SLOW LIGHT USING ELECTROMAGNETICALLY INDUCED TRANSPARENCY AND GAUSSIAN STATE FIDELTY IN OPTOMECHANICS

Project Report Submitted to Cochin University of Science and Technology in Partial Fulfilment of the Requirements of the Degree of Master of Science in Photonics (2008-2013)

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INTRODUCTION

The study of interaction between light and matter has a long history and it has been a very important tool in many new technologies. The interactions include absorption, emission, transmission, etc. The interaction of light on two level systems can be considered as a simplest example of atom light interactions. Besides the interactions in a two level system, three level systems have many applications in quantum optics and quantum information theory. Quantum information processing is a fast growing research field and extensive research is going on in three level systems because of its diverse applications.

The following thesis presents a study of basic atomic level systems and electromagnetically induced transparency (EIT) and its applications. Electromagnetically induced transparency is one of the interesting phenomena's in which an optically thick medium become transparent as the result of interaction of light with an atomic three level systems. One of the applications of electromagnetically induced transparency is slow light which is studied in detail. Other applications are quantum information processing, storage of data, enabling quantum computation, etc.

Apart from interaction of light with atomic systems, optomechanical systems for quantum state transfer were also studied. Like electromagnetically induced transparency, optomechanics has many applications in quantum information technology. Study of quantum state transfer in optomechanical cavity shows that high fidelity quantum state transfer can be performed using mechanical dark modes, which is analogous to three level atomic systems in atomic physics.

CHAPTER 1: THEORETICAL BACKGROUND

1.1 INTERACTIONS BETWEEN LIGHT AND MATTER

Light matter interactions in atomic physics are given below.

Absorption: It is a transfer of energy from the electromagnetic radiation to the atoms or molecules of medium. Energy transferred to an atom can excite electrons from lower energy to higher energy states.

Spontaneous emission: An atom in the excited state emits electromagnetic radiation when it transfers to the lower energy state spontaneously. The releasing energy is in the form of a photon, which is emitted in random direction.

Stimulated emission: It is the process of transfer an atom or molecule from a higher energy state to the lower energy state by incoming photon. For stimulated emission, an additional photon is emitted at exactly the same energy (or frequency) as that of the incident photon, and in exactly the same direction in phase with the incident photon.

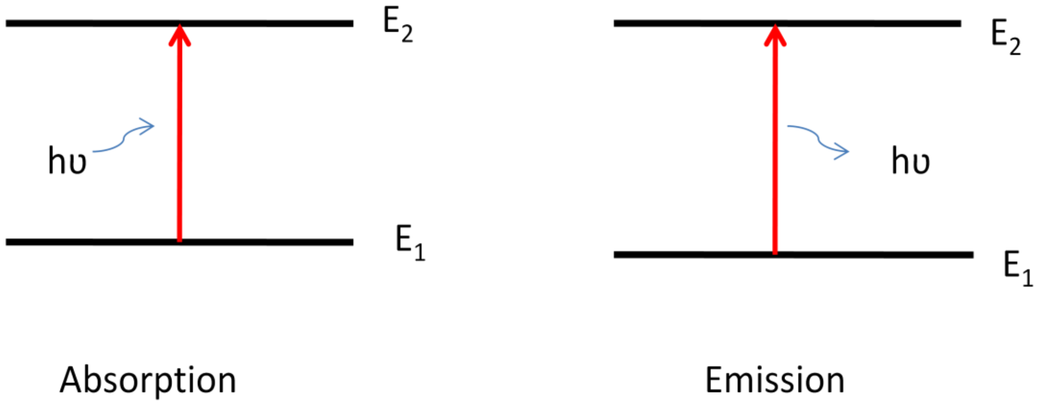


Figure 1: Absorption and Emission of light

Here hυ is the photon energy with h = planck's constant and υ = frequency of light. The energy of the emitted photon (hυ) is the difference between the energy level of the two states.

1.2 TWO LEVEL SYSTEM

A two level system is a system which has two possible quantum states. And two energy levels are separated by a transition with frequency ω_0. The atom is driven by monochromatic plane wave of frequency ω_0.

Hydrogen atom can be considered as two level system in which an electromagnetic wave is interacting with lower ground state and upper excited state. Two level atom is also analogous to a spin ½ system in an external magnetic field with two possible state. The two level atom undergoes optical Rabi- oscillatons under the action of the driving electromagnetic field.

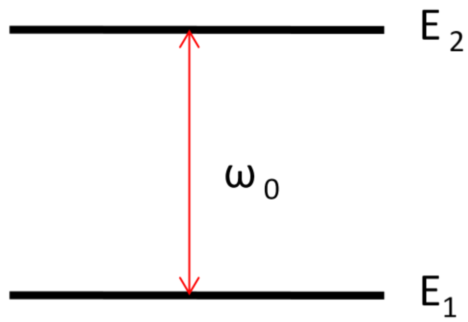


Figure 2: Two level system

Consider a two level system in which the eigenstates and eigenvalues of the Hamiltonian are given by $|\varphi_1\rangle$, $|\varphi_2\rangle$ and E_1,E_2 respectively. Now the time-independent external perturbation changes the Hamiltonian, which is given by

$$H = H_0 + W$$

Where, $W = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix}$

$$H = \begin{bmatrix} E_1 + W_{11} & W_{12} \\ W_{21} & E_2 + W_{22} \end{bmatrix}$$

Then the eigenvalues are given by

$$E\pm = \frac{1}{2}(E_1+W_{11}+E_2+W_{22}) \pm \frac{1}{2} + \sqrt{(E_1+W_{11}-E_2-W_{22})^2+4|W_{12}|^2}$$

And the eigenvectors are

$$|\Psi +\rangle = \cos\frac{\Theta}{2}\, e^{-i\phi/2}\, |\varphi_1\rangle + \sin\frac{\Theta}{2}\, e^{i\phi/2}\, |\varphi_2\rangle$$

$$|\Psi -\rangle = \sin\frac{\Theta}{2}\, e^{-i\phi/2}\, |\varphi_1\rangle + \cos\frac{\Theta}{2}\, e^{i\phi/2}\, |\varphi_2\rangle$$

Where $\tan\Theta = \frac{2|W_{12}|}{E_1+W_{11}-E_2-W_{22}}$ and $W_{12} = |W_{12}|\, e^{i\phi}$

If $W_{11}=W_{22}=0$,

then $E\pm = 1/2\ (E_1+E_2) \pm 1/2\ \sqrt{(E_1-E_2)^2+4|W_{21}|^2} = E_m \pm \sqrt{\Delta^2+|W_{12}|^2}$

Where $E_m = 1/2\ (E_1+E_2)$, and $\Delta = 1/2(E_1 - E_2)$

The graph corresponding to the values, $E_m=0$, $E_1=\Delta_1$ and $E_2=\Delta_2$ is shown below.

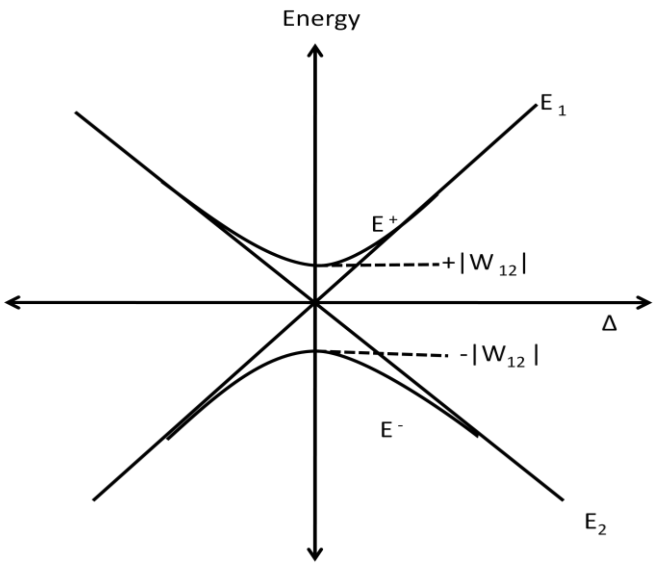


Figure 3: Energy spectrum versus Δ

Here 2Δ represents the difference between the eigenvalues of E_1 and E_2.We can see that the energy levels repel each other in the presence of perturbation.

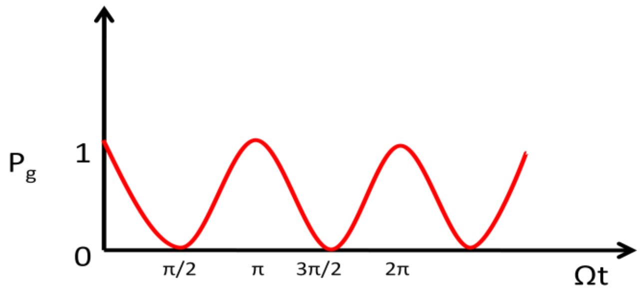
The probability of finding the system at time t in state $|\varphi_2\rangle$is given by

$$P_{12}(t) = |\langle\varphi_2|\Psi(t)\rangle|^2$$

$$\rho_{12}(t) = \frac{4|W12|^2}{4|W12|^2+(E1-E2)^2} \sin^2 \sqrt{(E_1 - E_2)^2\, t/2\hbar + 4|W12|^2} \qquad \text{Rabi's formula,}$$

$\rho_{12}(t)$ oscillates with frequency, $\Omega = (E_+ - E_-)/\hbar$ called as Rabi frequency.

It varies between 0 and some maximum value. If $E_1 = E_2$, then the system which starts in $|\varphi_1\rangle$ will at some later time be found in state $|\varphi_2\rangle$ with certainty. Rabi oscillations are damped. When the coupling is weak, oscillations between the state $|\varphi_1\rangle$ and $|\varphi_2\rangle$ do not have time to occure.



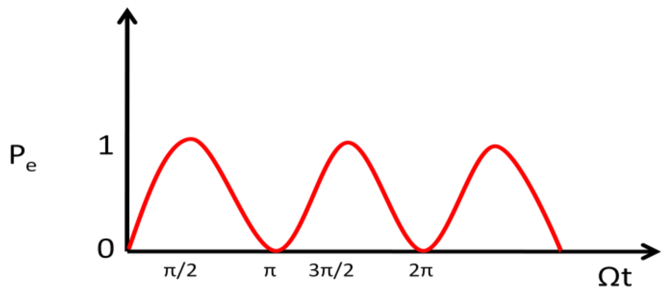


Figure 4: Evolution of occupation probabilities of ground and excited state

1.3 THREE LEVEL SYSTEM

The classic examples for the configuration of energy levels are Λ, V and Ladder/Cascade.

Lambda (Λ) scheme:

The atomic system in which two lower energy states are coupled optically with an excited state. Optical coupling between two lower energy states is forbidden.

Vee (V) scheme:

The system in which the lower energy state is optically coupled to two higher energy states and the optical coupling between two higher energy states is forbidden.

Ladder scheme:

The atomic system in which the lower energy state is coupled to the higher energy state through an intermediate state. Optical coupling between lower and higher energy states is forbidden.

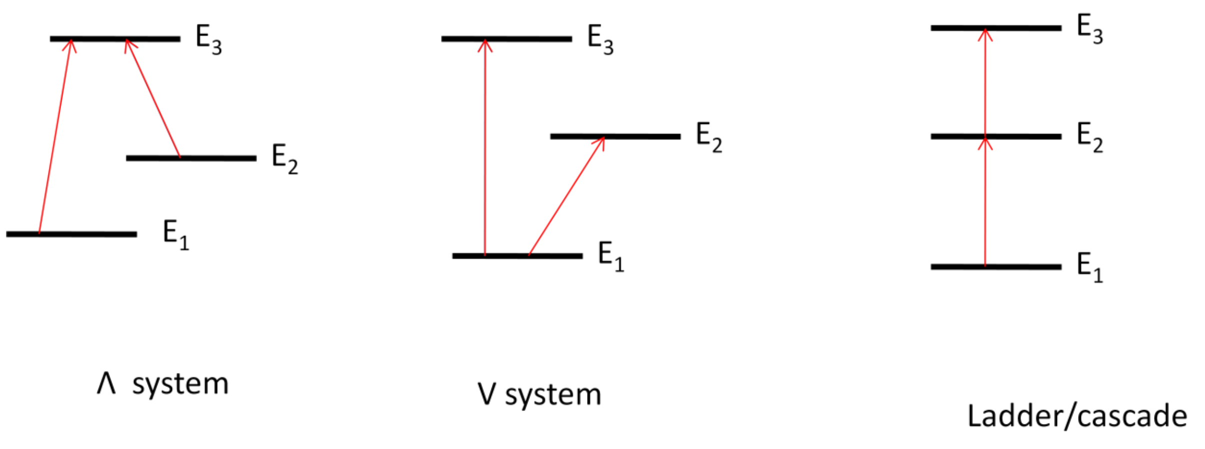


Figure 5: Three level atomic systems

1.4 THREE LEVEL LAMBDA (Λ) SYSTEM

Lambda (Λ) configuration in which two lower levels $|b\rangle$ and $|c\rangle$ are coupled to a single upper level $|a\rangle$. Consider a Λ scheme that consisting of the energy states $|a\rangle$, $|b\rangle$ and $|c\rangle$ coupled by two near resonance laser fields of strength obtained in terms of the Rabi frequency Ω_1 (at frequency ω_1) and Ω_2(at frequency ω_2). The dipole interaction between the two lower energy levels is forbidden. Then the Hamiltonian of the system is given by

$$H = H_0+V_1+V_2$$

Where, $H_0 = \hbar\omega_a |a\rangle\langle a| + \hbar\omega_b |b\rangle\langle b| + \hbar\omega_c |c\rangle\langle c|$

$V_1 = \hbar\Omega_1 \exp(-i\omega_1 t)/2 \, |b\rangle\langle a| + \hbar\Omega_1 \exp(i\omega_1 t)/2 \, |a\rangle\langle b|$

$V_2 = \hbar\Omega_2 \exp(-i\omega_2 t)/2 \, |c\rangle\langle a| + \hbar\Omega_2 \exp(i\omega_2 t)/2 \, |a\rangle\langle c|$

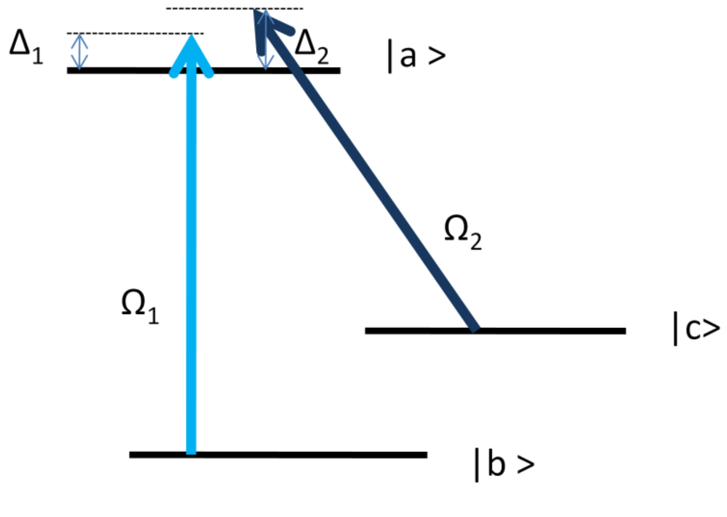


Figure 6: Three level lambda system

In the case when the two fields are in resonance, i.e..

$$\omega_1 = \omega_a-\omega_b \text{ and } \omega_2 = \omega_a-\omega_c$$

the three eigenstates of H are:

$|C_1\rangle = (1/2^{1/2})\,[-|\,a\rangle + (\Omega_1/(\,\Omega_1^{\,2} + \Omega_2^{\,2})^{1/2})\,|\,b\rangle + (\Omega_2/(\,\Omega_1^{\,2} + \Omega_2^{\,2})^{1/2})\,|\,c\rangle]$

$|C_2\rangle = (1/2^{1/2})\,[\;|\,a\rangle + (\Omega_1/(\,\Omega_1^{\,2} + \Omega_2^{\,2})^{1/2})\,|\,b\rangle + (\Omega_2/(\,\Omega_1^{\,2} + \Omega_2^{\,2})^{1/2})\,|\,c\rangle]$

$|\,NC\rangle = (\Omega_2/(\,\Omega_1^{\,2} + \Omega_2^{\,2})^{1/2})\,|\,b\rangle - (\Omega_1/(\,\Omega_1^{\,2} + \Omega_2^{\,2})^{1/2})\,|\,c\rangle$

$|\,NC\rangle$ contains no component of $|\,a\rangle$ and hence there is no coupling between $|\,NC\rangle$ and $|\,a\rangle$. Thus any population $|\,NC\rangle$ is trapped in that state. This state is called as dark state. Over a period of time, dependent upon the rate of spontaneous emission from the excited state, all of the population of the system will build up in $|\,NC\rangle$. Hence all of the population becomes coherently trapped in a dark state.

When $\Omega_1 << \Omega_2$

Then the coupling and non coupling states become:

$$|\,C\rangle = (1/2^{1/2})\,|C_1\rangle + (1/2^{1/2})\,|C_2\rangle$$

$$|\,C\rangle = (\Omega_2/(\,\Omega_1^{\,2} + \Omega_2^{\,2})^{1/2})\,|\,c\rangle$$

$$|\,C\rangle \approx |\,c\rangle$$

And $$|\,NC\rangle = (\Omega_2/(\,\Omega_1^{\,2} + \Omega_2^{\,2})^{1/2})\,|\,b\rangle$$

$$|\,NC\rangle \approx |\,b\rangle.$$

So the ground state itself becomes the dark state.

Coherent population trapping:

Coherent population trapping leads to the generation of ‘dark state’. The population is trapped in the lower states and there is no absorption even in the presence of the field. This is because of the reason that after being pumped into the dark state, the atoms cannot be excited by either laser field.

1.5 ELECTROMAGNETICALLY INDUCED TRANSPARENCY

Electromagnetically induced transparency is a destructive quantum interference phenomena in which optically thick medium become transparent. The first experimental observation of EIT was made in 1990 by Harris in a Strontium vapour. Electromagnetically induced transparency can be produced by the interaction of strong coupling beam and weak probe beam on a three level Λ system. At two photon resonance all the atoms are trapped in a lower energy state which is called as a dark state. Since the dark state does not involve the excited state, probe beam does not get absorbed further. So the transparency of the light is maximum. This is called Electromagnetically Induced Transparency (EIT).

Any optical process in a medium, driven by radiation field is determined by the polarisation, which is induced by the laser fields. Polarisation is given by the equation,

$$P(\omega) = \epsilon_0 \chi(\omega) E.$$

The real and imaginary parts of the linear susceptibility relates to dispersion and absorption of the medium respectively. At two photon resonance condition, absorption of the probe beam vanishes and then dispersion is modified, which leads to a large effect on refractive properties of the medium

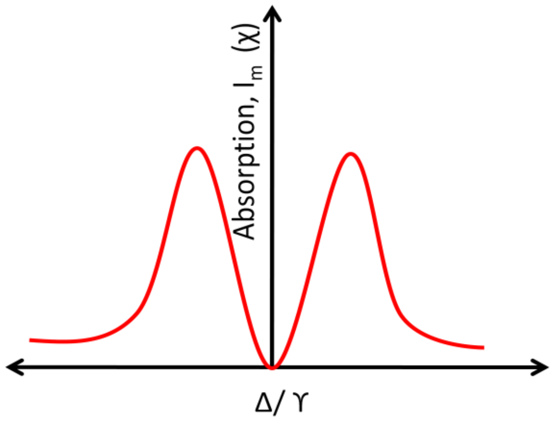


Figure 7: Absorption of probe beam versus optical detunin

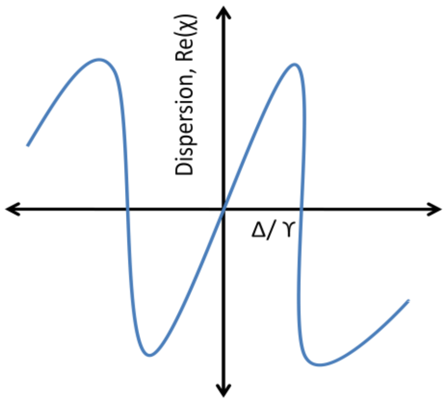


Figure 8: dispersion versus optical detuning

1.6 APPLICATIONS OF EIT

Electromagnetically induced transparency can be used in many fields like quantum optics, nonlinear optics, quantum information processing, etc., The applications of electromagnetically induced transparency include inducing transparency in optically thick medium, enabling four wave mixing, generating changes in the refractive index properties, promoting slow light, enabling quantum computation and information storage. The EIT is also used in lasing without inversion and high precision magnetometry.

CHAPTER 2: SLOWING OF LIGHT

2.1 INTRODUCTION

The reduction in group velocity of the light that passes through a medium is called slowing of light. Group velocity of light passing through a medium changes due to the refractive index of the medium. Slow light has applications in many diverse fields like optical communications, interferometry, sensing, etc. Electromagnetically induced transparency (EIT) is one of the mechanisms which can produce slow light, because EIT controls the refractive index of the medium driven by laser.

Slow group velocities have been achieved in a variety of media including atomic vapours and solids. Steep linear dispersion in the medium leads to large group index which in turn leads to small group velocity. By optically tuning the dispersion characteristics we can control group velocity of light in the medium driven by laser field. Hence, optical control has been applied to achieve slow light, stopped light and guided light. Our interest is only on slowing of light by electromagnetically induced transparency.

The work presented focuses on achieving slow light as a result of electromagnetically induced transparency. Here we have worked on a double lambda system which can produce a narrow transparency window. The experiment on Rubidium produces better results because of the fact alkali vapours have large interaction cross section per atom, well defined energy level structure, long interaction times, and the large optical depths that can be achieved.

2.2 THEORY OF SLOW LIGHT

Slowing of light means group velocity of light becomes smaller than velocity of light in vacuum, ie. $v_g < c$. The group velocity of the light relates to the group refractive index of the medium which is given as $v_g=c/n_g$. The rapid change of refractive index in a region of rapidly changing absorption associated with EIT leads to the steep linear dispersion at the centre of the transparency window which results in slowing of light. A narrow transparency window gives rise to strong dispersion which strongly reduces the group velocity for the light that passes through the region where EIT is produced.

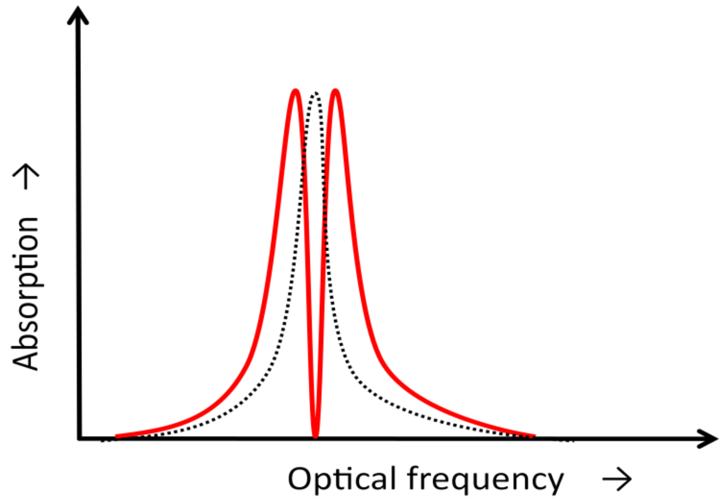


Figure 9: Absorption versus optical frequency

Consider a monochromatic plane wave propagating through a medium of refractive index n.

Electric field of wave is given by

$$E\ (z,t)= A\ e^{i(kz-\omega t)} + c.c$$

Where $k= n\omega/c$ and w is the angular frequency of the wave.

Phase velocity v_p is the velocity with which the points of constant phase move through the medium, and it is given as

$$v_p = \Delta z/\Delta t \quad \text{or} \quad v_p = \omega/k = c/n.$$

Group velocity gives the velocity with which a pulse of light propagates through a material or a medium. It is given by the equation,

$$v_g = d\omega/dk = c/(n+\omega \frac{dn}{d\omega}) = c/n_g \,,$$

where $n_g = n+\omega \frac{dn}{d\omega}$.

We can see that group index differs from the phase index by a term that depends on the dispersion $dn/d\omega$ of the refractive index.

Small group velocity for large dispersion implies that light travels more slowly. Slow light gives rise to a group delay which is defined as

$$\tau_d = l\,(1/v_g - 1/c) = l/c\,(n_g - 1).$$

Where l is the length of the dispersive region and $n_g = c/v_g$,group velocity index. For $n_g >> 1$, group velocity become very small so that $c\,\tau_d/l >> 1$. This corresponds to a situation in which a pulse experiences a long delay relative to a pulse travelling at c.

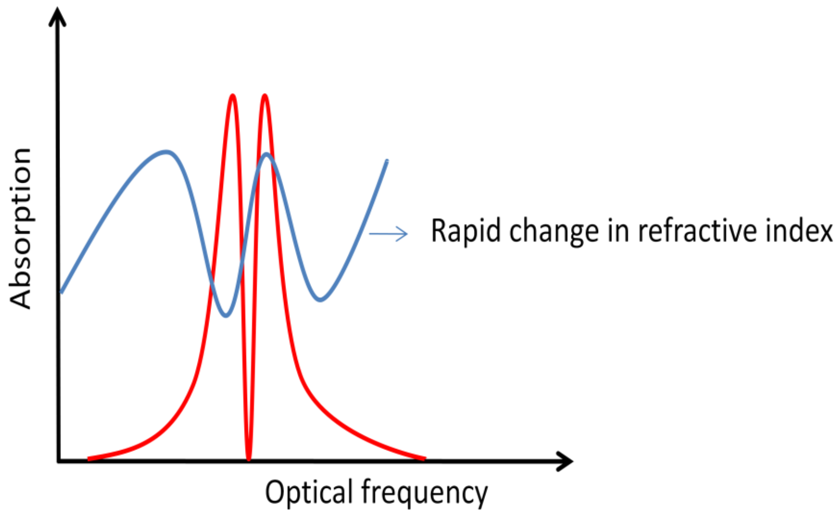


Figure 10: Rapid change of refractive index in the transparency window

Any optical process in a medium, driven by radiation field is determined by the polarisation, which is induced by the laser fields. Polarisation is given by the equation,

$$P(\omega) = \epsilon_0 \chi(\omega) E.$$

The imaginary part of the linear susceptibility, $I_m (\chi^1)$ gives the homogeneous evolution of the field amplitude and it determines spectroscopic features like absorption and gain. The real part of susceptibility, Re (χ^1) determines matters related to the index of refraction such as phase and group velocities.

The dispersion of the medium is given by $\frac{d\,Re\,(\chi 1)}{d\omega}$, and the refractive index of medium can be expressed as a function of linear susceptibility which is represented as,

$$n = \sqrt{1 + Re(\chi^1)}\ .$$

In EIT mechanism, absorption is nearly zero close to resonance and it modifies dispersion properties which has a significant effect on refractive index of the medium that leads to slow light within the transparency window.

2.3 ATOMIC STRUCTURE OF RUBIDIUM (^{85}Rb)

Rubidium is an alkali group atom with atomic number 37. Its electronic configuration is $1S^2\ 2S^2\ 2P^6\ 3S^2\ 3P^6\ 3d^{10}\ 4S^2\ 4p^6\ 5S^1$. ^{87}Rb (I=3/2) and ^{85}Rb (I=5/2) are stable isotopes of Rubidium. The abundance of ^{85}Rb is 72% and that of ^{87}Rb is 28%.

For ground state in ^{85}Rb, L=0 and S=1/2. Then hyperfine levels are given by $|J-I| < F < |J+I|$, where J=1/2 and I= 5/2, therefore F takes values $2 \leq F \leq 3$. For first excited state of ^{85}Rb, L= 1, S = ½ and J=1/2 or 3/2. For J= ½, hyperfine level are $2 \leq F \leq 3$. F takes values $1 \leq F \leq 4$ for J= 3/2.There are two prominent transitions called D_1 line ($5S_{1/2} \leftrightarrow 5P_{1/2}$) transition and D_2 line

($5S_{1/2} \leftrightarrow 5P_{3/2}$) transition. It is easier to build up ground state coherences using D_1 line rather than D_2 line. However, due to electric dipole selection rules, only transitions with $\Delta S=0$, $\Delta J=0, \pm 1$ are allowed. And hyperfine transitions must have $\Delta F=0, \pm 1$.

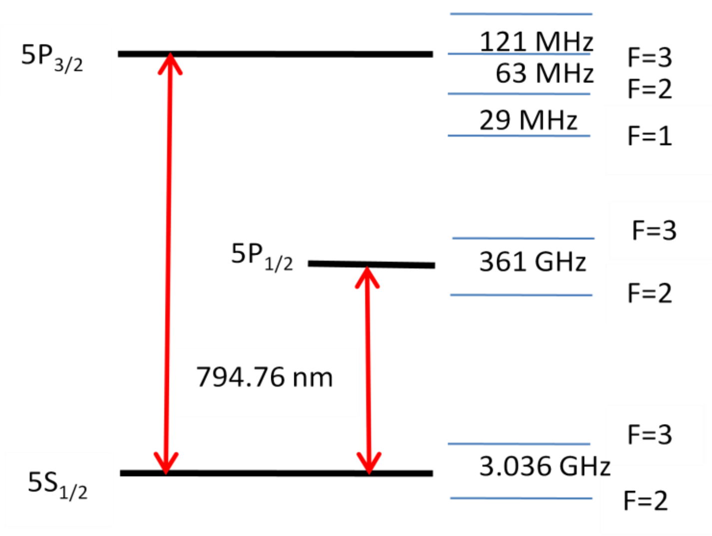


Figure 11: Atomic structure of ^{85}Rb

2.4 DOUBLE LAMBDA SYSTEM

Here we are only concerned about D_1 line ($5S_{1/2} \leftrightarrow 5P_{1/2}$) transition. For $5S_{1/2}$, F can take two values 2 and 3. For F=2, there are 2F+1 levels from -2 to +2. And for F=3, values are -3 to +3. The double lambda system can be formed by F=2 and F=3 of $5P_{1/2}$ and F=3 state of $5S_{1/2}$. It is shown in the diagram.

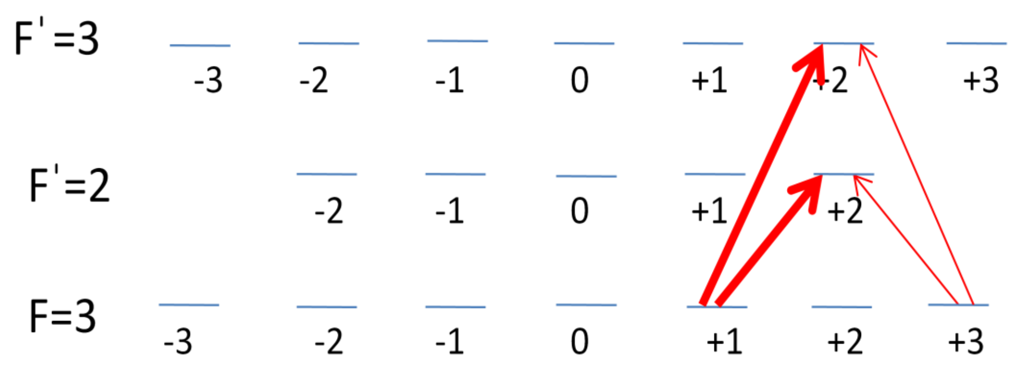


Figure 12: Hyperfine level splitting

Now we can select a lambda system by considering selection rule $\Delta m_F = \pm 1$. We know that different polarisations drive different energy level transitions. So Δm_F =-1and Δm_F = +1 transitions are driven by left (σ^-) circularly polarised and right (σ^+) polarised light respectively.

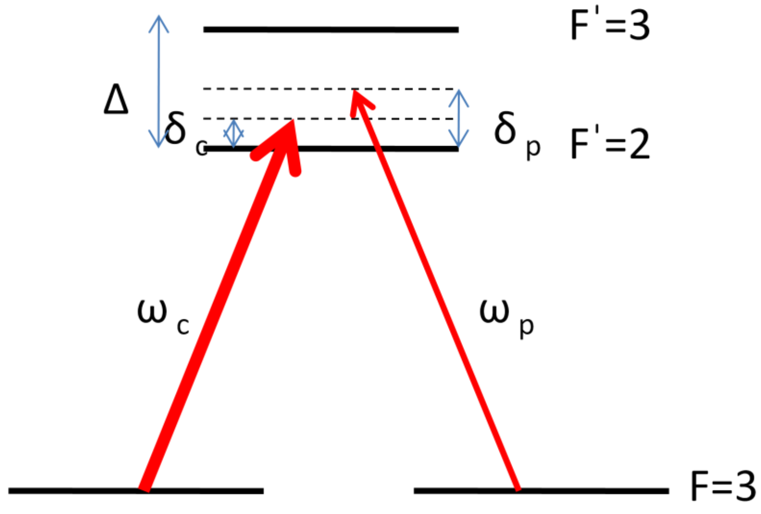


Figure 13: Double lambda system

The detuning of first lambda system formed by F=3 and F'=2is given by $\delta_1=\delta_p-\delta_c$. Then the detuning of second lambda system is given as $\delta_2=\Delta-\delta_c-(\Delta-\delta_p) = \delta_p-\delta_c = \delta_1$. We can see that both the detuning are equal and they will satisfy two photon resonance condition. So this system gives a resultant electromagnetic induced transparency at two photon resonance condition. Since it comes from a double lambda system, it results in a narrow EIT peak at two photon resonance condition and it leads to steep dispersion and slow light in the medium.

2.5 COMPONENTS OF EXPERIMENTAL SET UP

Our aim of this experiment is to produce narrow EIT signal which provides an indication of slow light in the medium.

Diode laser

External cavity diode laser is tunable laser which can produce narrow band light. This is used in the experimental set up because of its small size and high efficiency. This configuration can reduce the bandwidth of the laser output to less than 1MHz. But laser is still prone to a slow drift across frequencies. This will give rise to problems, because the laser can excite atoms to different transitions, since the energy level separation between the hyperfine levels of Rubidium are narrow. This can be overcome by a technique called Doppler free saturation spectroscopy which is used to stabilise and lock the laser to a particular frequency.

AOM

Acousto-optic modulator can be used as frequency shifter. It uses the acousto-optic effect to diffract and shift the frequency of light using sound waves. Acousto-optic modulators are made up of transparent material like glass and it is attached by a piezoelectric transducer driven by electric signal. A piezoelectric transducer is able to send acoustic (vibrational) waves through the material that would change the index of refraction and also diffract incident light passing through the material.

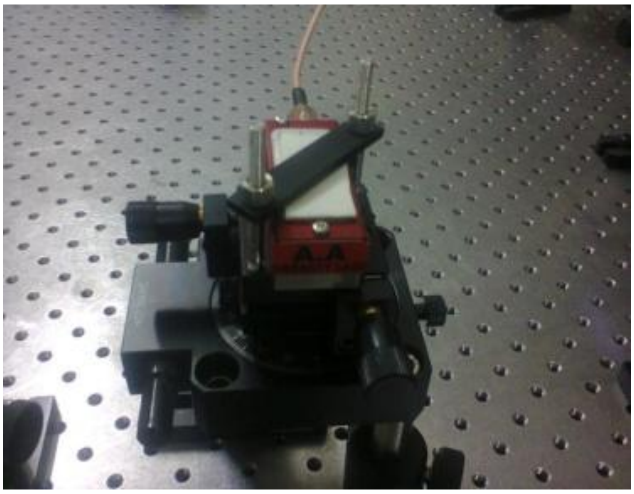


Figure 14: Acousto-optic modulator

Wave plates

Wave plates are made up of material that transmits light of different polarisations at different speeds. These are used for controlling, analyzing and optimizing polarized light. They shift the phase of light polarized along its optic axis with respect to light polarized across its optic axis. So they can be used for transforming one state of polarisation to another- linear to circular, vertical to horizontal, etc.

A quarter wave plate is used to transform plane polarized light into circularly polarized light and vice versa. And a half wave plate transforms a linearly polarized light into linearly polarised light but rotated such that, the rotation angle is twice that of the incident angle to the optic axis.

2.6 EXPERIMENTAL SETUP

The experimental set up is shown below.

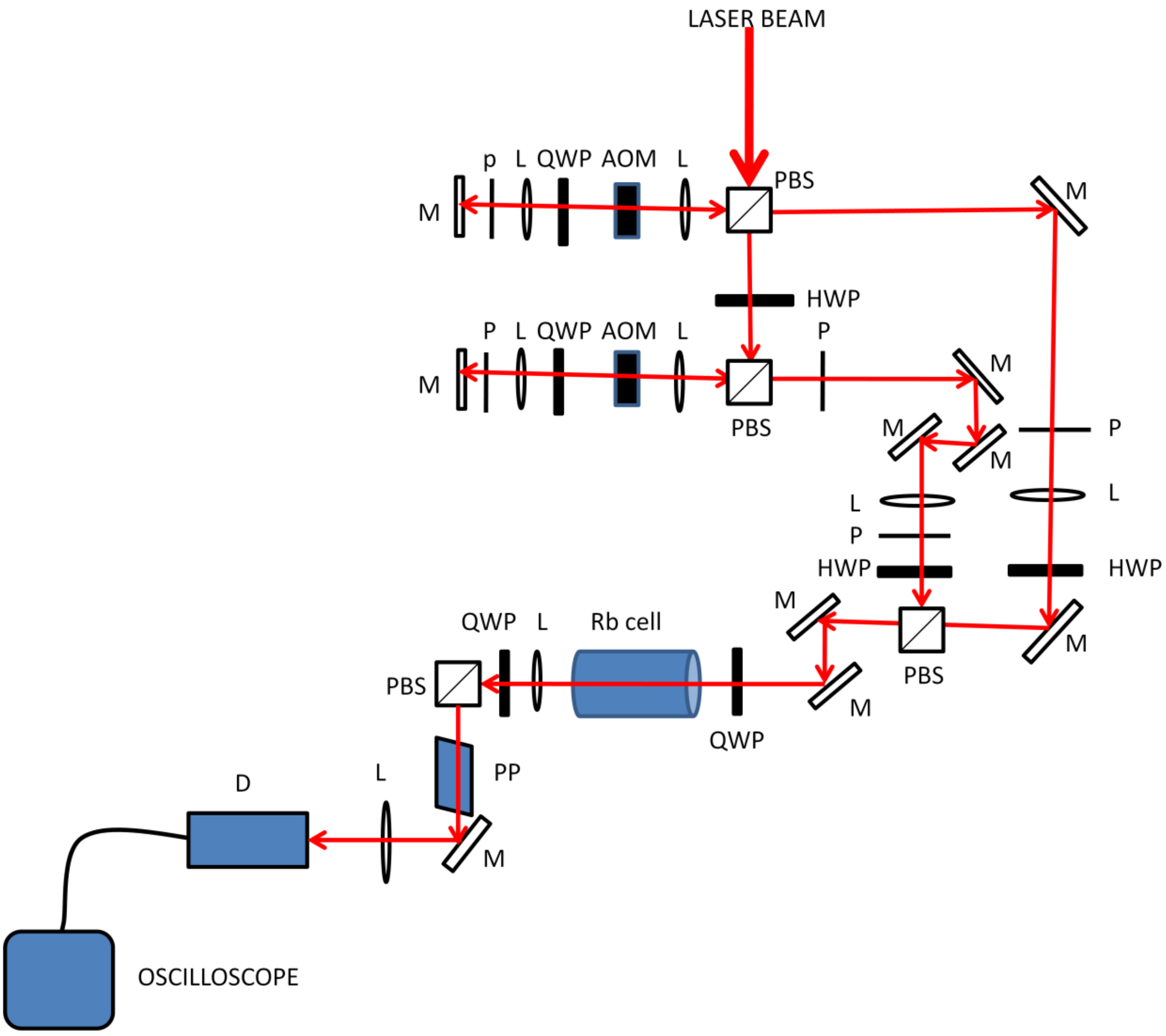


Figure 15: Slow light experimental setup (a)

M=MIRROR
P=PIN HOLE
L=LENS
D=DETECTOR
PP=POLARISING PRISM

QWP=QUARTER WAVEPLATE
HWP=HALF WAVEPLATE
AOM=ACOUSTO-OPTIC MODULATOR
PBS=POLARISING BEAM SPLITTER

Calibration of laser output can be obtained by observing SatAbs from the Doppler free saturation absorption technique. By adjusting offset value we can choose a particular transition on rubidium atom. The laser output from saturation spectroscopy can be collected by an optical fiber and later it is split into two beams by a polarising beam splitter. Then each beam is double passed through different AOM and their frequency is shifted by 160MHz. One of the beam is called probe beam and other is called coupling beam.

Figure 16: Slow light experimental setup (b)

The probe beam and coupling beam take different path and afterwards they overlap and pass through rubidium cell. Rubidium vapour cell is a glass cell filled with natural rubidium having two isotopes ^{85}Rb and ^{87}Rb. The vapour pressure of rubidium inside the cell is determined by the cell temperature. Our laser is tuned such a way as to attain two photon resonance condition. Rotating the quarter wave plate will control the amount of light that is transmitted through the polarising beam splitter to the detector after the light has travelled back through the rubidium cell.

The probe beam is ramping so that we can observe EIT signal at rising and falling of the signal. After making sure that EIT peak is at the centre of rising and falling edge of the ramped signal, ramping of the signal is stopped so that probe beam and coupling beam are tuned at two photon resonance condition. Then output signal from the experimental setup is collected by a photodetector and can be displayed on an oscilloscope.

Figure 17: Rubidium cell

We can measure the delay of light in the medium by taking readings with and without coupling beam.

2.7 RESULTS AND DISCUSSIONS

The diagrams show the delay between signal output from the experimental set up with and without coupling beam.

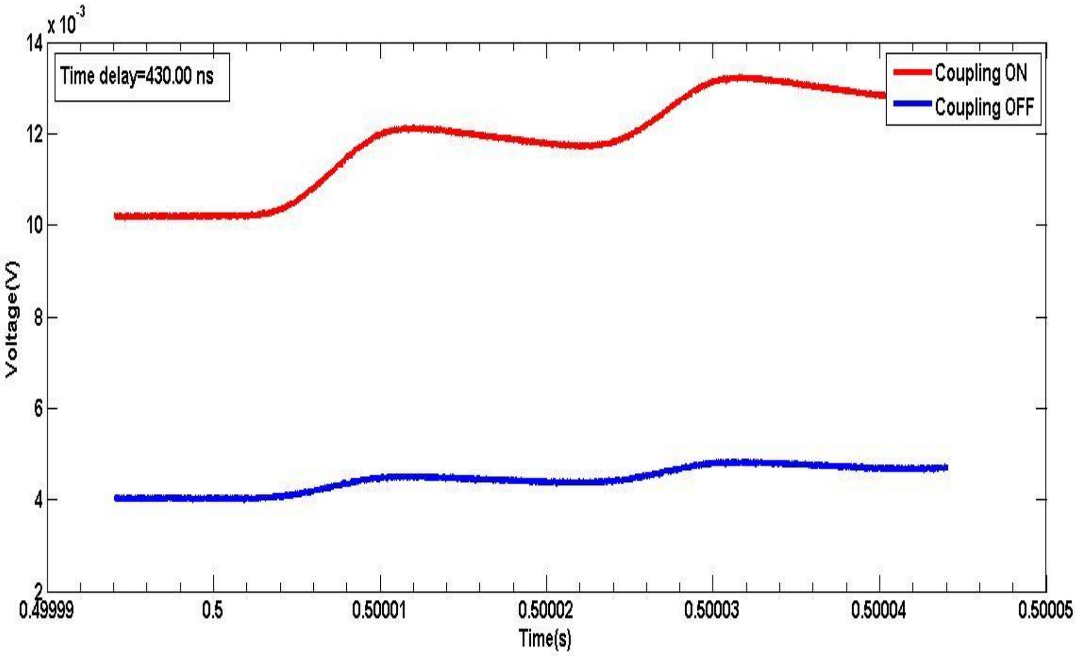


Figure 18: modulated output signal from the detector (a)

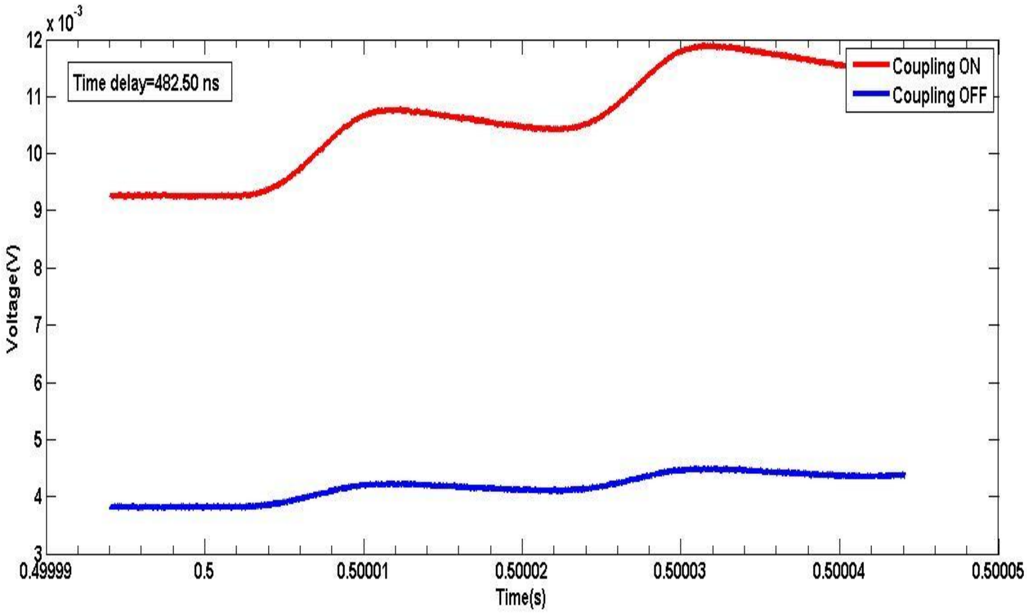


Figure 19: Modulated output signal from the detector (b)

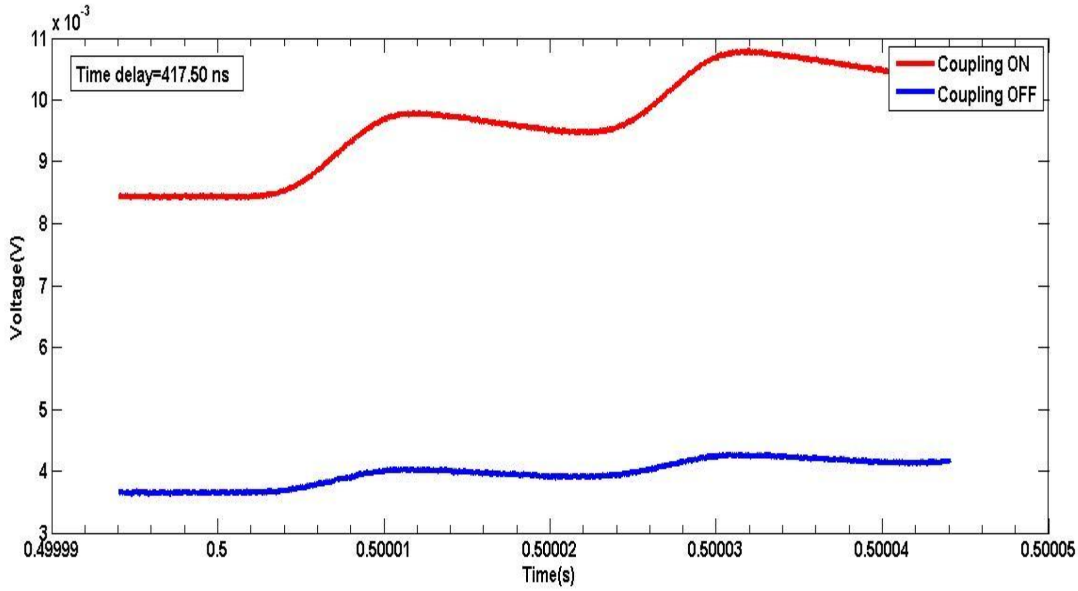


Figure 20: modulated output signal from the detector(c)

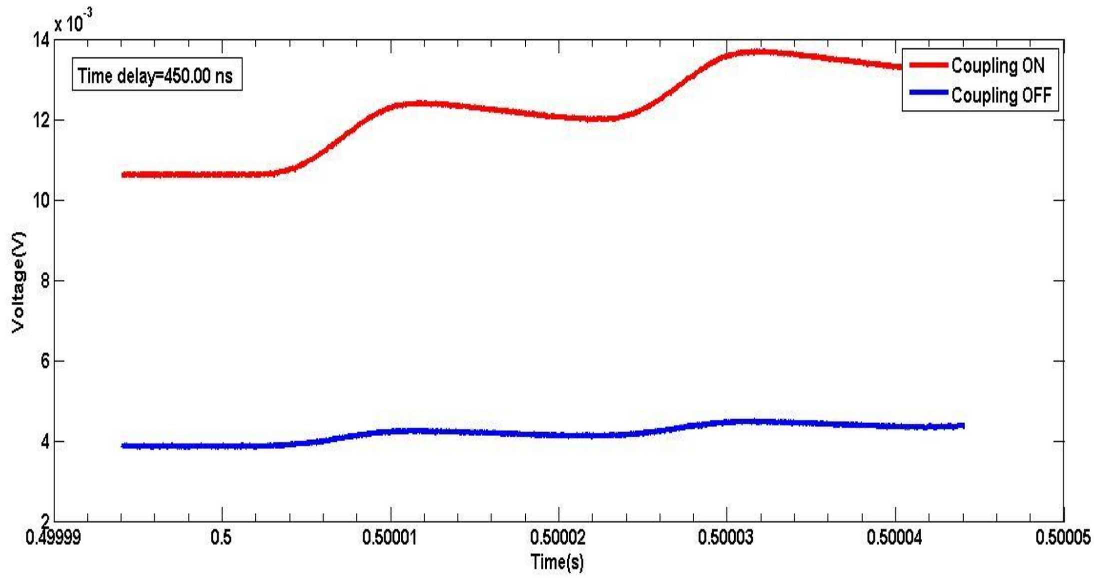


Figure 21: modulated output signal from the detector (d)

Average Time delay=4.45×10^{-7} s

Group velocity= 1.1231×10^{5} m/s

Experimental results suggest that the group velocity in the medium is lesser than the velocity of light in vacuum. Larger the delay in group velocity, the lesser it becomes than c. Narrow transparency window can induce more delay which can result in slowing of light. This can be obtained in a double lambda system with alkali vapours.

One of the essential criteria to produce slow light is the choice of atomic energy level configuration and other is strong coupling to overcome the inhomogeneous broadening.

2.8 APPLICATIONS

Slow light has many potential applications in telecommunications. Some of the effects of light matter interaction used in slowing down light can be used to create entangled photon pairs, which leads better quantum computing capabilities.

The applications of slow light include controllable optical delay, optical buffers and true time delay methods for synthetic aperture radar. Also slow light pulses can be stored in the medium for duration of 1ms.This can be used as an optical memory.

2.9 CONCLUSION

We demonstrated a slow light system based on electromagnetically induced transparency in double lambda atomic system of rubidium atom. We can obtain a low dispersive and absorptive broadening on signal pulses compared to the output from a single lambda system at two photon resonance condition. This experiment confirmed that the slow light is inversely proportional to the width of the electromagnetically induced transparency resonance.

2.10 FUTURE PROSPECTS

It is very difficult to observe a slow light in the medium than getting an EIT signal. The narrow EIT helps to observe propagation delay of light. The modification on experimental set up can produce narrower EIT signal. The experiment with fast response AOM is be carried out in the lab.

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CHAPTER 3: GAUSSIAN STATE FIDELITY

3.1 INTRODUCTION

Quantum states are the carriers of quantum information or classical information in quantum information processing. The current trend in information processing is based on quantum state transfer, because photons are the fastest and simplest carriers of quantum information for transmission and light can be transmitted over a long distance efficiently.

There are two forms of quantum information carriers, discrete and continuous. A discrete quantum state contains two distinguishable states, called quantum bit or qubit. Examples are spin ½ particles such as electrons and many nuclear spin. And a continuous variable system has an infinite dimensional Hilbert space described by observables with continuous eigenspectra. The continuous variable quantum systems are particularly relevant for quantum communication and quantum limited techniques for sensing, detection and imaging.

Gaussian quantum information processing is a technique using continuous variables. It is very useful for quantum computation, quantum cryptography, and quantum communication. Applications of Gaussian light are classified into two: one is weak Gaussian light as approximate qubits or two photon entangled states used in quantum teleportation, quantum cryptography in quantum entanglement based experiments. And second one is strong Gaussian light for continuous variable quantum information processing.

This chapter focuses on high fidelity quantum state transfer through micro- optomechanical system using Gaussian state. This includes Gaussian states and their phase space representation and performance of transferring Gaussian states into Gaussian states with fidelity equation.

3.2 OPTOMECHANICAL QUANTUM STATE TRANSFER

Quantum optomechanics is a system in which a mechanical resonator is coupled to photons in a cavity. To transfer a quantum state, we have to consider a two cavity optomechanical system. Ying-Dan Wang and Aashish A. Clerk developed a system in which a single mechanical resonator is coupled to both an optical cavity and a microwave cavity simultaneously (PRL 108, 153603, 2012). They showed that one can perform high fidelity optomechanical quantum state transfer by using dark mode with minimum mechanical dissipation. This technique is based on the presence of a dark mode which is analogous to the three level atomic system's dark state formation as described before in electromagnetically induced transparency.

Transfer of quantum state between cavities is the process of transfer of photons which is incident on cavity 1 to the output of the cavity 2 faithfully. This is called intracavity state transfer. The three basic scheme used to implement intracavity state transfer are double swap scheme, adiabatic passage scheme, and hybrid scheme which are explained by Ying-Dan Wang and Aashish A. Clerk (PRL 108, 153603,2012).

Double swap method is a scheme which does not use the dark mode and dark mode is entirely used in adiabatic passage scheme. The hybrid scheme is more advantage compare, to the other two schemes which partially uses the dark mode. The dark mode is a superposition of the two optical modes and is decoupled from the mechanical oscillator.

3.3 GAUSSIAN STATE

A Gaussian state is defined as a state whose distribution function in phase space is in the Gaussian form, or its characteristics function is Gaussian. Gaussian states are continuous variable states in which Gaussian transformation take Gaussian states to Gaussian state.

Examples of Gaussian states are quantum vacuum state, coherent states, squeezed state, and thermal state.

3.4 PURE STATE

A pure state is a state which can be represented by a ket vector, $|\Psi>$. And the mixed state is the statistical ensemble of pure states which is described by density operator. The density operator ρ is pure state if and only if $\rho = \rho^2$.

3.5 WIGNER FUNCTION

Wigner function is a quasi-probability distribution function in phase space, which is analogous to classical probability distribution function. We cannot measure position and momentum of a particle at a point simultaneously, so that we cannot define a trace phase distribution in quantum mechanics. E. P Wigner suggested a representation of quantum state to study quantum correction to classical statistical mechanics.

Consider a one dimensional system and $\hbar$=1, then for a solution Ψ of the Schrodinger equation, the Wigner function is defined as a real function which is given by the equation,

$$W(x,p,t) = 1/\pi \int \Psi^*(x+y,t)\, \Psi(x-y,t) \exp(2ipy/\hbar)\, dy$$

W (x,p) is a real function, it may take negative values in certain domains of the phase space. But Gaussian Wigner functions are positive and definite for pure states. The marginal distributions for x and p are given by

$$W(x)= \int_{-\infty}^{+\infty} W(x,p)dp \quad \text{and}$$

$$W(p)= \int_{-\infty}^{+\infty} W(x,p)dx$$

3.6 GAUSSIAN STATE TRANSFER FIDELITY

We are considering an optomechanical system in which a Gaussian state is transferring through it. The performance of a quantum state transfer through a cavity is defined by the fidelity. Fidelity is the transition probability between the quantum states.

Let ρ_i and ρ_f are be the initial and final state of the quantum state represented by its density operator. We are assuming that one of the states is a pure state such that $\rho = \rho^2$. Then the fidelity of the system is given by the Uhlmann fidelity equation, which is given by

$$F(\rho_i,\rho_f) = [\mathrm{Tr}\sqrt{\sqrt{\rho_i}\,\rho_f\,\sqrt{\rho_i}}\ ]^2$$

If ρ_i is a pure state, i.e., $\rho_i = |\varphi\rangle\langle\varphi|$ and $\sqrt{\rho_i} = \rho_i$

$$\text{Then } F(\rho_i,\rho_f) = [\ \mathrm{Tr}\sqrt{\ |\varphi\rangle\langle\varphi|\,\rho_f\,|\varphi\rangle\langle\varphi|}\ ]^2$$

$$F(\rho_i,\rho_f) = \sqrt{\left|\varphi\rangle\ \rho_f\langle\varphi\right|}\ [\ \mathrm{Tr}|\varphi\rangle\langle\varphi|\ ]$$

$$F(\rho_i,\rho_f) = \sqrt{\left|\varphi\rangle\,\rho_f\langle\varphi\right|}$$

$$F(\rho_i,\rho_f) = \mathrm{Tr}(\rho_i\,\rho_f)$$

We know that Wigner function is a probability distribution function which is analogous to classical probability distribution. For an arbitrary density operator, Wigner function is defined as

$$W_\rho(\xi) = 1/\pi \int_{-\infty}^{+\infty} d\xi \exp\left(-\frac{i}{\hbar}p\xi\right) < x + \frac{1}{2}\xi|\rho|x - \frac{1}{2}\xi >$$

Now let us define a function,

$$R = \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dp\, W\rho_1(\xi)\, W\rho_2(\xi)$$

$$R=\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} d\xi_1 \int_{-\infty}^{+\infty} d\xi_2 \left(\frac{1}{\pi}\right)^2 \int_{-\infty}^{+\infty} dp \exp\left(-\frac{i}{\hbar} p(\xi_1+\xi_2)\right) < x+\frac{1}{2}\xi_1|\rho_1|x-\frac{1}{2}\xi_1 >< x+\frac{1}{2}\xi_2|\rho_2|x-\frac{1}{2}\xi_2 >$$

Substituting $\xi_2 = -\xi_1 = \xi$ and $1/\pi \int_{-\infty}^{+\infty} dp \exp\left(-\frac{i}{\hbar} p\xi\right) = \delta(\xi)$. Then

$$R=1/\pi \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} d\xi < x+\frac{1}{2}\xi|\rho_1|x-\frac{1}{2}\xi >< x-\frac{1}{2}\xi|\rho_2|x+\frac{1}{2}\xi >$$

After putting $x'' = x + \xi/2$ and $x' = x - \xi/2$, R becomes

$$R= 1/\pi \int_{-\infty}^{+\infty} dx' \int_{-\infty}^{+\infty} dx'' < x''|\rho_1|x' >< x'|\rho_2|x'' >$$

Using completeness theorem, $\int_{-\infty}^{+\infty} dx'|x' >< x'| \equiv 1$

Therefore $R=\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dp\, W\rho_1(\xi)\, W\rho_2(\xi) = 1/\pi \int_{-\infty}^{+\infty} dx'' < x''|\rho_1\rho_2|x'' >$

Then we can write,

$$\pi \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dp\, W\rho_1(\xi) W\rho_2(\xi) = \int_{-\infty}^{+\infty} dx'' < x''|\rho_1\rho_2|x'' \geq \; = Tr(\rho_1\rho_2).$$

So that fidelity of the quantum state transfer can be written as

$$F(\rho_i,\rho_f)= Tr(\rho_i\rho_f)= \pi \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dp\, W\rho_i(\xi) W\rho_f(\xi) .$$

Now consider a continuous variable system having N mode and $\vec{\xi}$ is a vector formed by the quadratures of the n modes. $\vec{\xi}=\{q_1,p_1,q_2,p_2,\ldots\ldots\ldots\ldots,q_n,p_n\}$ and $\hat{\xi}= \{\hat{q}_1,\hat{p}_1,\hat{q}_2,\hat{p}_2,\ldots\ldots\ldots\ldots,\hat{q}_n,\hat{p}_n\}$. And quadratures of each mode is given by $\hat{q}_i = \frac{\hat{a}_i+\hat{a}_i^\dagger}{2}$ and $\hat{p}_i = -i\frac{\hat{a}_i-\hat{a}_i^\dagger}{2}$, where $\hat{a}_i$ is the canonical annihilation operator of the mode. A Gaussian state is defined as such a state that its characteristic function is Gaussian. The Gaussian state characteristic function χ_ρ , which is given by the equation

χ_ρ (η)= exp[iη^T. J_n.d-(1/4) $\eta^T J_n^T V J_n \eta$], where, and d is 2n real vector called displacement vector. $J_n = \bigoplus_{i=1}^{n} J$; J≡ $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ is called symplectic matrix which defines the symplectic scalar product and describes the geometry of the phase space. And V is a symmetrized covariance matrix defined as V_{ij} = $\frac{1}{2} < \Delta\hat{\xi}_i \, \Delta\hat{\xi}_j + \Delta\hat{\xi}_j \Delta\hat{\xi}_i >$ with $\Delta\hat{\xi}_i = \hat{\xi}_i - < \hat{\xi}_i >$.Then Wigner function can be defined as the symplectic fourier transform of the Gaussian characteristic function.

ie.

$$W_\rho\,(\vec{\xi}) = (1/2\pi)^{2n} \int d^{2n}\, \chi_\rho(\eta) \exp\,[-i\eta^T . J_n . \xi\,]$$

$$W_\rho\,(\vec{\xi}) = (1/2\pi)^{2n} \int d^{2n} \exp[\, i\eta^T . J_n . d - 1/4\, \eta^T J_n^T V J_n \eta - i\eta^T . J_n . \xi]$$

For n= 1, $\eta = \binom{q}{p}$,V= $\begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}$, d= $\binom{<x>}{<y>}$ and $\xi = \binom{x}{y}$

By substituting these values,

$$W_\rho(\vec{\xi}) = \frac{1}{2\pi \quad \sqrt{\det V}} \exp - \frac{1}{2 \det V} \,[-V_{11}(y-< y >)^2 - V_{22}(x-< x >)^2 + V_{12}\,(x-< x >)(y-< y >]$$

$$W_\rho\,(\vec{\xi}) = \frac{1}{2\pi \quad \sqrt{\det V}} \exp\,[-\frac{1}{2}(\xi - d)^T \;\; V^{-1}\,(\xi - d)]$$

For n mode, Wigner function can be written as

$$W_\rho\,(\vec{\xi}) \; = 1/(2\pi)^n \,\frac{1}{\sqrt{\det V}} \exp\,[-\frac{1}{2}(\xi - d)^T \;\; V^{-1}\,(\xi - d)]$$

Or

$$W_\rho(\vec{\xi}) = 1/(2\pi)^n \frac{1}{\sqrt{\det V}} \exp\left[-\frac{1}{2}\left(\vec{\xi} - <\hat{\xi}>\right)^T V^{-1} (\vec{\xi} - <\hat{\xi}>)\right]$$

Plugging this equation into fidelity equation, we obtain

$$F(\rho_i, \rho_f) = \mathrm{Tr}(\rho_i \rho_f) = \pi \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dp\, W\rho_i(\xi) W\rho_f(\xi)$$

$$F(\rho_i, \rho_f) = \pi \int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dp\, 1/(2\pi)^n \frac{1}{\sqrt{\det V_i}} \exp\left[-\frac{1}{2}\left(\vec{\xi}_i - <\hat{\xi}_i>\right)^T V_i^{-1} (\vec{\xi}_i - <\hat{\xi}_i>)\right]$$

$$1/(2\pi)^n \frac{1}{\sqrt{\det V_f}} \exp\left[-\frac{1}{2}\left(\vec{\xi}_f - <\hat{\xi}_f>\right)^T V_f^{-1} (\vec{\xi}_f - <\hat{\xi}_f>)\right]$$

$$F(\rho_i, \rho_f) = 1/(2\pi)^n \; 1/\sqrt{\det(V_i + V_f)} \exp\left[-\frac{1}{2}\left(<\hat{\xi}_i> - <\hat{\xi}_f>\right).\left(V_i + V_f\right)^{-1}\left(<\hat{\xi}_i> - <\hat{\xi}_f>\right)\right].$$

3.7 CONCLUSION

Gaussian states are type of robust quantum state. In Gaussian quantum information processing, Gaussian transformation taking part in information processing in which transformations take Gaussian states to Gaussian states. Fidelity of the quantum state transfer system can be related to a semi classical distribution function called Wigner function which is used to connect classical variables in quantum physics.

Performance of a quantum state transfer is quantified by fidelity. We can optimize the fidelity over simple rotations in phase space. Maximum fidelity can be obtained if ρ_f is a rotated version of ρ_i.

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CERTIFICATE

This is to certify that the project work entitled "***Slow Light using Electromagnetically Induced Transparency And Gaussian State Fidelity in Optomechanics*** " being submitted by Vineetha N, in partial fulfilment of the requirements for the award of the degree of Five Year Integrated M.Sc. in Photonics is a record of the bonafide work done by her during the period from December 2012 - April 2013 under the guidance of Dr. Andal Narayanan at Raman Research Institute, Bangalore.

Dr. Kailasnath M.

The Director

International School of Photonics

Cochin University of Science and Technology

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DECLARATION

I hereby declare that that the research work entitled "***Slow Light using Electromagnetically Induced Transparency And Gaussian State Fidelity in Optomechanics*** " being submitted to International School of Photonics, Cochin university of Science and Technology for partial fulfilment of the Five Year Integrated Master of Science degree in Photonics, is based on the original research work done by me at Raman Research Institute, Bangalore, under the guidance and supervision of Dr. Andal Narayanan and has not been included in any thesis submitted previously for the award of any other degree.

Date: 30-04-2013 Vineetha N

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MOTIVATION

The theory of quantum mechanics developed in the early 20^{th} century gives us a complete new understanding of physical world and provides the foundation for the latest developments in quantum information processing and computation. Finding new ways to store and process information has become a necessity in growing quantum information technology.

The process of slowing a pulse of light using the EIT effect can be tailored to perform critical tasks in the field of quantum information processing. Realizing a high fidelity quantum state transfer is also very helpful in quantum information processing and quantum computation. The future of quantum information processing appears bright in spite of the many remaining challenges. As a bonus, overcoming these challenges will probably also advance basic research.

ABSTRACT

This thesis presents theoretical derivations and experimental demonstrations of electromagnetically induced transparency, slow light and their dynamics. The slow light is studied based on electromagnetically induced transparency and it has applications in quantum information technology. The second part of the thesis discusses Gaussian state fidelity in optomechanics which has an important role in quantum information processing.

OUTLINE OF THESIS

The first chapter starts with reviewing some important theoretical basics from quantum mechanics. This chapter contains derivation and expressions for three level atomic lambda system in dark state picture, which gives a clear idea of electromagnetically induced transparency.

The second chapter presents theory of EIT and slow light with experimental set up using hot rubidium vapour gas.

In third chapter, fidelity of a quantum system is derived using Wigner function which is a quasi probability distribution function and it also presents a method for deriving an expression for Gaussian state fidelity in terms of covariance matrix.