneous Maxwell equations. Then reduce the equation to a time-independent situation by assuming a monochromatic wave of frequency ω travelling in a constant background. Replace time derivatives by $-i\omega$ ($\frac{\partial}{\partial t} \to -i\omega$) and so the electric field $\vec{E}$ is replaced by $i\omega\vec{A}$. For simplicity assume that there is no optical activity or magnetic effects in space, only a refracting medium like glass or optical fibre. Set the magnetic permeability to 1, $\mu = 1$, and assume $\epsilon(x)$ to be slowly varying function of space (constant over many wavelengths). Plug in the form

$$\vec{A}(\vec{x}) = \vec{\alpha}(\vec{x})e^{i\omega\sigma(x)}$$

and use the high frequency eikonal approximation (assume ω is large). The leading equations (highest order in ω) tell us that $\vec{\alpha}\cdot\vec{k} = 0$ (the wave is transverse) and that the wave vector $\vec{k} = \omega\vec{\nabla}\sigma$ satisfies $\vec{k}\cdot\vec{k} = \omega^2\epsilon$ (this gives the dispersion relation). Computing $k^a\partial_a k_b = k^a\partial_a\partial_b\omega\sigma = k^a\partial_b k_a = \partial_b(k^a k_a/2)$ we find

$$k^a\partial_a k^b = \partial_b\left(\frac{\omega^2\epsilon}{2}\right) \tag{i}$$

that gradients in $\epsilon(x)$ causes refraction. Invoking the subleading terms we get the transport law for the polarisation

$$2k^b\partial_b\alpha^a = k^a(\partial_b\alpha^b) - \alpha^a\partial_b k^b\,, \tag{ii}$$

which describes the turning of the polarisation vector and the amplification of the wave due to focussing. From the subleading equations, we also have $\alpha^a\partial_a\epsilon(x) + \epsilon\partial_a\alpha^a = 0$ which can be used to express the divergence of α in terms of gradients of $\epsilon(x)$. We find

[1] G Călugăreanu, *Czech. Math. J.*, Vol.11, p.588, 1961.

Box 1. Continued...

Box 1. Continued...

$$k^b \partial_b \alpha^a = -\frac{k^a \alpha^b \partial_b \epsilon}{2\epsilon} - \alpha^a \partial_b k^b \,. \qquad \text{(iii)}$$

Defining unit vectors $(\hat{k}, \hat{\alpha})$ we find that $\frac{d\hat{k}_b}{ds} = \hat{k}^a \partial_a \hat{k}_b = -1/2\mathcal{P}_k \partial_b(\ln \epsilon)$ and $\hat{k}^a \partial_a \hat{\alpha}_b = -\hat{k}_b \frac{d\hat{k}_a}{ds} \hat{\alpha}_a$, the Fermi transport law in three-dimensional form. This form was given by S M Rytov[2]. We remark that this derivation can be repeated for any waves, e.g., seismic waves (the P waves are transverse and Fermi transported) or electron waves described by the Dirac equation. The derivation is tedious, but entirely straightforward.

[2] L D Landau and E M Lifshitz, *Electrodynamics of Continuous Media,* Pergamon Press, 1960.

Let us now move from light rays propagating in space to massive particles propagating in flat space–time. The direction of propagation is now a four-vector p^μ (where $\mu = 0, 1, 2, 3$, we use signature $(+, -, -, -)$) and the spin of the particle is given by a vector S^μ orthogonal to p^μ $(p \cdot S = 0)$. The particle could be an electron, a gyroscope or a planet. As is usual we define spin as a spatial vector in the rest frame of the particle and move this description to other frames by Lorentz transformation. The four-momentum p^μ satisfies $p^\mu p_\mu = m^2$, where m is the rest mass of the particle, which we will henceforth set to 1. If the four momentum p^μ changes, the spin vector S^μ must change with it maintaining orthogonality $S \cdot p = 0$. As one can guess, the natural rule for transporting S along the world line of the spinning massive particle is (remember, we set $m = 1$, and also the speed of light $c = 1$, it makes life simpler and τ is the proper time along the particle world line)

$$\frac{dS^\mu}{d\tau} = A^{\mu\nu} S_\nu, \qquad (9)$$

where $A^{\mu\nu} = \left(\frac{dp^\mu}{d\tau}\, p^\nu - p^\mu\, \frac{dp^\nu}{d\tau}\right)$ is a second rank antisymmetric tensor. This rule (9) is Fermi transport. One can view $A^{\mu\nu} d\tau$ as the generator of the Lorentz transformation that 'rotates' $(p^\mu(\tau), S^\mu(\tau))$ to the pair $(p^\mu(\tau + d\tau),\ S^\mu(\tau + d\tau))$, in the plane containing both p's, maintaining orthogonality $p \cdot S = 0$. This rule (9)

Spin is a four-vector orthogonal to the momentum four-vector.

actually describes the behaviour of gyroscopes in accelerated frames, the spin of an electron in an atom and the spin of an orbiting neutron star.

Apart from being physically relevant, the rule is geometrically natural. Let p^μ be a four-momentum of a massive particle: a time-like, future-pointing vector of unit size: $p^0 > 0$, ${p^0}^2 - \vec{p} \cdot \vec{p} = 1$. Let H_p be the set of vectors orthogonal to p^μ : $H_p = \{S | S \cdot p = 0\}$. H_p can be identified with 'space' for an observer in the rest frame of the particle.

Suppose now we have two such four-momenta p_1 and p_2, how can one compare vectors in H_{p_1} and H_{p_2}? The natural choice is to identify[1] the two spaces by means of the unique Lorentz transformation that takes p_1 to p_2 and leaves the subspace orthogonal to both vectors unchanged. This transformation is given by the matrix

[1] To identify two points means to regard them as the same. For example, identifying the two ends of your shoe lace would make it into a loop. In the present case we identify points in the two spaces H_{p1} and H_{p2} by the rule described in the text.

$$\Lambda^\mu_{21\nu} = \delta^\mu_\nu - \frac{(p_1^\mu + p_2^\mu)(p_{1\nu} + p_{2\nu})}{1 + p_1 \cdot p_2} + 2 p_2^\mu \, p_{1\nu}. \qquad (10)$$

Λ is a Lorentz transformation: it preserves the Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$. That is, $\Lambda^\mu_\alpha \, \Lambda^\nu_\beta \eta_{\mu\nu} = \eta_{\alpha\beta}$. It takes p_1 to p_2: $\Lambda^\mu_{21\nu} p_1^\nu = p_2^\mu$ and $\Lambda_{21} = \Lambda_{12}^{-1}$. It is the identity on vectors orthogonal to the linear subspace spanned by p_1 and p_2. As p_2 approaches p_1 along a curve $p(\tau)$ with tangent vector $\frac{dp}{d\tau}$, $\Lambda^\mu_{21\nu}(\tau)$ approaches the identity with tangent

$$A^\mu_\nu = \frac{d\Lambda^\mu_{21\nu}}{d\tau} = \left(\frac{dp^\mu}{d\tau} \, p_\nu - p^\mu \frac{dp_\nu}{d\tau} \right).$$

The rule for comparison of vectors reduces to the transport rule (9).

Given three vectors p_1, p_2, p_3 on the unit hyperboloid, one can identify H_{p_1} with H_{p_2} and H_{p_2} with H_{p_3} using the above rule. However, this rule does not close; the result of combining $\Lambda_{13} \, \Lambda_{32} \, \Lambda_{21} = \Gamma$ is not the identity. Γ leaves p_1 invariant ($\Gamma p_1 = \Lambda_{13} \, \Lambda_{32} \, p_2 = \Lambda_{13} p_3 = p_1$)

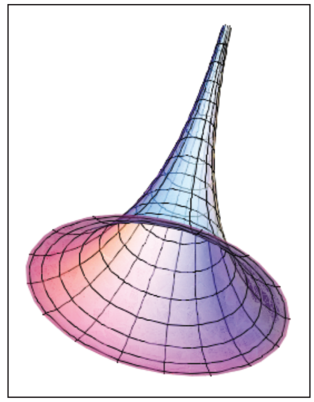

Figure 2. The Pseudosphere: It shows part of a two-dimensional surface of constant negative curvature embedded in three-dimensional space. Unlike the sphere, the whole of the pseudosphere cannot be described as a subset of Euclidean space without self intersections. We show a part of the pseudosphere. However, the whole of the pseudosphere can be embedded in Minkowski space. In fact the mass shell of a relativistic particle is an example of such embedding. The sum of the angles of a geodesic triangle on a pseudosphere add up to less than π radians. Triangles have angle deficits rather than angle excesses due to the negative curvature. The pseudosphere is an example of hyperbolic geometry, which has fascinated artists like M C Escher, whose work you can find on the internet.

Geometrically natural answer gives the physically correct answer for transport.

and is therefore a pure rotation of H_{p_1}. This is called the Thomas rotation. The lack of integrability can be traced to the curvature of the unit hyperboloid $p_0^2 - p_1^2 - p_2^2 - p_3^2 = 1$. Unlike the sphere which has uniform positive curvature, the hyperboloid has uniform negative curvature. (See *Figure* 2.) It is striking that the geometrically natural argument gives the physically correct answer for the transport of spins! This is no accident, the geometry encodes the symmetry of the Lorentz group and any relativistic equations of motion that one writes down for the spin will repect Lorentz invariance.

This discussion can be moved to the curved space–time of general relativity *in toto*. The metric of space–time is no longer Minkowskian, but one still has a Lorentzian metric in each tangent space. The transport law given by Fermi is:

$$\frac{DS^\mu}{d\tau} = A^{\mu\nu} S_\nu \,,$$

where all that has been done is to replace the ordinary derivative by the covariant derivative.

The earth's axis precesses ever so slightly due to Fermi transport.

What are the physical consequences of Fermi transport? Just as a helical fibre rotates the plane of polarisation of light, the spin of a massive particle precesses, if the particle is in accelerated motion *even if no torques are applied in the instantaneous rest frames of the particle.* This is an elementary consequence of the fact that

Lorentz transformations do not in general commute. As an example, consider a massive spinning particle following a helical world line. In equations $x(\tau) = \gamma R \cos \omega\tau$, $y(\tau) = \gamma R \sin \omega\tau$, $z(\tau) = 0$, $t(\tau) = \gamma\tau$. This describes a particle in circular orbit of radius R and speed $v = \tanh \eta$, (where η is the rapidity) and $\gamma = \cosh \eta = 1/\sqrt{1 - v^2}$). We would expect that every time the particle completes a circle, the spin direction would rotate in the $x - y$ plane by an angle $\alpha = 2\pi(1 - \cosh \eta) = 2\pi(1 - \gamma)$. Note that α is negative here since γ exceeds one. As one can see from the formula for α, the effect is appreciable only at relativistic speeds. Actually, all of us follow helical world lines since we travel with the earth around the Sun in a nearly circular orbit at 30km/sec. Also we all have a gyroscope, in fact we live on one: the spinning earth itself is a gyroscope, whose axis points very nearly to the North Star. The plane of the earth's orbit is called the ecliptic (where eclipses occur) and the earth's rotation axis is tilted by about 23^0 from the normal to the ecliptic. Due to Fermi transport you would find that every time your birthday comes around, the spin axis of the earth rotates by about four millionth of a degree about the normal to the ecliptic. It would take you a million birthdays to work up an appreciable effect. Good luck with that!

There are much larger precession effects that will overwhelm geometric precession, but it is the principle of the thing we want to emphasise here. The effect is tiny since the speed of the earth around the Sun is small compared to the speed of light. However, electrons in the inner shells of heavy atoms move with relativistic speeds. Such geometric precession effects have been observed in atomic physics. In order to correctly account for the fine structure of atomic spectra (the relativistic correction to the spectra) one has to take into account the fact that the electron spin is subject to geometric precession. This effect is called Thomas precession: it

We all live on a spinning gyroscope.

Abstractions help us see what is common to diverse situations.

is a geometrical correction to the precession of electron spin. For a corresponding discussion of massless particles, see *Box* 2.

Fermi was 21-years old when he wrote his paper on Fermi Transport, generalising the idea of parallel transport in general relativity. Fermi's original paper appeared in Italian [1], and is not accessible to most of us, but you can find English language treatments in secondary sources (see suggested reading [2, 3]). The idea was extended in [4] by A G Walker (of Robertson–Walker fame) to set up coordinate systems in general relativity. The transport law (9) is sometimes called Fermi–Walker transport. Although the law was discovered in general relativity, it can be understood in Minkowski space, as we have done. It does not involve curvature of space–time, but only needs non-geodesic time-like curves (in physical terms, accelerated world lines). Its geometric nature can be understood in terms of the curvature of the mass shell hyperboloid or the fact that Lorentz transformations in different directions do not commute. This last feature of special relativity – that Lorentz transformations in different directions do not commute – apparently came as a shock to Albert Einstein when he heard about it!

In this article we have moved from polarised light to relativistic gyroscopes to spinning neutron stars to spinning electrons in the inner shells of atoms. Fermi transport even appears in the theory of DNA elasticity [5, 6] (see *Box* 3). That is quite a range of phenomena, described by the idea of Fermi transport! This is a good example of mathematical abstraction. The power of abstraction is that it 'abstracts' the essence of a phenomenon and then one finds that this essence applies in many different situations. The idea that is abstracted here is the idea of a 'connection', one that dominates many areas of theoretical physics these days. It is curious that the same idea had been arrived at earlier by mathematicians like Riemann, Weyl, Levi-Civita and Cartan.

The geometrical idea of a connection permeates many branches of modern physics.

Box 2.

Fermi transport is a rule for transporting polarisation vectors along a time-like space–time curve. It (i) preserves inner products between vectors, (ii) reduces to parallel transport on geodesic curves and (iii) is not integrable, in the sense that the transport depends on the entire curve and not just the end points. In this respect it differs from the more familiar Serret–Frenet equations of elementary differential geometry which are integrable. Is there a notion of transport of polarisation vectors along *null* non-geodesic curves? The answer, it turns out is yes, but this takes us well beyond the scope of this article. See reference below[1], which describes the transport of polarisation vectors along null, non-geodesic curves. The corresponding transport is locally integrable, unlike Fermi transport. In this respect, it is closer to the Serret–Frenet equations. Unlike Fermi transport (and like Serret–Frenet), transport along null curves does not have a smooth geodesic limit. For a light illustration see *Figure* 3.

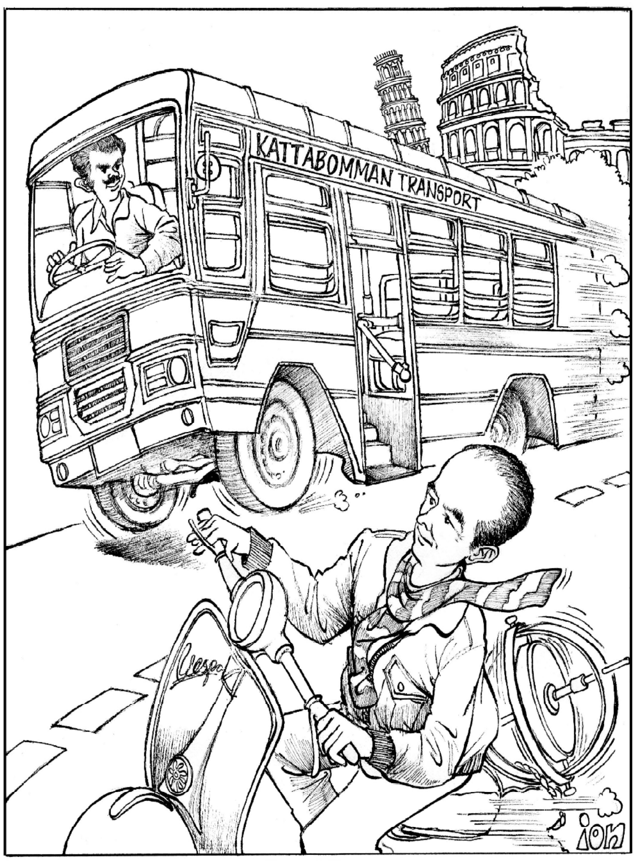

Figure 3. Fermi transport and Kattabomman transport.

[1] J Samuel and Rajaram Nityanada, Transport along Null Curves, *J. Phys*. A., Vol.33, p.2895, 2000.

Box 3.

Take a belt, buckle its ends together and set it on a table. The centerline of the belt (join the holes in the belt and extend this line around) lies in a plane half the belt's thickness above the table. Now unbuckle it, twist the buckle one turn (that is two half turns) buckle it again and set it down on the table. You will notice that the centerline is no longer in a plane. If you twist the buckle two turns, the deviation of the centerline from planarity is even greater.

If you twist the buckle Lk times before buckling it you generate a configuration where the edges of the belt (which are closed circles) are linked Lk times. To see this substitute the belt with a strip of paper or a ribbon and cut down the centerline to see that the two halves of the strip (or ribbon) are linked Lk times. In order to accommodate the impressed link Lk, the belt (or strip or ribbon) responds in two ways: the centerline 'writhes' and the belt twists around the centerline. The impressed link decomposes into twist and writhe

$$\mathrm{Lk} = \mathrm{Tw} + \mathrm{Wr}\,, \tag{i}$$

where Wr is a property of the centerline. Tw is the twisting of the width of the belt around the Fermi transported frame. The manner in which Lk is divided into twist and writhe depends on the elastic constants that resist bending and twisting. The above relation holds for any configuration of the belt. You can see similar effects by wringing a towel.

This result (i) was proved by Călugăreanu. It was independently found by Fuller. Brock Fuller was approached by Vinograd, a molecular biologist who asked if a quantitative description of the configurations of overwound DNA was possible. Fuller's treatment is now used in understanding the molecular biology of DNA molecules. DNA in its cellular environment is subject to torques and forces. Fermi transport appears naturally in this biological context!

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Suggested Reading

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