PHOTOELASTIC EFFECT IN CRYSTALS

1. INTRODUCTION

A thorough and exhaustive account of Crystal Photoelasticity up to the year 1928 is given by Szivessy in his article in Handbuch der Physik (62). Since then numerous papers have appeared on this subject, partly because of its important relationship to other phenomena like the optical effects arising from mechanical oscillations in a crystal and the thermal diffusion of light. In this article an attempt is made to summarise the present state of knowledge in this subject both from the theoretical and experimental aspects.

2. PHENOMENOLOGICAL THEORY OF PHOTOELASTICITY

(a) **ELASTO AND PEIZO-OPTIC CONSTANTS**:—The index ellipsoid for any crystal can be represented by the equation

$$\frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} = 1 \qquad . (1)$$

where n_1, n_2 and n_3 are the three principal refractive indices for any wavelength λ. This equation is not valid for all wavelengths for crystals exhibiting monoclinic and triclinic symmetry. For these two cases, however, a transformation is necessary leading to an equation for the index ellipsoid in terms of six parameters. For the sake of completeness we shall use the equation for the index ellipsoid, in the general form

$$\frac{x^2}{n_{11}^2} + \frac{y^2}{n_{22}^2} + \frac{z^2}{n_{33}^2} + \frac{2yz}{n_{23}^2} + \frac{2zx}{n_{31}^2} + \frac{2xy}{n_{12}^2} = 1 \qquad (2)$$

as our starting equation.

In general, a mechanical stress both deforms and rotates the index ellipsoid with consequent changes in the birefringence and in the direction of the principal axes. In the development of the phenomenological theory of photoelasticity it is customary to assume that the Fresnel's laws hold good in a homogeneously deformed crystal and that the differences between the optical parameters in the deformed and the original states are linear functions of the six stress components X_x, Y_y, Z_z, Y_z, Z_x, X_y or of the six strain components $x_x, y_y, z_z, y_z, z_x, x_y$.

If we express the optical parameters of the underformed crystal by n_{ij} ($i, j = 1, 2, 3$) and the corresponding quantities in the deformed state by ν_{ij} then

$$\left.\begin{aligned}
\frac{1}{\nu_{11}^2} - \frac{1}{n_{11}^2} &= p_{11}x_x + p_{12}y_y + p_{13}z_z + p_{14}y_z + p_{15}z_x + p_{16}x_y \\
\frac{1}{\nu_{22}^2} - \frac{1}{n_{22}^2} &= p_{21}x_x + p_{22}y_y + p_{23}z_z + p_{24}y_z + p_{25}z_x + p_{26}x_y \\
\frac{1}{\nu_{33}^2} - \frac{1}{n_{33}^2} &= p_{31}x_x + p_{32}y_y + p_{33}z_z + p_{34}y_z + p_{35}z_x + p_{36}x_y \\
\frac{1}{\nu_{23}^2} - \frac{1}{n_{23}^2} &= p_{41}x_x + p_{42}y_y + p_{43}z_z + p_{44}y_z + p_{45}z_x + p_{46}x_y \\
\frac{1}{\nu_{31}^2} - \frac{1}{n_{31}^2} &= p_{51}x_x + p_{52}y_y + p_{53}z_z + p_{54}y_z + p_{55}z_x + p_{56}x_y \\
\frac{1}{\nu_{12}^2} - \frac{1}{n_{12}^2} &= p_{61}x_x + p_{62}y_y + p_{63}z_z + p_{64}y_z + p_{65}z_x + p_{66}x_y
\end{aligned}\right\} \qquad (3)$$

or in terms of stress

$$
\left.\begin{aligned}
\frac{1}{\nu_{11}{}^2} - \frac{1}{n_{11}{}^2} &= -\,[q_{11}X_x + q_{12}Y_y + q_{13}Z_z + q_{14}Y_z + q_{15}Z_x + q_{16}X_y] \\
\frac{1}{\nu_{22}{}^2} - \frac{1}{n_{22}{}^2} &= -\,[q_{21}X_x + q_{22}\,Y_y + q_{23}Z_z + q_{24}Y_z + q_{25}Z_x + q_{26}X_y] \\
\frac{1}{\nu_{33}{}^2} - \frac{1}{\ } &= -\,[q_{31}X_x + q_{32}Y_y + q_{33}Z_z + q_{34}Y_z + q_{35}Z_x + q_{36}X_y] \\
\frac{1}{\nu_{23}{}^2} - \frac{1}{n_{23}{}^2} &= -\,[q_{41}X_x + q_{42}Y_y + q_{43}Z_z + q_{44}Y_z + q_{45}Z_x + q_{46}X_y] \\
\frac{1}{\nu_{31}{}^2} - \frac{1}{n_{31}{}^2} &= -\,[q_{51}X_x + q_{52}Y_y + q_{53}Z_z + q_{54}Y_z + q_{55}Z_x + q_{56}X_y] \\
\frac{1}{\nu_{12}{}^2} - \frac{1}{n_{12}{}^2} &= -\,[q_{61}X_x + q_{62}Y_y + q_{63}Z_z + q_{64}Y_z + q_{65}Z_x + q_{66}X_y].
\end{aligned}\right\} \quad (4)
$$

The negative sign in equation (4) arises due to the convention according to which stress is taken to be positive when it is compressional and negative when extensional, while the strain components are positive for elongation and negative for compression. In other words a positive strain corresponds to a negative stress.

The first group of constants p_{ij} are called the Pockels elasto-optic constants and the second group q_{ij} the Pockels piezo-optic constants. They are related as follows.

$$
p_{ij} = \sum_{1}^{6} q_{ik}c_{kj}\,; \qquad q_{ij} = \sum_{1}^{6} p_{ik}s_{kj} \qquad (5)
$$

where c_{kj} and s_{kj} are the elastic constants and moduli respectively. The set of 36 constants composed of the p_{ij}'s and q_{ij}'s completely define the behaviour of a crystal when subjected to known strains or stresses. Only in the crystals of lowest symmetry do all the 36 constants have values differing from zero. The number decreases with increasing symmetry, becoming 20 for monoclinic, 12 for orthorhombic and so on. The number of independent constants and the scheme for the various crystal classes are entered in Table I. The schemes given in Table I are those of Pockels which have been revised by Bhagavantam (4). In the first scheme published by Pockels in 1897 some serious errors existed for the crystal classes C_3, S_6, C_{3h}, C_6, C_{6h}, C_4, S_4, C_{4h}, T and T_h. These important corrections were made by Bhagavantam and have been confirmed by group theoretical methods (Bhagavantam 4, 7 ; Jahn 27) and also experimentally (5-13, 37, 38, 51). Here it may be mentioned that if one were to adopt Raman's (55) new theory of elasticity the number of independent photoelastic constants for a triclinic crystal will be 54 instead of 36, and this number will again be reduced with ascending symmetry of the crystal.

The first three equations in formulae (3) and (4) express changes in the principal velocities which the crystal experiences on deformation and these can be measured by the usual methods by making observations with the incident light having its electric vectors parallel to the three axes of the undeformed crystal. The last three equations represent a rotation of the principal axes of the ellipsoid on deformation. This rotation is determined by measuring the change in refractive index under stress with the incident electric vector perpendicular to any one of the axes and bisecting

the angle between the other two. From the change in length of the corresponding radius vector of the ellipsoid, the amount of rotation can be calculated. Thus by measuring the changes in the principal refractive indices and the rotation of the principal axes for various directions of pressure and observation, one gets a system of linear equations in q_{ij} in terms of measurable quantities, from which q_{ij} and hence p_{ij} can be evaluated. From a knowledge of q_{ij} and p_{ij} the three principal refractive indices and the orientation of the principal axes of the deformed crystal can be evaluated as follows :—

(b) **THE POSITION OF THE OPTICAL SYMMETRY AXES AND THE PRINCIPAL REFRACTIVE INDICES OF THE DEFORMED CRYSTAL :**—Let the direction cosines between the optical symmetry axes of the undeformed crystal $X°Y°Z°$ and the axes XYZ, to which $X°Y°Z°$ are transformed on deformation, be given by the adjoining matrix.

	X	Y	Z
X°	α_1	β_1	γ_1
Y°	α_2	β_2	γ_2
Z°	α_3	β_3	γ_3

Then the six polarisation parameters $\dfrac{1}{\nu_{hk}^2}$ of the deformed crystal are given by

$$\frac{1}{\nu_{hk}^2} = \frac{\alpha_h \alpha_k}{n_1^2} + \frac{\beta_h \beta_k}{n_2^2} + \frac{\gamma_h \gamma_k}{n_3^2} \quad (h, k = 1, 2, 3) \tag{7}$$

From (7) and the orthogonality relations

$$a_1 b_1 + a_2 b_2 + a_3 b_3 = 0, \qquad a \neq b \ (a, b = \alpha, \beta, \gamma) \tag{8}$$
$$= 1 \qquad a = b$$

we get

$$\frac{\beta_1 \gamma_1}{\nu_{11}^2} + \frac{\beta_2 \gamma_2}{\nu_{22}^2} + \frac{\beta_3 \gamma_3}{\nu_{33}^2} + \frac{\beta_2 \gamma_3 + \beta_3 \gamma_2}{\nu_{23}^2} + \frac{\beta_3 \gamma_1 + \beta_1 \gamma_3}{\nu_{31}^2} + \frac{\beta_1 \gamma_2 + \beta_2 \gamma_1}{\nu_{12}^2} = 0.$$

$$\frac{\gamma_1 \alpha_1}{\nu_{11}^2} + \frac{\gamma_2 \alpha_2}{\nu_{22}^2} + \frac{\gamma_3 \alpha_3}{\nu_{33}^2} + \frac{\gamma_2 \alpha_3 + \gamma_3 \alpha_2}{\nu_{23}^2} + \frac{\gamma_3 \alpha_1 + \gamma_1 \alpha_3}{\nu_{31}^2} + \frac{\gamma_1 \alpha_2 + \gamma_2 \alpha_1}{\nu_{12}^2} = 0. \tag{9}$$

$$\frac{\alpha_1 \beta_1}{\nu_{11}^2} + \frac{\alpha_2 \beta_2}{\nu_{22}^2} + \frac{\alpha_3 \beta_3}{\nu_{33}^2} + \frac{\alpha_2 \beta_3 + \alpha_3 \beta_2}{\nu_{23}^2} + \frac{\alpha_3 \beta_1 + \alpha_1 \beta_3}{\nu_{31}^2} + \frac{\alpha_1 \beta_2 + \alpha_2 \beta_1}{\nu_{12}^2} = 0.$$

$$\frac{1}{\nu_{11}^2} - \frac{1}{n_{11}^2}, \ \frac{1}{\nu_{22}^2} - \frac{1}{n_{22}^2}, \ \frac{1}{\nu_{33}^2} - \frac{1}{n_{33}^2}, \ \frac{1}{\nu_{23}^2}, \ \frac{1}{\nu_{31}^2}, \ \frac{1}{\nu_{12}^2}$$ are of the same order

of magnitude as the deformation and hence very small compared to $\dfrac{1}{n_1^2}, \dfrac{1}{n_2^2}, \dfrac{1}{n_3^2}.$

It can then be shown that the direction cosines are given by

$$\alpha_1 = \beta_2 = \gamma_3 = 1.$$

$$\beta_3 = -\gamma_2 = \frac{\dfrac{1}{\nu_{23}^2}}{\dfrac{1}{n_2^2} - \dfrac{1}{n_1^2}} \ ; \quad \gamma_1 = -\alpha_3 = \frac{\dfrac{1}{\nu_{31}^2}}{\dfrac{1}{n_3^2} - \dfrac{1}{n_1^2}}. \tag{10}$$

$$\alpha_2 = -\beta_1 = \frac{\dfrac{1}{\nu_{31}^2}}{\dfrac{1}{n_3^2} - \dfrac{1}{n_1^2}}.$$

When the direction cosines have been determined as above, the principal refractive indices can be calculated from

$$\frac{1}{\nu_1{}^2} = \frac{\alpha_1{}^2}{\nu_{11}{}^2} + \frac{\alpha_2{}^2}{\nu_{22}{}^2} + \frac{\alpha_3{}^2}{\nu_{33}{}^2} + \frac{2\alpha_2\alpha_3}{\nu_{23}{}^2} + \frac{2\alpha_3\alpha_1}{\nu_{31}{}^2} + \frac{2\alpha_1\alpha_2}{\nu_{12}{}^2}$$

$$\frac{1}{\nu_2{}^2} = \frac{\beta_1{}^2}{\nu_{11}{}^2} + \frac{\beta_2{}^2}{\nu_{22}{}^2} + \frac{\beta_3{}^2}{\nu_{33}{}^2} + \frac{2\beta_2\beta_3}{\nu_{23}{}^2} + \frac{2\beta_3\beta_1}{\nu_{31}{}^2} + \frac{2\beta_1\beta_2}{\nu_{12}{}^2}$$

$$\frac{1}{\nu_3{}^2} = \frac{\gamma_1{}^2}{\nu_{11}{}^2} + \frac{\gamma_2{}^2}{\nu_{22}{}^2} + \frac{\gamma_3{}^2}{\nu_{33}{}^2} + \frac{2\gamma_2\gamma_3}{\nu_{23}{}^2} + \frac{2\gamma_3\gamma_1}{\nu_{31}{}^2} + \frac{2\gamma_1\gamma_2}{\nu_{12}{}^2}. \qquad (11)$$

Denoting the small rotations which the principal polarisation axes experience with respect to $X°$, $Y°$, $Z°$ by ϕ_x, ϕ_y, ϕ_z we get

$$\tan 2\phi_x = \frac{\dfrac{2}{\nu_{23}{}^2}}{\dfrac{1}{\nu_{22}{}^2} - \dfrac{1}{\nu_{33}{}^2}} \; ; \quad \tan 2\phi_y = \frac{\dfrac{2}{\nu_{31}{}^2}}{\dfrac{1}{\nu_{33}{}^2} - \dfrac{1}{\nu_{11}{}^2}} ; \quad \tan 2\phi_z = \frac{\dfrac{2}{\nu_{12}{}^2}}{\dfrac{1}{\nu_{11}{}^2} - \dfrac{1}{\nu_{22}{}^2}}. \qquad (12)$$

Through the combination of the three individual rotations (Eqn. 12) one obtains the total rotation which the principal axes of the crystal experience on deformation. In biaxial crystals ϕ_x, ϕ_y, ϕ_z are in general, very small and hence it is immaterial in which order the successive rotations are carried out. But in the case of uniaxial crystals the rotation about the optic axis (say ϕ_z) will be finite, due to the fact that the denominator and numerator are of the same order of magnitude in the equation for ϕ_z (Eqn. 12); on the other hand, ϕ_x and ϕ_y will be small to a first degree of approximation. In such cases the rotation about the optic axis ϕ_z must be carried out first.

(c) **BEHAVIOUR OF CRYSTALS UNDER UNIDIRECTIONAL PRESSURE**:—Let us now consider in some detail the various optical properties of crystals under unidirectional pressure. To start with the simplest case namely cubic crystals of T_d, O, O_h crystal classes equation (4) can be written as

$$\left.\begin{array}{l} \dfrac{1}{\nu_{11}{}^2} - \dfrac{1}{n^2} = - [q_1 X_x + q_2 (X_x + Y_y + Z_z)] \\[2mm] \dfrac{1}{\nu_{22}{}^2} - \dfrac{1}{n^2} = - [q_1 Y_y + q_2 (X_x + Y_y + Z_z)] \\[2mm] \dfrac{1}{\nu_{33}{}^2} - \dfrac{1}{n^2} = - [q_1 Z_z + q_2 (X_x + Y_y + Z_z)] \\[2mm] \dfrac{1}{\nu_{23}{}^2} = - q_3 Y_z \\[2mm] \dfrac{1}{\nu_{31}{}^2} = - q_3 Z_x \\[2mm] \dfrac{1}{\nu_{12}{}^2} = - q_3 X_y \end{array}\right\} \qquad (13)$$

where

$$\left.\begin{array}{l} q_1 = q_{11} - q_{12} = (p_{11} - p_{12})\,(s_{11} - s_{12}) \\ q_2 = q_{12} = p_{11} s_{12} + p_{12}\,(s_{11} + s_{12}) \\ q_3 = q_{44} = p_{44}\, s_{44} \end{array}\right\} \qquad (14)$$

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For any general direction of pressure the crystal will become biaxial and the optic axial angle 2V is given by

$$\sin V = \sqrt{\frac{1/v_1{}^2 - 1/v_2{}^2}{1/v_1{}^2 - 1/v_3{}^2}} \qquad (15)$$

where v_1, v_2, v_3 are the principal refractive indices of the deformed crystal. It is seen from equations (13) and (11) that $(1/v_1{}^2 - 1/v_2{}^2)$ and $(1/v_1{}^2 - 1/v_3{}^2)$ are homogeneous linear functions of the pressure. Their ratio and hence the optic axial angle and the position of the principal axes of the deformed crystal are independent of the magnitude of the pressure, for the same type of stress distribution. Pockels (44) has shown that as long as the direction of the principal axes and the optic axial angle are concerned the photoelastic behaviour of these crystals depends only on the direction of pressure and constant $X = q_3/q_1$. According to Pockels the complete description of the optical behaviour of cubic crystals (of classes T_d, O and O_h) under unidirectional pressures in different directions can be had if one allows the pressure direction P to run through firstly (i) half a quadrant of a cube face (say from [100] direction to the [110] direction) and then (ii) a quadrant from the [110] to the [001] direction. It is clear from symmetry as also from equation (13) that two principal axes of polarisation lie in the first case in the plane $X°,Y°$ and in the second case in the plane $\{110\}$. If we represent the various angles involved by the following symbols.

ψ - the angle between the pressure direction P and $Z°$

ϕ - the angle between $X°$ and the projection of P in the plane $X° Y°$.

ξ - the angle between $X°$ and one of the principal axes of the deformed crystal in case (i)

η - the angle between $Z°$ and one of the principal axes of the deformed crystal in case (ii)

then according to Pockels

$$\tan 2\xi = X \tan 2\phi \; ; \; \tan 2\eta = \frac{4X \sin 2\psi}{1 - X + (3 + X) \cos 2\psi} \qquad (16)$$

When once ξ and η are determined by this equation, the principal refractive indices and hence the optic axial angle can easily be determined as explained previously. The formulae for the latter are given in Table II along with the corresponding positions of the biaxial plane for various values of X, ϕ and ψ. It is clear that the optical effect of unidirectional pressure on these cubic crystals depends on the magnitude and sign of X. The four possible cases have been listed in Table II. Further from the expressions for 2V, we find that 2V vanishes for types (1) and (2) for $\phi = 0$ or $\psi = 0$ or $\psi = \cos^{-1} \frac{1}{\sqrt{3}}$ whereas for types (3) and (4) 2V vanishes for $\psi = \cos^{-1} \frac{1}{\sqrt{3}}$ and becomes 180° for $\psi = 0$. In other words, these cubic crystals of all the four types become uniaxial by unidirectional pressure parallel to a cube normal, or to an octahedral normal.

One sided pressure in any other direction makes the cubic crystal biaxial, and one of the principal axes of the strained crystal coincides with the pressure direction only if the latter is perpendicular to a dodecahedral face. Table III gives the optical

behaviour of four typical crystals, (belonging to the four different groups) for various directions of pressure.

The phenomena in the case of the cubic crystals of T and T_h crystal classes are slightly more complicated because for these crystals q_{12} and q_{13} are not equal to each other as in T_d, O, O_h classes. Consequently the crystal becomes biaxial even by a simple compression along a cube axis, with one of the principal axes coinciding with the pressure direction. From (4) it can be seen that for unidirectional pressure along a cube axis (say $X°$), $\frac{1}{\nu_{23}^2}$, $\frac{1}{\nu_{31}^2}$ and $\frac{1}{\nu_{12}^2}$ vanish and hence the principal axes do not experience any rotation. If $q_{12} > q_{13}$ (as is the case with all the crystals studied thus far in these crystal classes) the biaxial plane will be $X°OY°$ and the optic axial angle 2V is given by

$$\sin V = \sqrt{\frac{q_{12} - q_{13}}{q_{11} - q_{13}}} \tag{17}$$

from (11). Again we see that this is independent of the magnitude of the pressure as in T_d, O and O_h crystal classes.

If now the crystal is stressed along the normal to a dodecahedral face, the crystal becomes biaxial just as crystals of T_d, O, O_h classes, but with one important difference, in that the principal axis of the deformed crystal does not coincide with the direction of pressure as in the latter crystal classes. And the angular displacement θ of the principal axes from the direction of stress, can be shown to be

$$\tan 2\theta = \frac{q_{12} - q_{13}}{2q_{44}} \tag{18}$$

In fact Bhagavantam and Suryanarayana (10, 11) have used this tilt of the principal axes for ditinguishing T and T_h classes from the T_d, O, O_h classes.

By the methods outlined before it can be shown that crystals of T and T_h classes will become uniaxial only when the pressure is applied along the normal to an octahedral face. For all other directions of pressure the crystal becomes biaxial.

In conclusion one can state the following as a general rule applicable to all the five classes of the cubic system. A pressure applied along any trigonal or tetragonal axis of symmetry makes the crystal optically uniaxial with the pressure direction as the unique axis and a pressure applied along any other axis makes the crystal biaxial (Bhagavantam and Suryanarayana 10).

The above rule is valid for uniaxial crystals as well. As can be seen from Table I and (4) a unidirectional pressure along the Z-axis of any uniaxial crystal does not rotate the principal axes (since $q_{43} = q_{53} = q_{63} = 0$). Further as $q_{23} = q_{13}$ the principal refractive indices ν_1 and ν_2 of the deformed crystal will be maintained equal. In other words, under unidirectional pressure the uniaxial crystal becomes biaxial unless the pressure direction coincides with the optic axis. Let us now consider in detail the particular case where the pressure direction is normal to the optic axis $Z°$. In addition if it were to coincide with one of the crystallographic axes (say $X°$ axis) then the biaxial angle of the deformed crystal, to a first approximation is given by

$$\sin V = \sqrt{\left| \frac{q_{11} - q_{12}}{1/n_1^2 - 1/n_3^2} \right| P} \tag{19}$$

where n_1 and n_3 are the ordinary and extraordinary refractive indices of the undeformed crystal. Unlike the cubic crystal we see that the biaxial angle is proportional to the square root of the pressure. For all crystals excepting those belonging to Group VII of Table I one of the bisectrices would not coincide with the optic axis of the undeformed crystal, but would be rotated slightly from this position by an angle determined by the first two equations in (12). Further the position of the biaxial plane is determined by $q_{11} \gtreqless q_{12}$. If $q_{11} < q_{12}$, the binormal plane is parallel to the pressure direction for a positive uniaxial crystal and perpendicular to the pressure direction for a negative uniaxial crystal. When $q_{11} > q_{12}$ the behaviour would be the opposite.

For biaxial crystals the phenomena are naturally more complex; but when once all the p_{ij}'s or q_{ij}'s have been determined, the position of the axes, the biaxial angle, etc. of the crystal under unidirectional pressure of any orientation can be worked out by the methods described in Sec. 2 (b).

(d) **CRYSTALS UNDER HYDROSTATIC PRESSURE**:— If in equation (4) we introduce the condition that $X_x = Y_y = Z_z = P, X_y = Y_z = Z_x = 0$ then the left hand side of the equation gives a measure of the change in the principal velocities and also of the rotation of the principal axes of the crystal under hydrostatic pressure. Again, as outlined in Sec. 2 (b) the various properties of the deformed crystal can be evaluated from a knowledge of p_{ij}'s and q_{ij}'s.

It is interesting to note from the schemes given in Table I that all crystals excepting those belonging to the monoclinic and triclinic systems do not experience rotation of the principal axes on hydrostatic pressure. This is true in spite of the fact that some of these groups like IV, V, VI and VIII have coefficients like q_{41}, q_{51}, q_{61} etc. One also notices that a cubic crystal remains cubic and a uniaxial crystal uniaxial under hydrostatic pressure.

3. EXPERIMENTAL METHODS

(a) OPTICALLY INACTIVE CRYSTALS

The various methods that have been adopted for the complete determination of the photoelastic constants may be broadly classified as follows :— (i) Relative retardation methods, (ii) Interference method and (iii) Ultrasonic diffraction method. In the first two, the changes in the optical properties of the crystal under compression are measured, while in the third the ratios of the photoelastic constants are obtained from a study of the polarisation characteristics of the light diffracted by ultrasonic waves in the crystal. As complete details of the experimental arrangements are given in various publications (10, 49, 52, 63, 66) only some general remarks are given below.

(i) **RELATIVE RETARDATION METHODS** :— The crystal specimen of suitable orientation is subjected to a unidirectional stress and the change in the relative retardation produced by stress, between the rays with electric vectors parallel and perpendicular to the direction of stress, is measured by the usual techniques employing Babinet compensator etc. One important condition that must be satisfied to obtain reliable results from these measurements is that the stress distribution in the specimen must be uniform. This is usually achieved by making the length of the experimental

specimen, which is also the direction of pressure at least three times the breadth, and by confining the measurements to the middle of the specimen. Normally measurements are carried out at a large number of points and the average value is taken for the evaluation of the photoelastic constants. Lead spacers are sometimes employed on both sides of the specimen as the medium for the transmission of stress from the compressing apparatus to the specimen. Lead is found to introduce additional shearing stresses in the specimen (Coker and Filon 18) while card board or paper is found to be very satisfactory (20).

Stresses of the order of 100 Kg./cm.² are normally employed in these experiments and within this range the law of proportionality (Brewster's law) is found to hold good in general except in a few cases like alkali halides, (Maris 32) AgCl (Goodman et al 23). In these cases even stresses of the order of 10 to 50 Kg./cm.² produce translational gliding and thus lead to irreversible effects.

Recently Ramaseshan and Chandrasekharan (57) have evolved a new method of measuring small birefringence usually encountered in photoelastic experiments by measuring the decrease in the apparent Faraday rotation with stress. This method has been further elaborated and perfected (54, 56) so that this method is now capable of measuring the dispersion of the mechanical birefringence very accurately even though the absolute values at any wavelength may not be as accurate as the Babinet compensator measurements. The application of this method is however normally limited to cubic crystals.

Theoretical expressions relating the birefringence produced by stress to the piezo-optic constants can be easily derived from equation (4). For example, consider the case when the pressure is applied along the X-axis and the direction of observation is Z. Then the relative retardation between the two rays polarised parallel and perpendicular to the direction of stress is given by

$$t \left(\triangle n_{11} - \triangle n_{22} \right) = t \left(\frac{n^3_{11}}{2} q_{11} - \frac{n^3_{22}}{2} q_{21} \right) P + \triangle t \left(n_{11} - n_{22} \right)$$

The second term on the right hand side represents the birefringence introduced due to the change in thickness of the specimen due to the pressure P. It is seen that the mechanical birefringence is a linear function of the two constants q_{11} and q_{21}. Thus observations made purely on the birefringence produced by stress cannot yield the absolute values of q_{11} and q_{21}. In general similar reasonings hold for other q_{ij}'s.

(*ii*) **INTERFEROMETRIC METHODS** :— In this two plane parallel crystal plates of the same orientation and dimensions are kept in the two beams of a Jamin interferometer. When one of the prisms is subjected to a unidirectional stress the interference fringes shift due to the changes in the thickness and the refractive index of the specimen. From a study of the shift of the interference fringes for light polarised parallel and perpendicular to the direction of stress, the changes in the refractive index can be evaluated after making due allowance for the change in the thickness of the specimen. An important modification of this method has been described by Ramachandran (49) which has been used for all recent interferometric measurements. In this, the shift of the localised Newtonian fringes formed between the two surfaces of the crystal specimen is measured for a known

unidirectional stress. From this the photoelastic constants can be evaluated after taking into consideration the additional path retardation introduced due to the thickness change.

Recently Vittoz (67, 68) has described a method of measuring optical path differences accurate to $\lambda/1000$. The method of measurement is based on the exploration by means of a photomultiplier of the Fraunhoffer diffraction pattern produced by three similar slits illuminated by a monochromatic parallel beam. Before being incident on the slits the light traverses three identical cubes of the specimen, of which the central one is subjected to stress during a measurement. A comparison of the luminous flux at a certain point of the diffraction pattern due to the three beams, with the intensity at the same place of the pattern due to the two lateral beams alone, yields a very accurate measure of the variation of the optical path in the central beam.

In all these methods the precautions to be observed for obtaining uniform stress distribution in the specimen are the same as described in Section 3 (i). Here again the relations between the observed changes in refractive indices and the piezo-optic constants can easily be derived from any of the equations (4). For example for the particular case mentioned in Sec. 3 (i).

$$\triangle n_{11} = \frac{n^3_1}{2} q_{11}P \; ; \; \triangle n_{22} = \frac{n^3_{22}}{2} q_{12}P.$$

Thus by measuring the absolute changes in refractive indices for various directions of pressure and observation, one obtains a number of linear relations between q_{ij}'s in terms of measurable quantities, from which all q_{ij}'s and hence p_{ij}'s can be evaluated.

(iii) **ULTRASONIC DIFFRACTION METHODS :**— An elegant method in which the uncertainty of non-uniform stress distribution is altogether eliminated has been devised by Mueller (35). In this method plane polarised light of suitable orientation is diffracted by progressive or stationary ultrasonic waves in a crystal ; and the azimuth of the polarisation of the diffracted beam with respect to that of the incident beam is determined by an analysing nicol. From a study of the rotation of the plane of polarisation of the diffracted light and its dependence on the amplitude or the intensity of the sound wave, the ratios of the elasto-optic constants can be evaluated [Mueller (35), Galt (22), Vedam (63)]. In recent years the method has been extensively used for the case of cubic crystals. For the less symmetric crystals upto orthorhombic system this method can also be used for a few particular directions of observation and propagation of ultrasonic waves in the crystal. For any general direction of propagation of the ultrasonic waves and observation the equations involved are too complicated to be of any use even in the case of cubic crystals.

(b) OPTICALLY ACTIVE CRYSTALS

It is obvious that the above methods cannot be directly used in the case of circularly or elliptically birefringent crystals. This is because, in these crystals the only vibrations that are propagated without change are circularly or elliptically polarised vibrations and not linearly polarised vibrations as in the case of linearly birefringent crystals. The actual computation of the changes of the optical characteristics of these crystals on deformation, are extremely complicated if one were to use the electromagnetic theory of the propagation of light in crystals. Even

so, using this method, Pockels (40), who was the first to study the photoelastic behaviour in such crystals, did succeed in evaluating all the constants for quartz.

Recently Ramachandran and Chandrasekharan (52) have determined the photoelastic constants of $NaClO_3$, making use of the Poincaré sphere concept of the propagation of light in an optically active crystalline medium. This method completely obviates the necessity to use extremely cumbersome algebraic formulae. Space does not permit us to give a full description of the Poincaré sphere and its particular utility in photoelastic studies in optically active media but reference may be made to the various publications on this topic (52, 54).

Here again the method essentially consists in (i) determining the linear birefringence produced by a known stress by measuring the changes in the polarisation characteristics of the light transmitted by the deformed crystal (ii) and by measuring the ratios of the photoelastic constants by the ultrasonic method. For the latter, Mueller's theory and also the experimental technique have to be suitably modified to take into effect the optical activity of the medium (Vedam and Ramachandran 66).

(c) ACCURACY

In the relative retardation measurements the accuracy obtainable is chiefly dependent on the uniformity of stress distribution. However, under favourable conditions one can get results accurate to 1% or even better. On the other hand, in the interferometric method, the accuracy of the results is rather discouragingly poor for the following reasons. In this method, the major contribution to the path retardation is due to the change in the thickness of the specimen and hence the change in refraction which is the object of the investigation, is obtained as a small difference between two large quantities. Further, the thickness change itself cannot be evaluated accurately, for the elastic moduli $s_{ij} (i, j = 1, 2, 3)$ on which the thickness change depends, are usually had with an error of 10% or more. Thus even though one can determine the absolute path retardation to $\lambda/1000$ (Vittoz 68) the value of the piezo-optic constants evaluated is susceptible to a quite large error. However, since this error due to the thickness will be constant, for a study of the dispersion of the photoelastic constants the method described by Vittoz would be found very suitable. Using the ultrasonic method measurements have so far been carried out visually and the accuracy obtainable is not very high. However, with the aid of photoelectric devices, one can obtain the ratio of the photoelastic constants to within 0·5% or even better.

4. DISCUSSION OF RESULTS

A glance at the tables reveals immediately the marked disagreement between the values of various authors for the same crystal. As has been mentioned before, the accuracy obtained in the values of the absolute constants p_{ij} is not very high. But one might expect tolerable agreement within reasonable limits of error at least between the values of $(p_{11} - p_{12})$, and p_{44} which are determined directly from relative retardation measurements. Again we find the discrepancies between the results far exceed the experimental errors. From a series of experiments carried out in this laboratory (Vedam) it is found that the experimental values are dependent on (a) the inequality of the stress distribution in the specimen during static measure-

ments (b) conditions under which the crystals were grown and annealed (c) previous thermal and elastic history of the specimen and (d) the impurities, defects and dislocations present in the crystals. In this connection, mention may be made of the observations of Ballard et al (1) on the large variations of the mechanical and optical properties of air grown and vacuum grown LiF crystals. The refractive index and optical dispersion of KBr is also found to vary markedly from specimen to specimen (Rodney and Spindler 59). Further, it is well known that the values of the elastic constants of any crystal determined by various authors, do not agree amongst themselves. As the values of the elastic constants used in photoetastic studies by different workers are different from one another, it is not surprising to find that discrepancies are exhibited in a pronounced form in the photoelastic constants. It would have been preferable if each observer had mentioned in his paper the actual values of the elastic constants used in the evaluation of the photoelastic constants. In spite of the limitations mentioned above, a few general observations can be made regarding the photoelastic behaviour of crystals.

As was mentioned earlier, Pockels (42), purely from the phenomenological theory predicted that four possible types of cubic crystals of T_d, O, O_h classes exist (Table II) depending on the sign and magnitude of χ. He could get experimental evidence for the existence of crystals belonging to the second, third and fourth groups. Mueller (34) on the other hand classified cubic crystals of these classes into four possible groups purely from the signs of $(p_{11} - p_{12})$ and p_{44} as is shown in Table XI. The classification of Pockels is also given in the same table for comparison. There seems to be a tendency amongst workers in this field to identify the four groups of Mueller with the four proposed by Pockels. From Table XI, one notices that Pockels' classification not only depends on the signs of $(p_{11} - p_{12})$ and p_{44} but also on their respective magnitudes, and on the magnitudes of the elastic constants. This difference in the classification is brought out very clearly in NH_4Cl which belongs to group III according to Mueller but to group (4) of Pockels since for this substance $(p_{11} - p_{12})$ $= -0.095$, $p_{44}=0.025$, $s_{44}/(s_{11}-s_{12}) = 147/31\cdot6$ and hence $\chi = -1\cdot2$. It seems that as far as the complete description of the optical behaviour of crystals under stress is concerned Pockels' classification is more useful.

Purely from some arbitrary limits of a few parameters set from data on crystals like NaCl, KCl and glasses Mueller (34) made some very general conclusions about the classification of cubic crystals by photoelastic means. He predicted that crystals of the NaCl type, such as K, Rb, NH_4 halides and the bivalent salts BaO, CaO, etc. would belong to group IV while the Li and Na halides would belong to group II. Recent measurements on KBr, KI, LiF, AgCl, have confirmed the above predictions. He also stated that the crystals of the CsCl and CaF_2 type should belong to group III. This has been found to be true in NH_4Cl, a crystal belonging to the CsCl type. However, West and Makas (69) have found that thallium halides (CsCl type) actually belong to group I, i.e., they behave like positive uniaxial crystals when strained along the [100] and [111] directions. These crystals belong to group (1) of Pockles as well and indeed are the only examples for this group for which experimental evidence was lacking so far. Since lead glasses of high refractivity are known to show positive birefringence, one could naturally expect a similar behaviour in such highly refracting substances like the thallium halides (West and Makas 69).

Further, according to Mueller, all crystals of the ZnS and Na_2O type should belong to group III irrespective of whether the binding is homopolar or ionic. The existing data for the photoelastic constants of diamond and for p_{44} of ZnS do not support Mueller's predictions which need some modification (Burstein and Smith 15). However, the agreement in the case of NH_4Cl suggests that more experimental data are necessary to clarify this question. It is not out of place to point out here the enormous discrepancy between the two experimental determinations of p_{44} of ZnS. While West and Makas (69) using Schramm's (61) data find the p_{44} value to be the largest on record, Kara, Mathieu and Poullet (29) find it to be the least.

Some of the other interesting results that one notices is that the values of $(p_{11} - p_{12})$ and p_{44} for KCl>KBr>KI. It is very interesting to note that as AgCl resembles KCl (as the radii of Ag and K ions are nearly the same), photoelastically it behaves like KCl and KBr and belongs to group IV. MgO is the only crystal which has negative values of both p_{11} and p_{12} while diamond has a —ve value for only p_{11}. The exact significance of these results will become apparent when they are examined from the point of view of Mueller's theory as has been done in the next section.

Amongst the isomorphous nitrates of Ba, Sr and Pb, $(p_{11} - p_{12})$ and $(p_{11} - p_{13})$ are much higher for the Ba salts than for the others. But this type of anomaly is found to exist in other properties like melting point, elastic constants, thermal expansion, solubility, magnetooptic anomaly, etc. The reason for this behaviour is probably due to the presence of slight covalent binding in $Ba(NO_3)_2$ which is indicated by the appearance of the lattice lines in its Raman spectrum more intensely than those for Pb and Sr nitrates, (Couture and Mathieu 19).

In potassium alum, where the refraction is mainly due to the oxygens of SO_4 ions, the symmetry of this group combined with the small elastic anisotropy makes the photoelastic anisotropy very small. Amongst the crystals of T_h class thallium alum alone has positive values of p_{44}.

A comparison of the results of NaCl and $NaClO_3$ shows that the magnitudes of the photoelastic constants are very similar to each other. This is obviously due to the fact that the $NaClO_3$ structure is based on slightly distorted NaCl lattice and the refractive indices of the two are also nearly the same.

Even though five out of the seven q_{ij}'s of the tetragonal system (group 7 of Table I) and also a linear term involving the remaining two constants can be determined purely from relative retardation measurements, values are reported for only two crystals, namely ADP and KDP. Further as all the q_{ij}'s have not been measured, the p_{ij}'s could not be evaluated and hence only q_{ij}'s have been reported in Table VI.

Beryl of the hexagonal system, quartz and calcite of the trigonal system are the only uniaxial crystals for which complete data are available. The various predictions deduced from the phenomenological theory mentioned in Sec. 2 have been experimentally verified in all these crystals by Pockels. For a unidirectional pressure applied normal to the optic axis, these crystals become biaxial. In addition, if the directions of applied pressure coincides with one of the crystallographic axes, the biaxial angle 2V can be calculated using eqn. (19) and for a pressure of 1 Kg/mm.^{2}

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they have the following values:—beryl 6°15′, quartz 5°54′, calcite 1° 30′. The optic axial angle in the case of deformed quartz has been directly measured by Pockels (40) and Beaullard (3) and the values are in very good agreement with the calculated values.

Amongst the optically biaxial crystals the photoelastic behaviour of topaz (21) and barite (64) both crystallising in the orthorhombic holohedral class has been reported. It is seen from Table (IX) that the wide disparity between the elastic constants of these two crystals is reflected in the photoelastic behaviour as well. The p_{ij}'s of topaz are in general much smaller than those of barite. On applying hydrostatic pressure, the principal refractive indices of both these crystals are found to increase. In topaz the optic axial angle increases on hydrostatic pressure while the opposite effect is found in barite. It may be pointed out that while the acute bisectrix in topaz is the c-axis that in barite is the a-axis.

A survey of the results given in Tables (IV to IX) for all systems reveals that p_{hk} ($h, k = 1, 2, 3$) of ionic crystals are in general, of the same order of magnitude and are much larger than p_{1l} ($l = 4, 5, 6$) ; but such differences in values do not exist in covalent crystals like, diamond, topaz and MgO.

An exceedingly interesting point that has emerged from the studies on quartz by Pockels (40) and on $NaClO_3$ by Ramachandran and Chandrasekharan (52) is that the rotatory power of the substance is sensibly constant in the range of stress usually employed in photoelastic studies.

We shall next mention, in passing, the relationship of the photoelastic properties of crystals with other closely allied phenomena. Since both on the application of compressive stress and on lowering of the temperature, the specific volume of a substance diminishes, it would be interesting to enquire whether the changes in refractive index for identical changes in volume produced by these effects would be the same. Calculations show that they are not and in fact in some crystals even the signs of the refractive index changes are opposite in the two cases. This indicates that a factor affecting the refractive index purely caused by a change in temperature, is operative in addition to that due to a change in the lattice dimensions. The existence of this factor was discovered as early as 1902 by Pockels but a satisfactory explanation of the phenomenon has been developed only recently by Ramachandran (49, 50, 53).

The phenomenon of photoelasticity plays a significant role in thermal scattering of light in crystals. [Mueller (35), Chandrasekharan (17a)]. Due to the existence of elastic waves of thermal origin in a solid, say in a cubic crystal, each element of volume gets deformed and hence the optical polarisability of each ion in the element becomes anisotropic. The numerical computation of this anisotropy factor introduced in the polarisability of each ion by both longitudinal and transverse waves in NaCl has been worked out by Marck and Taurel (31) using the elasto-optic constants of Pockels.

5. THEORY OF PHOTOELASTICITY

No review on the subject of photoelasticity would be complete without a reference to the various theories that have been put forward to explain this phenomenon. In this section we shall briefly deal with the general theory developed

by Mueller (34) of which earlier attempts by Banerjee (2) and Herzfeld and Lee (24, 25) form a part. When a solid is stressed its symmetry is altered and the calculation of photoelastic constants is based on the evaluation of the changes in (1) the Coulomb field, (2) the Lorenz-Lorentz field and (3) the intrinsic polarisability of the ions. As these computations are extremely complicated for crystals of lower symmetry, the theory has so far been worked out only for isotropic solids and cubic crystals.

The contribution of the Coulomb field to the refractive index of an undeformed cubic crystal is zero by virtue of its symmetry and would be significant only when the crystal is deformed anisotropically. The changes in the Lorentz field, however, are dependent on the alteration of both the density and the symmetry. For example when a cubic crystal is subjected to hydrostatic pressure, while the contribution of the Coulomb field to the change in the refractive index would be zero, that due to the Lorentz field cannot be neglected. When a cubic crystal is unidirectionally stressed along the cube normal, the deformation reduces the crystal symmetry to a tetragonal one, causing anisotropy in the Coulomb and the Lorentz fields. The effects of these can be evaluated by using Bragg's method of calculating the birefringence (Banerjee) or the Lattice sum method of Madelung (Herzfeld and Lee) or from Born's theory of crystal lattices (Mueller).

However, the third effect, namely, the changes in the intrinsic atomic polarisabilities due to the strain cannot be computed so easily since it requires a knowledge of the optical strengths, the polarisabilities and also strain-polarisability constants of the ions. While attempts have been made to evaluate the former two, there is no way as yet for obtaining the strain-polarisability constant from fundamental considerations. This is because, in an elastically deformed solid, there is a distortion of the arrangements of the atoms, which strengthens some of the chemical bonds while weakening some others. It is most natural to expect that this in its turn would alter the electronic configuration of each atom. It should be possible in principle, to calculate the changes in the optical polarisability with the aid of quantum mechanics. Unfortunately this has not been possible as we do not know the state of the electronic atmosphere even in a simple undeformed crystal. However, an estimate of the strain-polarisability constant can be had from the measured values of the photoelastic constants and from the values of Lorentz and Coulomb contributions calculated from Mueller's theory.

We shall give below in the briefest outline the results of Mueller's theory.

(a) **LORENTZ FIELD IN STRAINED D* LATTICES** :—In a D lattice let the elementary cell contain several different atoms at positions r_j, r_k ...... etc. When a light wave is incident let the components of the moments induced in the ions be μ_{ix}, μ_{kx} ...... ; μ_{iy}, μ_{ky} ...... ; μ_{iz}, μ_{kz} ...... etc. along the x, y and z directions. The field at the atom i due to all the other dipoles, in the undeformed state is given by

$$F_{ix} = E_x + \frac{4\pi}{\triangle} \sum_k L_{ikx} N_{kx} \mu_{kx} \qquad (20)$$

* In diagonal or D lattices the atoms of the elementary cell are located on the body diagonal of the elementary cube. The face and body centred cubic lattices of NaCl, CsCl, CaF$_2$, ZnS and diamond are D lattices.

where $\triangle$ is the volume of the elementary cell, N_k the number of atoms of the kind k in it and $\sum L_{1kx}$ is the Lorentz factor which would be $1/3$ for the crystal in the undeformed state. When the lattice is deformed by a strain z_z the above equation becomes

$$\overset{L}{F_{1x}} = E_x + \frac{4\pi}{\triangle} \sum_k N_{kx}\, \mu_{xk}\, (1/3 + s_{1kx}\, z_z) \qquad (21)$$

where $s_{1x} = \dfrac{d\, L_{1x}}{dz_z}$ which to a first degree of approximation, is

$$s_{1x} = \{\, L_{1x}\,(1 + z_z) - 1/3\,\}\,/z_z \qquad (22)$$

Similarly for a shear,

$$x_x' = -y_y' = x_y/2$$

$$\overset{L}{F_{1x}'} = E_x + \frac{4\pi}{\triangle} \sum_k N_{kx}'\, \mu_{kx}'\, (1/3 + s_{1kx}'\, x_x') \qquad (23)$$

Mueller has numerically evaluated the values of L_{1kx}, s_{1kx}, s_{1kx}' for various D lattices for known values of small deformation. It must be mentioned that no account is taken here of the " inner displacements " of atoms of the crystal which are responsible for the piezo-electric effect and also for the invalidation of the well-known Cauchy relationships. This effect has been neglected as the magnitude of these displacements are not known.

(b) **THE COULOMB FIELD** :—Under the influence of the light wave with the electric vector in the X direction the atom i becomes an oscillator which can be considered as a vibrating electron with charge $f_i e$. The forces acting on the electron are the binding forces of the atom i, the Lorentz force of the light wave, and the Coulomb forces of all the ions.

In an elastically deformed D lattice, we get Coulomb field as

$$\overset{C}{F_{1x}} = \frac{4\pi\, \mu_{1x}\, z_z}{f_1\, \triangle} \sum z_k N_k\, s_{1kx} \qquad (24)$$

where z_k is the valence of the ion k.

(c) **STRAIN-POLARISABILITY EFFECTS** :—As has been mentioned earlier, this problem can be tackled only from the phenomenological point of view. One could assume for small deformations a linear proportionality between polarisability and the strain. If the molar refraction of the kth atom is represented by R_k in the undeformed state and by R_{kx}, R_{ky}, R_{kz} in the deformed state we have for a strain

$$R_{kx} = R_{ky} = \frac{4\pi\, A\, \alpha_{kx}}{3} = R_k\,(1 + \lambda_{kx}\, z_z) \qquad (25a)$$

$$R_{kz} = \frac{4\pi\, A\, \alpha_{kz}}{3} = R_k\,(1 + \lambda_{kz}\, z_z)$$

Here α_k represents the polarisability of the kth atom and A the Avagadro's number, and λ_k is the strain polarisability or atomic anisotropy factor.

Similarly for a shear

$$R_{kx}' = R_k\,(1 + \lambda_{kx}'\, x_x')\ ;\ R_{ky}' = R_k\,(1 + \lambda_{ky}'\, x_x') \qquad (25b)$$

For a hydrostatic deformation, therefore, the change in the refractive index due to strain polarisability is given by

$$\lambda_0 = 1/3 \,(\lambda_z + 2\lambda_x) \tag{26}$$

Under the influence of the Lorentz and Coulomb forces only, the dipole moments induced in the atom i is

$$\mu_{ix} = \alpha_{ix} \,(\overset{L}{F_{ix}} + \overset{C}{F_{ix}}) \tag{27}$$

If the strain polarisability effect is also considered the altered value of the polarisability α_i is given by equation (25). Taking this and substituting the values of $\overset{L}{F_{ix}}$ and $\overset{C}{F_{ix}}$ from (21) and (24) in equation (27) the moment induced in the ion is given by

$$\mu_{ix} = 3R_i \,(1 + \lambda_{ix}\, z_z) \, A^{-1} \left[\frac{E_x}{4\pi} + \frac{1}{\triangle} \, \sum N_k \,\mu_{kx}\, (1/3 + s_{ikx}\, z_z) \right.$$
$$\left. + \frac{\mu_{ix}\, z_z}{f_i\, \triangle} \, \sum Z_k\, N_k\, s_{ikx} \right] \tag{28}$$

This with

$$\frac{n^2{}_x - 1}{4\pi} = \frac{P_x}{E_x} = \sum N_k \,\mu_x/E_x \tag{29}$$

yields the requisite equation for the refractive indices of the deformed solid. For $z_z = 0$ this equation reduces to the familiar Lorentz-Lorenz equation. On differentiation of this equation for the deformed solid we get

$$\left.\begin{aligned}
p_x &= p^d + \overset{L}{p_x} + \overset{C}{p_x} + \overset{A}{p_x} \\
p_z &= p^d + \overset{L}{p_z} + \overset{C}{p_z} + \overset{A}{p_z} \\
p'_{x'} &= \overset{L}{p_{x'}} + \overset{C}{p_{x'}} + \overset{A}{p_{x'}}
\end{aligned}\right\} \tag{30}$$

where

$$\left.\begin{aligned}
p_x &= \frac{-2n}{(n^2-1)^2} \frac{dn_x}{dz_z} = n^4 p_{12}/(n^2 - 1)^2 \\
p_z &= \frac{-2n}{(n^2-1)^2} \frac{dn_z}{dz_z} = n^4 p_{11}/(n^2 - 1)^2 \\
p'_{x'} &= \frac{-2n}{(n^2-1)^2} \frac{dn_{x'}}{dx_{x'}} = 2n^4 p_{44}/(n^2 - 1)^2
\end{aligned}\right\} \tag{31}$$

and p^d is the contribution due to the change in density and is given by

$$p^d = (n^2 + 2)/3 \,(n^2 - 1) \tag{32}$$

$\overset{L}{p_x}$, $\overset{C}{p_x}$ and $\overset{A}{p_x}$ are respectively the contributions due to the *anisotropy* of the Lorentz force, Coulomb force and the strain-polarisability effect and have the following values

$$\overset{L}{p_x} = -\sum_{ik} S_{ikx}\, N_i N_k R_i R_k/[\sum N_k R_k]^2 \tag{33}$$

$$\overset{C}{p_x} = -\sum_i \frac{R_i{}^2 N_i}{f_i} \sum_k N_k z_k s_{ikx}/[\sum N_k R_k]^2 \tag{34}$$

$$\overset{A}{p_x} = \frac{-(n^2 + 2)}{3\,(n^2 - 1)} \lambda_x \tag{35}$$

and $\qquad \lambda_x = \sum \lambda_{ix}\, N_i R_i/\sum N_i R_i. \tag{36}$

Equations analogous to these can be written for p_z and $p'_{x'}$.

In equation (30), p_x is known experimentally from (32). The quantities p^d, p_x^C and p_x^L can be calculated from formulae (32, 33 and 34) using the values of s_{1kx} data given by Mueller. The strain polarisability constant λ_x can therefore be evaluated. It must, however, be borne in mind that values computed for p_x^L, p_z^L, p_x^C and p_z^C are not very accurate as one has to use empirical values of f and the molar refraction of the ions in the solid state.

If one is interested only in the birefringence produced by a strain z_z we get from (6·11)

$$p = p_z - p_x = \frac{n^4 (p_{11} - p_{12})}{(n^2 - 1)} = p^L + p^C + p^A \qquad (37)$$

where

$$p^L = -3 p_x^L ; \quad p^C = -3 p_x^C ; \quad \text{and } p^A = \lambda (n^2 + 2)/3(n^2 - 1)$$
$$\text{and } \lambda = \lambda_z - \lambda_x.$$

Similarly the birefringence introduced by a shear is given by

$$p' = p_{x'}^L + p_{x'}^C + p'^A$$

where

$$p'^A = - \lambda' (n^2 + 2)/3(n^2 - 1)$$

and

$$\lambda' = \sum \tfrac{1}{2} [\lambda_{1x}' - \lambda_{1y}'] N_1 R_1 / \sum N_1 R_1$$

We shall now briefly discuss the experimental data on cubic crystals in the light of the above theory. It is well known that the refractivity of a substance is less in the solid state than in the liquid or gaseous states. Recent computations by Shockley (60) have shown that the polarisability of a negative ion decreases and that of a positive ion increases as they go from an independent gaseous state to the solid state. One could therefore expect further changes in the polarisability of the ions in the same direction when the lattice constant of the crystal is reduced by stress. As the contribution to the refractive index by the anions far exceed that of the cations the net effect would be to reduce the polarisability, *i.e.*, p_x^A in equation (30) would in general be —ve which is actually found to be the case.

No such general conclusion can be drawn as regards the Lorentz and Coulomb contributions. The magnitudes and signs of these are dependent on the directions of stress and observation and also on the particular crystal. Finally the density contribution p^d is always positive and in most D lattices it overwhelms the combined effects of p_x^L, p_x^C and p_x^A thereby yielding positive values of p_{11} and p_{12}. This is not a general rule ; for example, in MgO, since the atomic anisotropy effect is much larger than the combined effect of the other three one gets negative values for p_{11} and p_{12}.

One notices from equations (30) and (37) that the density effect p^d does not play any role in the birefringence produced by unidirectional or shearing stresses. Hence the magnitude and the sign of the birefringence are determined by the relative magnitudes and signs of the contributions p_x^L, p_x^C and p_x^A. Actual computation of p_x^L, p_x^C etc. have been carried out by Mueller for NaCl, KCl, and CaF$_2$ making the assumption that the f values of the ions are the same as those of the corresponding inert gases. The values of λ evaluated by him for these substances are given in Table (XII). The λ values of CaF$_2$ have not been included in this table, as a rather questionable value of f had to be used in the computation. The λ values of a typi-

cally covalent crystal like diamond have also been calculated making use of the revised data of the photoelastic constants (Ramachandran 51) and are included in the table.

It is seen from the Table (XII) that NaCl and KCl in spite of having completely different photoelastic properties, have practically identical values of $\lambda = (\lambda_z - \lambda_x)$. This is as is to be expected since the refraction of these substances is largely due to Cl ions. One notices that in ionic crystals λ is very much greater than λ'. Mueller explains this as due to the fact that strain alters the distance between the nearest neighbours, while in a shear the Na-Cl or K-Cl distance remains unchanged. On the other hand in the completely homopolar crystal Diamond λ' is greater than λ. For stress along [111] or [110] directions corresponding to the measurements of the shear constant p_{44} one notices from the structure of diamond that the C-C bond lengths and angles are altered to a maximum degree. Hence it is not surprising to find that the polarisability changes are very much larger for a shear than for a linear strain for diamond. In the calculation of λ for diamond the Coulomb field has naturally been taken as zero. In the case of a partly co-valent crystal like MgO one is faced with the serious problem of assigning proper values for z and f in the determination of the Coulomb contribution and thence the final evaluation of λ.

Here it may be mentioned that Mueller, setting arbitrary limits for the values of the atomic anisotropy factor obtained from NaCl, KCl, and a few glasses, was able to predict the photoelastic behaviour of various cubic crystals depending on their values of R_1 and f_1. As has been remarked earlier many of these predictions have been found to be true.

The uncertainties introduced in the numerical evaluations of p_X^L, p_X^C etc. due to the use of empirical values of R_1 and f_1. are eliminated if one considers the refractive index change due to hydrostatic pressure. In the case of hydrostatic pressure, since the symmetry of the crystal is not altered, both p_X^C and p_X^L which are dependent on the anisotropy, are zero. The change in the refractive index with density may be obtained as (from 26, 30, 32, 35).

$$\frac{dn}{d\rho} = (1 - \lambda_0)\frac{(n^2 + 2)(n^2 - 1)}{6n\rho}$$

while the value calculated from the experimental data is given by

$$\frac{dn}{d\rho} = \frac{n^3}{6\rho}(p_{11} + p_{12} + p_{13}).$$

From these two equations, the value of λ_0 can be evaluated. Table (XIII) gives the values of $dn/d\rho$ observed and calculated and the values of λ_0 for different cubic crystals. In most crystals the calculated values of $dn/d\rho$ are much higher than the observed values showing the negative contribution of the strain-polarisability effect. Further it is seen that diamond and MgO stand apart in having $-$ve values of $dn/d\rho$, i.e., an increase in density actually decreases the refractive index. Similar anomalous behaviour is exhibited by these crystals in the variations of their refractive indices with temperature, viz., dn/dt is positive (48) (50). It will be most interesting to investigate whether ZnS which is another cubic crystal exhibiting $+$ve dn/dt also shows a similar behaviour in $dn/d\rho$.

Before we proceed to the discussion of the significance of λ_0 in different crystals it must be remembered that much importance should not be attached to individual values of λ_0 as they have been derived from not too accurate data of p_{11}, p_{12} and p_{13}.

One notices that the λ_0 value for most ionic crystals is much less than that for crystals that are known to be covalent. In other words the changes in polarisability with density in covalent crystals are much larger than those for ionic crystals. Even amongst ionic crystals one notices that LiF, has the largest value of λ_0. This is quite in accord with the fact that it exhibits the maximum covalent character amongst the alkali halides (vide solubility, melting point, elastic constants, etc.).

According to Burstein and Smith (16) the decrease in the polarisability of an ion with interatomic distance in ionic crystals is primarily due to a change in the ion overlap and consequently to a change in the relative amounts of ionic and homo-polar binding. One could, therefore, expect these effects to depend on the ionic and homopolar binding already present in the crystal in the undeformed state. Thus λ_0 can be considered to be a measure of ionic overlap and the homopolar binding in crystals. This accounts for the large variation of λ_0 from crystal to crystal. Burstein and Smith have also shown in a comparative table the parallelism between the values of λ_0 for various crystals and physical constants like C_{44}/C_{12}, volume expansion melting point and Debye temperature which are also considered measures of the ionic overlap and homopolar binding.

One notices from Table (XIII) that ammonium alum, thallium alum and lead nitrate are the only crystals that exhibit negative values of λ_0. The reason for this is not far to seek. In these crystals, the polarisability of the cation is of the same order of magnitude or sometimes even much larger than that of the anion. As discussed earlier a decrease in the lattice parameter, while diminishing the polarisability of the anions increases that of the cations. It is quite evident that in these crystals the latter effect more than compensates that of the former thus yielding negative values of λ_0.

The calculated values of $\left(\dfrac{dn}{d\rho}\right)$ and λ_0 in Table XIII are on the basis of the Lorentz dispersion formula. The values of $(dn/d\rho)$ and λ_0 calculated using a formula of the Drude type have also been entered in the Table [Burstein and Smith (16), Vedam (65).] One notices that while the actual value of λ_0 is dependent on the type of the dispersion equation used, the magnitude and sign of λ_0 show in general the same trend. Hence the conclusions drawn above will hold equally well for the Drude type of formula.

In the case of K-alum one notices that while λ_0 calculated from the Lorentz formula is positive, the Drude formula gives a negative value of λ_0. This indicates the importance of the correct type of dispersion equation to be used in these studies. We shall not take up this question in this present review.

6. DISPERSION OF PHOTOELASTIC CONSTANTS

From what has been said in the previous section it is clear that the effect of the change in the Lorentz field, Coulomb field and the atomic anisotropy would be to alter the energy levels and the transition probabilities of the electrons for any

particular substance. Hence if one is just interested in the dispersion of the stress optic coefficient and not in the individual effects one could assume (Ramaseshan and Sivaramakrishnan 58) that the stress alters the three parameters N_i the number, f_i the transition probability and ν_i the frequency of the dispersion electron. For a cubic crystal if one starts with a dispersion formula of the type

$$n^2 - 1 = \sum \frac{A_i N_i f_i}{\nu_i^2 - \nu^2} \tag{38}$$

then for unidirectional pressure the changes in the refractive index for light with electric vector parallel to the direction of pressure is obtained by differentiating equation (38) as

$$\frac{dn_\parallel}{dp} = \frac{1}{n} \sum \frac{A_i N_i f_i \nu_i^2}{(\nu_i^2 - \nu^2)^2} [K_i]'_\parallel + \frac{1}{2} \frac{A_i N_i f_i}{(\nu_i^2 - \nu^2)} [K_i]''_\parallel + \frac{1}{2} \frac{A_i N_i f_i}{2(\nu_i^2 - \nu^2)} [K_i]'''_\parallel \tag{39}$$

where $[K_i]'_\parallel$, $[K_i]''_\parallel$ and $[K_i]'''_\parallel$ are respectively

$\left(\frac{1}{\nu_i} \frac{d\nu_i}{dp} \right)_\parallel$, $\left(\frac{1}{f_i} \frac{df_i}{dp} \right)_\parallel$ and $\left(\frac{1}{N_i} \frac{dN_i}{dp} \right)_\parallel$. The last term $\frac{1}{N} \frac{dN}{dp}$ is equal to $-\frac{1}{\rho} \frac{d\rho}{dp}$ which can be calculated from the elastic constant data. Since the sum of the transition probabilities must necessarily be equal to unity, we get $\sum [K_i]'' f_i = 0$ in equation (39).

An equation similar to 39 is valid for $dn_\perp/dp$. From these two formulae one can easily calculate the dispersion formula for the birefringence produced by unidirectional pressure and for changes in the refractive index due to hydrostatic pressure. The values of K will naturally depend on the direction of observation and pressure. The extension of this theory to crystals of lower symmetry is obvious. Using accurate dispersion formulae, the photoelastic dispersion of some glasses [Ramaseshan and Sivaramakrishnan (58)] and of fused quartz (Jog 28) have been explained satisfactorily with the aid of the above formula.

The only data on cubic crystals available are those of Iyengar (26) which are given in Table (XIV). As the measurements are confined to only three wavelengths in the visible region, the verification of the dispersion formula could not be undertaken. It may be mentioned that the dispersion of the photoelastic constants of most of these substances is of the order of magnitude to be expected from the above formula. The values of the dispersion of p_{44} of LiF, however, whose absorption wavelength responsible for dispersion is much below $0 \cdot 1$ μ seems to be inordinately large.

Eppendahl (21) has also recorded the dispersion of some of the photoelastic constants of topaz in the region $\lambda\, 6880 - \lambda\, 4470$.

7. PHOTOELASTIC CONSTANTS DATA

In the following Tables the data on the elasto-optic constants of the various crystals belonging to different crystal classes have been collected. The references of the authors whose elastic constants data have been used for the calculation of the constants have also been included in the Tables. All these values unless otherwise stated are for $\lambda\, 5893$. Galt (22) has measured the ratios of the photoelastic constants by the ultrasonic method for NaCl, KCl, KBr and his values are given in Table X.

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The photoelastic constants of most of the cubic crystals given in Tables (IV & V) have been evaluated by combining the measurements of $(p_{11}-p_{12})$ and $(p_{11}-p_{13})$ by relative retardation methods and of p_{11}/p_{12} and p_{11}/p_{13} by ultrasonic method. As the errors involved in these measurements may be about 2-3%, the absolute values of p_{11}, p_{12} and p_{13} are accurate to only within 5-10%. In some cases the errors involved may be even as much as 20%.

TABLE I

GROUP 1. TRICLINIC SYSTEM, C_1, C_i (1, $\bar{1}$) (36 Constants)

p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}	q_{11}	q_{12}	q_{13}	q_{14}	q_{15}	q_{16}
p_{21}	p_{22}	p_{23}	p_{24}	p_{25}	p_{26}	q_{21}	q_{22}	q_{23}	q_{24}	q_{25}	q_{26}
p_{31}	p_{32}	p_{33}	p_{34}	p_{35}	p_{36}	q_{31}	q_{32}	q_{33}	q_{34}	q_{35}	q_{36}
p_{41}	p_{42}	p_{43}	p_{44}	p_{45}	p_{46}	q_{41}	q_{42}	q_{43}	q_{44}	q_{45}	q_{46}
p_{51}	p_{52}	p_{53}	p_{54}	p_{55}	p_{56}	q_{51}	q_{52}	q_{53}	q_{54}	q_{55}	q_{56}
p_{61}	p_{62}	p_{63}	p_{64}	p_{65}	p_{66}	q_{61}	q_{62}	q_{63}	q_{64}	q_{65}	q_{66}

GROUP 2. MONOCLINIC SYSTEM, C_S, C_2, C_{2h} (m, 2, 2/m) (20 Constants)

p_{11}	p_{12}	p_{13}	0	0	p_{16}	q_{11}	q_{12}	q_{13}	0	0	q_{16}
p_{21}	p_{22}	p_{23}	0	0	p_{26}	q_{21}	q_{22}	q_{23}	0	0	q_{26}
p_{31}	p_{32}	p_{33}	0	0	p_{36}	q_{31}	q_{32}	q_{33}	0	0	q_{36}
0	0	0	p_{44}	p_{45}	0	0	0	0	q_{44}	q_{45}	0
0	0	0	p_{54}	p_{55}	0	0	0	0	q_{54}	q_{55}	0
p_{61}	p_{62}	p_{63}	0	0	p_{66}	q_{61}	q_{62}	q_{63}	0	0	q_{66}

GROUP 3. ORTHORHOMBIC SYSTEM, C_{2v}, V, V_h (mm, 222, mmm) (12 Constants)

p_{11}	p_{12}	p_{13}	0	0	0	q_{11}	q_{12}	q_{13}	0	0	0
p_{21}	p_{22}	p_{23}	0	0	0	q_{21}	q_{22}	q_{23}	0	0	0
p_{31}	p_{32}	p_{33}	0	0	0	q_{31}	q_{32}	q_{33}	0	0	0
0	0	0	p_{44}	0	0	0	0	0	q_{44}	0	0
0	0	0	0	p_{55}	0	0	0	0	0	q_{55}	0

GROUP 4. TRIGONAL SYSTEM, C_3, C_{3i} (3, $\bar{3}$) (12 Constants)

p_{11}	p_{12}	p_{13}	p_{14}	$-p_{25}$	p_{62}	q_{11}	q_{12}	q_{13}	q_{14}	$-q_{25}$	$2q_{62}$
p_{12}	p_{11}	p_{13}	$-p_{14}$	p_{25}	$-p_{62}$	q_{12}	q_{11}	q_{13}	$-q_{14}$	q_{25}	$-2q_{62}$
p_{31}	p_{31}	p_{33}	0	0	0	q_{31}	q_{31}	q_{33}	0	0	0
p_{41}	$-p_{41}$	0	p_{44}	p_{45}	p_{52}	q_{41}	$-q_{41}$	0	q_{44}	q_{45}	$2q_{52}$
$-p_{52}$	p_{52}	0	$-p_{45}$	p_{44}	p_{41}	$-q_{52}$	q_{52}	0	$-q_{45}$	q_{44}	$2q_{41}$
$-p_{62}$	p_{62}	0	p_{25}	p_{14}	$\frac{(p_{11}-p_{12})}{2}$	$-q_{62}$	q_{62}	0	q_{25}	q_{14}	$(q_{11}-q_{12})$

GROUP 5. TRIGONAL SYSTEM, C_{3v}, D_3, D_{3d} ($3\,m$, 32, $\bar{3}\,m$)
(8 Constants)

p_{11}	p_{12}	p_{13}	p_{14}	0	0	q_{11}	q_{12}	q_{13}	q_{14}	0	0
p_{12}	p_{11}	p_{13}	$-p_{14}$	0	0	q_{12}	q_{11}	q_{13}	$-q_{14}$	0	0
p_{31}	p_{31}	p_{33}	0	0	0	q_{31}	q_{31}	q_{33}	0	0	0
p_{41}	$-p_{41}$	0	p_{44}	0	0	q_{41}	$-q_{41}$	0	q_{44}	0	0
0	0	0	0	p_{44}	p_{41}	0	0	0	0	q_{44}	$2q_{41}$
0	0	0	0	p_{14}	$\frac{(p_{11}-p_{12})}{2}$	0	0	0	0	q_{44}	$(q_{11}-q_{12})$

GROUP 6. TETRAGONAL SYSTEM., C_4, S_4, C_{4h} (4, $\bar{4}$, $4/m$)
(10 Constants)

p_{11}	p_{12}	p_{13}	0	0	p_{16}	q_{11}	q_{12}	q_{13}	0	0	q_{16}
p_{12}	p_{11}	p_{13}	0	0	$-p_{16}$	q_{12}	q_{11}	q_{13}	0	0	$-q_{16}$
p_{31}	p_{31}	p_{33}	0	0	0	q_{31}	q_{31}	q_{33}	0	0	0
0	0	0	p_{44}	p_{45}	0	0	0	0	q_{44}	q_{45}	0
0	0	0	$-p_{45}$	p_{44}	0	0	0	0	$-q_{45}$	q_{44}	0
p_{61}	$-p_{61}$	0	0	0	p_{66}	q_{61}	$-q_{61}$	0	0	0	q_{66}

GROUP 7. TETRAGONAL SYSTEM

V_d, C_{4v}, D_4, D_{4h}, $(\bar{4}2m, 4mm, 42, 4/mmm)$ (7 Constants)

p_{11}	p_{12}	p_{13}	0	0	0	q_{11}	q_{12}	q_{13}	0	0	0
p_{12}	p_{11}	p_{13}	0	0	0	q_{12}	q_{11}	q_{13}	0	0	0
p_{31}	p_{31}	p_{33}	0	0	0	q_{31}	q_{31}	q_{33}	0	0	0
0	0	0	p_{44}	0	0	0	0	0	q_{44}	0	0
0	0	0	0	p_{44}	0	0	0	0	0	q_{44}	0
0	0	0	0	0	p_{66}	0	0	0	0	0	q_{66}

GROUP 8. HEXAGONAL SYSTEM, C_{3h}, C_6, C_{6h} $(\bar{6}, 6, 6/m)$ (8 Constants)

p_{11}	p_{12}	p_{13}	0	0	$-p_{61}$	q_{11}	q_{12}	q_{13}	0	0	$-2q_{61}$
p_{12}	p_{11}	p_{13}	0	0	p_{61}	q_{12}	q_{11}	q_{13}	0	0	$2q_{61}$
p_{31}	p_{31}	p_{33}	0	0	0	q_{31}	q_{31}	q_{33}	0	0	0
0	0	0	p_{44}	p_{45}	0	0	0	0	q_{44}	q_{45}	0
0	0	0	$-p_{45}$	p_{44}	0	0	0	0	$-q_{45}$	q_{44}	0
p_{61}	$-p_{61}$	0	0	0	$\dfrac{(p_{11}-p_{12})}{2}$	q_{61}	$-q_{61}$	0	0	0	$(q_{11}-q_{12})$

GROUP 9. HEXAGONAL SYSTEM, D_{3h}, C_{6v}, D_6, D_{6h} $(\bar{6}2m, 6mm, 62, 6/mmm)$ (6 Constants)

p_{11}	p_{12}	p_{13}	0	0	0	q_{11}	q_{12}	q_{13}	0	0	0
p_{12}	p_{11}	p_{13}	0	0	0	q_{12}	q_{11}	q_{13}	0	0	0
p_{31}	p_{31}	p_{33}	0	0	0	q_{31}	q_{31}	q_{33}	0	0	0
0	0	0	p_{44}	0	0	0	0	0	q_{44}	0	0
0	0	0	0	p_{44}	0	0	0	0	0	q_{44}	0
0	0	0	0	0	$\dfrac{(p_{11}-p_{12})}{2}$	0	0	0	0	0	$(q_{11}-q_{12})$

GROUP 10. CUBIC SYSTEM, T, T_h (23, m3) (4 Constants)

p_{11}	p_{12}	p_{13}	0	0	0	q_{11}	q_{12}	q_{13}	0	0	0
p_{13}	p_{11}	p_{12}	0	0	0	q_{13}	q_{11}	q_{12}	0	0	0
p_{12}	p_{13}	p_{11}	0	0	0	q_{12}	q_{13}	q_{11}	0	0	0
0	0	0	p_{44}	0	0	0	0	0	q_{44}	0	0
0	0	0	0	p_{44}	0	0	0	0	0	q_{44}	0
0	0	0	0	0	p_{44}	0	0	0	0	0	q_{44}

GROUP 11. CUBIC SYSTEM, T_d, O, O_h ($\overline{4}$3m, 43, m3m) (3 Constants)

p_{11}	p_{12}	p_{12}	0	0	0	q_{11}	q_{12}	q_{12}	0	0	0
p_{12}	p_{11}	p_{12}	0	0	0	q_{12}	q_{11}	q_{12}	0	0	0
p_{12}	p_{12}	p_{11}	0	0	0	q_{12}	q_{12}	q_{11}	0	0	0
0	0	0	p_{44}	0	0	0	0	0	q_{44}	0	0
0	0	0	0	p_{44}	0	0	0	0	0	q_{44}	0
0	0	0	0	0	p_{44}	0	0	0	0	0	q_{44}

TABLE II

Type	Pressure direction parallel to cube face	Pressure direction parallel to dodecahedral face	
	$0 < \phi < \pi/4$	$0 < \psi < \cos^{-1}\sqrt{\tfrac{1}{3}}$	$\cos^{-1}\sqrt{\tfrac{1}{3}} < \psi < \pi/2$
1	$0 < \xi < \pi/4$ $\xi > \phi$	$0 < \eta < \cos^{-1}\sqrt{\tfrac{1}{3}}$ $\eta > \psi$	$\cos^{-1}\sqrt{\tfrac{1}{3}} < \eta < \pi/2$ $\eta > \psi$
$x > 1$	Biaxial plane parallel to the cube face $\sin^2 V = $ $\dfrac{\cos 2\phi - \cos 2\xi}{2\cos 2\phi}$	Biaxial plane parallel to dodecahedral face $\sin^2 V =$ $\dfrac{\sin \eta . \sin(\eta - \psi)}{\cos \psi}$	Biaxial plane perpendicular to dodecahedral face $\sin^2 V =$ $\tan \eta . \tan(\psi - \eta)$
2	$0 < \xi < \pi/4$ $\xi < \phi$	$0 < \eta < \cos^{-1}\sqrt{\tfrac{1}{3}}$ $\eta < \psi$	$\cos^{-1}\sqrt{\tfrac{1}{3}} < \eta < \pi/2$ $\eta > \psi$
$0 < x < 1$	Biaxial plane perpendicular to cube face $\sin^2 V =$ $\tan(\phi+\xi).\tan(\phi-\xi)$	Biaxial plane perpendicular to dodecahedral face $\sin^2 V =$ $\tan \eta . \tan(\psi - \eta)$	Biaxial plane parallel to dodecahedral face $\sin^2 V =$ $\dfrac{\sin \eta \sin(\eta - \psi)}{\cos \psi}$
3	$0 > \xi > -\pi/4$ $-\xi < \phi$	$\pi/2 > \eta > \cos^{-1}\sqrt{\tfrac{1}{3}}$ $(\pi/2 - \eta) < \psi < \eta$	$\cos^{-1}\sqrt{\tfrac{1}{3}} > \eta > 0$ $\eta < \psi$
$-1 < x < 0$	Biaxial plane and $\sin^2 V$ as in Type 2	Biaxial plane and $\sin^2 V$ as in Type 2	Biaxial plane and $\sin^2 V$ as in Type 2
4	$0 > \xi > -\pi/4$ $-\xi > \phi$	$0 < \psi$ $< \tfrac{1}{2}\cos^{-1}\dfrac{(x+3)}{x-1}$ $\pi/2 > \eta$ $> \tfrac{1}{2}\tan^{-1}\dfrac{-x\sqrt{2}}{\sqrt{-(1+x)}};$ $\eta > \psi$ if $\psi < \cos^{-1}\sqrt{\tfrac{1}{3}};$	$\tfrac{1}{2}\cos^{-1} . \dfrac{(x+3)}{(x-1)}$ $< \psi < \pi/2$ $\tfrac{1}{2}\tan^{-1}\dfrac{-x\sqrt{2}}{\sqrt{-(1+x)}}$ $< \eta < \pi/2$ $\eta < \psi$ for $\psi > \cos^{-1}(\sqrt{\tfrac{1}{3}})$
$x < -1$	Biaxial plane and $\sin^2 V$ as in Type 1	Biaxial plane and $\sin^2 V$ as in Type 1	Biaxial plane an $\sin^2 V$ as in Type Id

TABLE III

DOUBLE REFRACTION OF CUBIC CRYSTALS UNDER UNIDIRECTIONAL PRESSURE

Pressure Direction	TlBr—I Type (1)	NaCl Type (2)	CaF$_2$ Type (3)	KCl Type (4)
Cube normal	+ve uniaxial	—ve uniaxial	—ve uniaxial	+ve uniaxial
Octahedral normal	+ve uniaxial	—ve uniaxial	+ve uniaxial	—ve uniaxial
Dodecahedral normal	+ve biaxial	—ve biaxial	—ve biaxial	+ve biaxial
	biaxial plane is the cube face normal to the pressure direction.	biaxial plane is the dodecahedral face perpendicular to the pressure direction.	biaxial plane is the dodecahedral face perpendicular to the pressure direction.	biaxial plane is the cube face parallel to the pressure direction.
	acute bisectrix is parallel to the pressure direction.	acute bisectrix is parallel to the pressure direction.	acute bisectrix is the dodecahedral normal, perpendicular to the pressure direction.	acute bisectrix is the dodecahedral normal, perpendicular to the pressure direction.
	$2V = 2\sin^{-1}\sqrt{\dfrac{x-1}{2x}}$ $= 81°32'$	$2V = 2\sin^{-1}\sqrt{\dfrac{1-x}{1+x}}$ $= 49°20'$	$2V = \left(\pi - 2\sin^{-1}\sqrt{\dfrac{1+x}{1-x}}\right)$ $= 72°10'$	$2V = \left(\pi - 2\sin^{-1}\sqrt{\dfrac{x-1}{x+1}}\right)$ $= 66°46'$

TABLE IV
STRAIN-OPTICAL CONSTANTS OF CRYSTALS BELONGING TO CUBIC SYSTEM (T_d, O, O_h, Classes)

No.	Crystal	p_{11}	p_{12}	$p_{11}-p_{12}$	p_{44}	Ref.	Elastic constants Reference	Remarks
1.	Ammonium Chloride	0·145	0·24	—0·095	0·025	(37)	87	
2.	Calcium Fluoride	0·056	0·228	—0·172	0·024	(41)	88	
3.	Diamond	—0·31	0·09	—0·40	—0·12	(51)	77	
4.	Lithium Fluoride	0·02	0·128	—0·108	—0·064	(14)	—	
				—0·107	—0·045	(69)	79	$\lambda = 560\ m\mu$
		0·02	0·130	—0·11	—0·045	(26)		
				—0·16		(47)		$\lambda = 550\ m\mu$
				—0·12		(65)	75	
5.	Magnesium Oxide	—0·32	—0·08	—0·24		(14)	—	
				—0·253	—0·096	(69)	79	$\lambda = 560\ m\mu$
6.	Potassium Bromide	0·22	0·171	0·049	—0·026	(14)	—	
				0·043	—0·029	(69)	79	$\lambda = 560\ m\mu$
		0·212	0·165	0·047	—0·022	(26)	—	
				0·050	—0·021	(65)	22	
7.	Potassium Chloride			0 0595	—0·0276	(42)	88	
		0·17	0·124	0·046		(14)	—	
		0·215	0·159	0·056	—0 024	(26)	—	
				0·065		(65)	22	
8.	Potassium Iodide	0·210	0·169	0·041		(14)		
		0·203	0·164	0·038		(65)	80	
9.	Silver Chloride			0·134	—0·056	(69)	73	
				0·225	—0·075	(23)	73	
10.	Sodium Chloride	0·1372	0·178	—0·0408	—0·0108	(42)	88	
		0·110	0·153	—0·043	—0·010	(14)	—	
				—0·045		(65)	22	
11.	Thallium Bromo-Iodide (KRS 5)			0·08	0·157	(69)	74	$\lambda = 610\ m\mu$
12.	Thallium Chloride			0·03	0·06	(14)	74	Values approximate
13.	Thallium Chloro-Bromide (KRS 6)				0·096	(69)	74	$\lambda = 610\ m\mu$
14.	Zinc Sulphide				—0·84	(61)&(69)	76	
					—0·0044	(29)	76	$\lambda = 546\ m\mu$

TABLE V
STRAIN OPTICAL CONSTANTS OF CRYSTALS BELONGING TO CUBIC SYSTEM (T, T$_h$ Classes)

No.	Crystal	p_{11}	p_{12}	p_{13}	p_{44}	$p_{11}-p_{12}$	$p_{11}-p_{13}$	Author Reference	Elastic Constants Reference	Remarks
1.	ALUMS				—0·006	—0·064		(46)		
	(a) Ammonium alum	0·38	0·46	0·45	—0·009	—0·08	—0·07	(13) & (26)	85	
	(b) Chromium alum				—0·009	—0·082	—0·07	(9)	84	
	(c) Potassium alum				—0·004	—0·06		(46)	84	
		0·26	0·34	0·33	—0·005	—0·08	—0·07	(10) & (26)	84	
	(d) Potassium ammonium alum				—5·06×10⁻¹³	—4·51×10⁻¹⁵	—1·02×10⁻¹³	(9)		q_{ij}'s only are available.
	(e) Thallium alum	0·43	0·58	0·57	+0·007	—0·088	—0·072	(9) & (30)	83	
2.	Barium nitrate				—0·019	—0·96	—0·72	(11)	78	
3.	Lead nitrate				—0·019	—0·28	—0·17	(8)	78	
		0·309	0·608	0·476	—0·039	—0·299	—0·167	(26)	78	
4.	Strontium nitrate				—0·02	—0·409	—0·289	(9)	78	
5.	Sodium bromate	0·185	0·218	0·213	—0·0139	—0·033	—0·028	(38)	86	
6.	Sodium chlorate	0·173	0·258	0·223	—0·0187	—0·083	—0·050	(52)	82	
		0·162	0·24	0·20	—0·0198	—0·078	—0·058	(38)	86	

Note: the scientific notation in row (d) uses exponents; as printed: $-5·06\times10^{-13}$, $-4·51\times10^{-15}$, $-1·02\times10^{-13}$.

TABLE VI

TETRAGONAL SYSTEM

(Only piezo-optic constants are known. Values are given in units of Brewsters 10^{-13}cm.^{2}/dyne.)

No.	Crystal	q_{11}	q_{12}	q_{31}	q_{44}	q_{66}	$q_{13}-q_{33}$	Author and Reference
1.	$NH_4H_2PO_4$ (ADP)					-17.7		West & Makas (70)
						-12.1		Willard (72)
		8.6	7.9	12.3	-5.8	-12.2	-1.6	Carpenter (17)
2.	KH_2PO_4	$0 > (q_{11}-q_{12}) > -0.3$				-11.25		West & Makas (71)

TABLE VII
HEXAGONAL SYSTEM

No.	Crystal	p_{11}	p_{33}	p_{12}	p_{13}	p_{31}	p_{44}	Author and Reference	Elastic Constants Reference
1.	Beryl	0·0099	0·023	0·0175	0·191	0·313	−0·152	Pockels (43)	(88)

TABLE VIII
TRIGONAL SYSTEM

No.	Crystal	p_{11}	p_{33}	p_{12}	p_{13}	p_{14}	p_{31}	p_{41}	p_{44}	Author and Reference	Elastic Constants Reference
1.	Calcite	0·095	0·178	0·189	0·215	−0·006	0·309	0·01	−0·090	Pockels (45)	(88)
2.	⍺-Quartz	0·138	0·098	0·250	0·259	−0·029	0·258	−0·042	−0·0685	Pockels (40)	(88)

TABLE IX
ORTHORHOMBIC SYSTEM

No.	Crystal	p_{11}	p_{22}	p_{33}	p_{12}	p_{13}	p_{21}	p_{23}	p_{31}	p_{32}	p_{44}	p_{55}	p_{66}	Author and Reference	Elastic Constants Reference
1.	Barite	0·21	0·24	0·31	0·25	0·16	0·34	0·19	0·275	0·22	0·002	−0·012	0·037	Vedam (64)	(75, 87)
2.	Rochelle salt										−0·009	0·006	−0·015	Pockels (44)	(86a)
3.	Topaz	−0·085	−0·120	−0·083	0·069	0·052	0·093	0·065	0·095	0·085	−0·095	−0·031	−0·098	Eppendahl (21)	(87)

TABLE X

RATIOS OF PHOTOELASTIC CONSTANTS

Crystal	p_{12}/p_{11}	p_{44}/p_{11}	$p_{44}/(p_{11} + p_{12})$
NaCl	$1{\cdot}35 \pm {\cdot}03$	$-0{\cdot}099$	$-0{\cdot}042 \pm {\cdot}004$
KCl	$0{\cdot}76 \pm {\cdot}01$	$-0{\cdot}12$	$-0{\cdot}069 \pm {\cdot}004$
KBr	$0{\cdot}77 \pm {\cdot}01$	$-0{\cdot}12$	$-0{\cdot}067 \pm {\cdot}005$

TABLE XI

CLASSIFICATION OF CUBIC CRYSTALS OF T_d, O, O_h CLASSES

Mueller				Pockels		
Group No.	$(p_{11}-p_{12})$	p_{44}	$p_{44}/(p_{11}-p_{12})$	Group No.	Sign of χ	Magnitude of χ
I	$+$	$+$	$+$	(1)	$+$	> 1
II	$-$	$-$	$+$	(2)	$+$	< 1
III	$-$	$+$	$-$	(3)	$-$	< 1
IV	$+$	$-$	$-$	(4)	$-$	> 1

TABLE XII

STRAIN POLARISABILITY CONSTANTS

Substance	λ_z	λ_x	λ'
NaCl	$1{\cdot}24$	$0{\cdot}18$	$0{\cdot}06$
KCl	$1{\cdot}20$	$0{\cdot}11$	$-0{\cdot}09$
Diamond	$1{\cdot}00$	$1{\cdot}16$	$2{\cdot}88$

TABLE XIII

$$\left(\frac{\rho \cdot dn}{d\rho}\right) \text{ AND } \lambda_0 \text{ OF CUBIC CRYSTALS}$$

Crystal	n_D	$\left(\frac{\chi \, dn}{d\rho}\right)$ obs.	L-L Equation		Drude Equation	
			$\left(\frac{\rho dn}{d\rho}\right)$ calc.	λ_0	$\left(\frac{\rho dn}{d\rho}\right)$ calc.	λ_0
LiF	1·392	0·1	0·44	0·7	0·34	0·7
NaCl	1·544	0·24	0·66	0·63	0·45	0·47
KCl	1·490	0·23	0·58	0·61	0·41	0·44
KBr	1·559	0·35	0·68	0·48	0·46	0·24
KI	1·667	0·44	0·85	0·48	0·53	0·19
CaF_2	1·434	0·25	0·50	0·50	0·37	0·32
MgO	1·736	−0·40	0·97	1·4	0·52	1·8
Diamond	2·417	−0·28	2·62	1·1	1·00	1·28
NH_4Cl	1·642	0·46	0·81	0·43	0·52	0·12
$NaClO_3$	1·513	0·38	0·61	0·38	0·42	0·10
$NaBrO_3$	1·594	0·42	0·73	0·43	0·48	0·12
K alum	1·456	0·49	0·53	0·06	0·38	−0·28
NH_4 alum	1·459	0·67	0·53	−0·25	0·39	−0·72
Tl alum	1·498	0·92	0·58	−0·44	0·42	−1·2
$Pb(NO_3)_2$	1·782	1·31	1·05	−0·25	0·61	−1·1

TABLE XIV

PHOTOELASTIC DISPERSION IN CUBIC CRYSTALS

Crystal	p_{11}			p_{12}			p_{13}			p_{44}		
	589 3Å	5461Å	4358Å	5893Å	5461Å	4358Å	5893Å	5461Å	4358Å	5893Å	5461Å	4358Å
LiF	0·020	0·020	0·020	0·130	0·130	0·130				0·045	0·051	0·068
KCl	0·215	0·211	0·182	0·159	0·156	0·134				0·024	0·024	0·026
KBr	0·212	0·208	0·194	0·165	0·162	0·151				0·022	0·022	0·024
Ammonium Alum	0·378	0·403	0·418	0·465	0·496	0·514	0·454	0·479	0·497	0·009	0·009	0·011
Potassium Alum	0·250	0·259	0·268	0·330	0·342	0·345	0·321	0·332	0·342	0·005	0·005	0·006
Lead Nitrate	0·309	0·348	0·398	0·608	0·666	0·742	0·476	0·537	0·613	0·039	0·045	0·070

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Strontium titanate becomes an optically negative uniaxial crystal when stressed along a crystallo-graphic axis and an optically negative biaxial crystal when under a crystallo-graphic diagonal stress.