PROJECT REPORT

A STUDY OF GRAVITATIONAL RADIATION FROM INSPIRALLING BINARY SOURCES AND DATA ANALYSIS

Submitted by

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We started by working through Chapters 8(Einstein Equations) and 9(Gravitational Radiation) of Schutz (2009). We solved various important problems from these chapters. Then we worked out the details of gravitational radiation from a binary in elliptical orbit, using Peters and Matthews (1963) as reference. We calculated the Energy and Angular momentum flux of the binary as a function of the masses of the bodies and the eccentricity and semi major axis of the orbit.

Using [1] for reference we learnt about various approximants for waveform templates such as TaylorT1,TaylorT4,TaylorT2,TaylorT3,TaylorEt andTaylorF2. Assuming the expressions for Energy and Flux of binaries in circular orbit in the adiabatic approximation, we reproduced the results of the paper (up to 3.5 PN order) using Mathematica.

We learnt how to calculate the Energy of system of a particle orbiting a massive compact body in circular motion to the required PN order. In the case of a test mass in circular orbit around aSchwarzschild black hole one gets the following exact expression for the Energy function,*e*(*x*);

$$e(x;\eta=0)=-x\frac{1-4x}{1-3x}$$

By expanding this function in a Taylor series we get the following Energy function to up to14PN order.

$$\begin{aligned}\mathrm{E14(v)} = (-1/2)\eta v\text{^}2[1 &- \frac{3v^2}{4} - \frac{27v^4}{8} - \frac{675v^6}{64} - \frac{3969v^8}{128} - \frac{45927v^{10}}{512} - \frac{264627v^{12}}{1024}\\ &- \frac{12196899v^{14}}{16384} - \frac{70366725v^{16}}{32768} - \frac{813439341v^{18}}{131072} - \frac{4710988269v^{20}}{262144}\\ &- \frac{109344517191v^{22}}{2097152} - \frac{635640105429v^{24}}{4194304} - \frac{7402640979375v^{26}}{16777216}\\ &- \frac{43172202191715v^{28}}{33554432}]\end{aligned}$$

Then using the result of [2]for the flux of such a system we extended the approximants to 5.5 PN order. Using the results of Fujita (2012) for the Flux and the Energy expression previously determined we calculated the approximants up to 14 PN order. The results up to 6 PN order were tabulated.

Referring to [3] we began working on the data analysis of Newtonian waveform buried in a random noise. The case considered was of two compact stars with masses m_1 and m_2 in circular orbit. The emitted gravitational waveform is a “chirp” waveform with both amplitude and frequency increasing with time. In the leading order (Newtonian) the Gravitational wave signal is given by

$$h(t) = A(t)\cos[\varphi(t) + \varphi_0] \quad t \geq t_0$$

Where ϕ(t) is the GW signal phase and A(t) is the amplitude given by

$$A(t) = 7.68\text{x}10^{-21}\left(\frac{D}{100\text{kpc}}\right)^{-1}\left(\frac{\mathcal{M}}{1.22M_\odot}\right)^{5/3}\left(\frac{F(t)}{10\text{Hz}}\right)^{2/3}$$

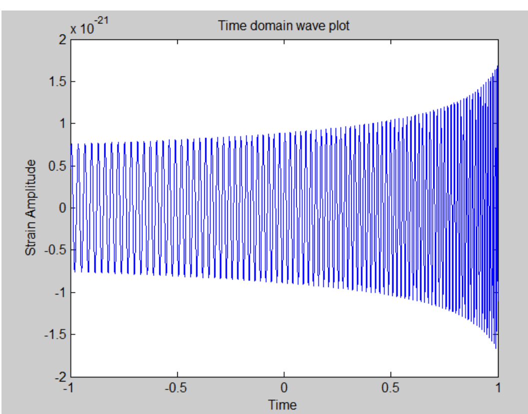
$$\varphi(t) = \varphi_0 + 2\pi\int_0^{t-t_0} F(t')\,dt'.$$

$\mathcal{M}$ Isthe chirp mass, D is the luminosity distance to the source and F(t) is the gravitational wave frequency.

$$F(t) = F_0\left(1 - \frac{t-t_0}{T_c}\right)^{-3/8}, \qquad T_c \equiv \frac{5}{256\,(\pi F_0)^{8/3}\mathcal{M}^{5/3}}$$

T_c is the chirp duration. $t_o$is the time the signal enters the detector,F_0 is the GW frequency and ϕ_0 the GW phase at that time.

As an exercise we built a Newtonian Chirp signal for the case; $\mathcal{M} = 8M_\odot$,D = 50 Mpc,F_0 = 40Hz and plotted it against time with sampling rate 2048 Hz using MATLAB. We got the following result



As expected the Amplitude and frequency increase with time.

We carried out some of the exercises given in [3] which include generating random numbers drawn from a Gaussian distribution and calculating the probability for one of them to be greater than a given threshold. We did the same calculations for another set of numbers which are the squares of the previous random numbers (their probability distribution function comes out to be Chi squared).

We worked out the Fast Fourier transform and estimated the power spectral density of a time series data vector sampled at a given sampling rate. To a sinusoidal signal h given by

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h = sin(2*pi*50*t) + sin(2*pi*120*t)
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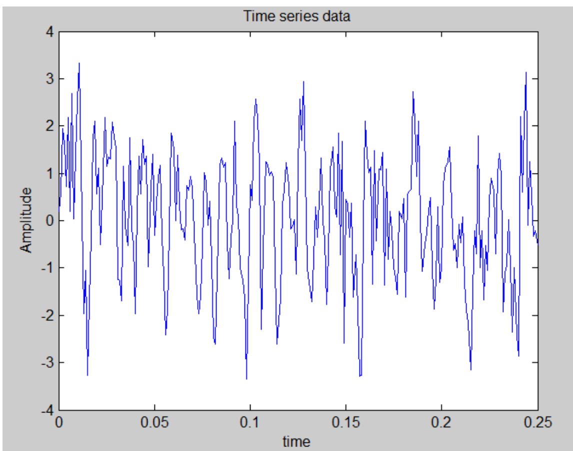
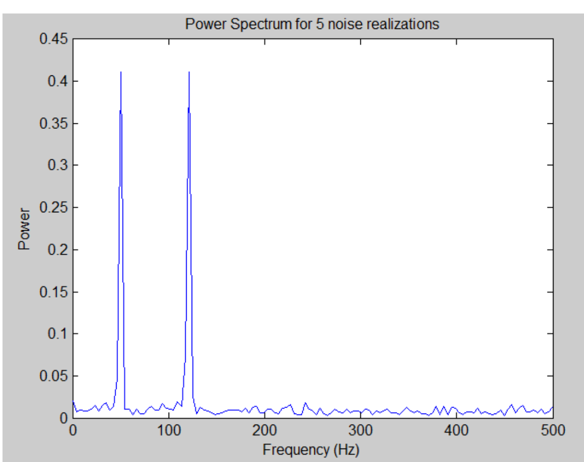
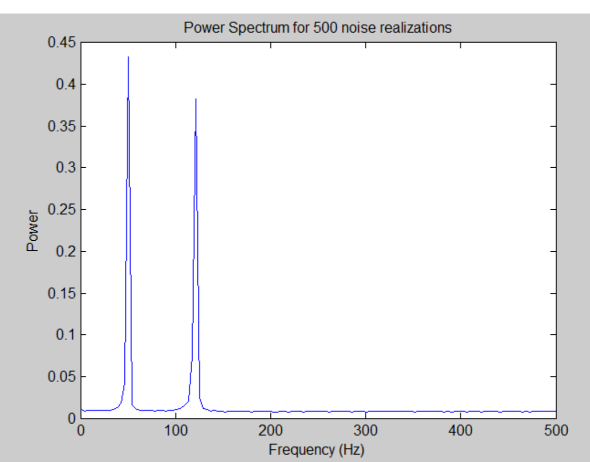
a random Gaussian noise with mean 0 and variance 1 was added. The sampling rate was taken to be 1000 Hz. The resulting data looked as follows



We estimated the Power Spectral Density(PSD) of the given data vector by averaging over many noise realizations. We got the following plots for PSD by averaging over 5 and 500 noise realizations respectively. As can be seen clearly from the plots, the peaks are at 50Hz and 120Hz and are sharper (with less background noise) when averaged over many noise realizations.





References

[1] Comparison of post-Newtonian templates for compact binary inspiral signals ingravitational-wave detectors, Bala Iyer et al. (2012)

[2]Analytic black hole perturbation approach togravitational radiation, Misao Sasaki and Hideyuki Tagoshi (2008)

[3] A Gravitational-Wave Data Analysis Primerfor the IndIGO Mock Data Challenge,P. Ajith, Satya Mohapatra and Archana Pai (2011)