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# FARADAY ROTATION AND RESIDUAL BIREFRINGENCE

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## SUMMARY

The Faraday rotation in an isotropic substance is known to be vitiated by the presence of accidental birefringence which is very common in many cubic crystals, grown either from solution or from melt. A simple method of eliminating the errors due to the birefringence is discussed here. It is shown that the actual value of the rotation could be accurately evaluated from the measured values of the apparent rotation. This method is exemplified in the case of MgO and $Pb(NO_3)_2$ crystals and is found to give most satisfactory results. This method now permits the extension of magneto-optic measurements to a large number of crystals that have a habit of exhibiting residual birefringence.

## INTRODUCTION

Recent studies by the authors on calcite and $NaClO_3$ have indicated that the study of the magneto-optic rotation in a substance may throw some light on the spectroscopic behaviour of the optical absorption frequency.[1] The systematic measurement of the Faraday rotation and its dispersion with wavelength could therefore yield results of some significance. The collection of such data, particularly in the case of isotropic solids, would in principle be a simple matter, but in practice the experimenter is hampered by a rather serious difficulty. This is the unavailability of suitable specimens for study. It is only in the case of optical glasses and a few crystals like NaCl and KCl are large flawless specimens procurable. The crystals that one commonly encounters are usually quite small and as a rule they exhibit a small amount of strain or residual birefringence. The smallness of the crystal by itself is no great disadvantage. Powerful magnetic fields, together with accurate photoelectric techniques, are capable of yielding, even in the case of tiny crystals, very precise values of the magnetic rotation over quite a wide range of wavelengths. The problem therefore, is one of eliminating the effects of residual birefringence. This paper deals with the practical methods of overcoming this obstacle, so that useful data may be obtained in the case of a large number of crystals.

## THE EFFECT OF BIREFRINGENCE ON FARADAY ROTATION

It is well known that the measurement of the Faraday rotation is affected by the presence of a small amount of birefringence. This error, even if it is small in the visible region becomes significant in the ultraviolet end of the spectrum. For example if an error of 1 per cent. in the rotation at $\lambda$ 5893 Å is introduced

due to this cause, then the error at $\lambda$ 3000 Å would be 4 per cent. and that at $\lambda$ 2000 Å would be 9 per cent. Such a variation would be disastrous if one is fitting up a dispersion formula to detect differences in the anomaly factors for the different absorption frequencies. When measuring the rotation in a substance exhibiting strain birefringence using any conventional apparatus, what is determined is the position of the major axis of the emergent elliptic vibration with respect to the plane of polarisation of the incident light. This can be termed as the apparent rotation. This apparent rotation is smaller or larger than the true rotation depending on the relative disposition of the plane of the incident vibration and the axis of strain.

Wiener,[2] Chauvin,[3,4] Pockels[5] and recently Ramachandran and Ramaseshan[6] have applied the general theory of light propagation to this problem and have calculated the parameters of emergent ellipse. The general formula for the case of any arbitrary angle ($\alpha$) between the plane of vibration of the incident light and a principal axis of the birefringent solid has been worked out.[6] From this it is quite easy to derive the particular cases when $\alpha = 0$ and $\alpha = 45°$. If the true rotation per unit length for a solid (of thickness $t$) in the absence of birefringence is $\rho_0$ (*i.e.*, total rotation $\rho = t\rho_0$) and if $\delta_0$ is the phase retardation per unit length in the absence of rotation, then the apparent rotation $\psi$ is given by

*Case I.*—

$$\alpha = 0, \tan 2\psi_0 = \frac{\sin 2\gamma \sin \Delta}{\cos^2 2\gamma + \sin^2 2\gamma \cos \Delta} \tag{1}$$

*Case II.*—

$$\alpha = 45°, \tan 2\psi_{45} = \sin 2\gamma \tan \Delta \tag{2}$$

where

$$\tan 2\gamma = 2\rho_0/\delta_0$$

and

$$\Delta = t\Delta_0 = t\sqrt{\delta_0^2 + (2\rho_0)^2}$$

when the total birefringence $\delta = (t\delta_0)$ is small then by suitable algebraic manipulation[6] equations (1) and (2) can be reduced to

$$2\psi_0 = 2\rho\left(1 - \frac{\delta^2}{3!}\right) \tag{3}$$

and

$$2\psi_{45} = 2\rho\left(1 + \frac{\delta^2}{3}\right) \tag{4}$$

From equations (3) and (4) one gets the true rotation to be

$$\rho = \frac{2\psi_0 + \psi_{45}}{3} \tag{5}$$

We get the most interesting result that when the birefringence is small the true rotation $\rho$ can be obtained from just the measurements of the apparent rotations $\psi_0$ and $\psi_{45}$ without the actual magnitude of the birefringence being known.

From equations (1) and (2) one can see that the apparent rotation varies with the angle $\alpha$. Hence it would appear that any missetting of the polariser or the specimen, or slight variations in the strain axis in the specimen, would considerably affect the measurement. It has however been rigorously proved that the usual practice of taking the mean of the measurements for the two directions of the field largely eliminates the errors caused by these effects.[6]

In the case of birefringence where the axes of strain are completely at random it is shown[6] that for small $\delta$

$$2\psi_m = 2\rho\left(1 + \frac{\delta^2}{12}\right) \tag{6}$$

where $\psi_m$ is the mean apparent rotation of a series of settings with either the polariser rotated or the crystal rotated in its own plane through a range of 180°. In this case the magnitude of the birefringence must be measured for a series of points on the crystal with a Babinet compensator.

## THE EXPERIMENTAL RESULTS

The problem of eliminating the effect of birefringence is made comparatively simpler in a practical case because for some strange reason crystals with residual birefringence grown from either melt or solution, invariably exhibit a preferred axis of strain, which does not vary by more than 5°. Hence formula (5) can be applied directly.

The crystals used for experimental verification were specimens of MgO and $Pb(NO_3)_2$. Both had fairly well-defined axes of strain, although the strain axis in the latter case was much sharper.

An A.C. compensation photoelectric polarimeter capable of measuring rotations to 0·01° was used. The crystal was first set so that there was minimum restoration of light. This ensured that the incident vibration was parallel to a principal axis of the crystal. The apparent rotations were determined for both directions of the magnetic field and the mean was taken to be $\psi_0$. The polariser was rotated through 45° and $\psi_{45}$ was measured.

Table I gives the value of $\psi_0$, $\psi_{45}$ and $\rho$ calculated from equation (5). Column (6) gives the values of the Verdet constant, column (7) gives the value of $\delta$ calculated from the formula

$$\delta = \sqrt{\frac{6\,(\psi_{45} - \psi_0)}{2\psi_0 + \psi_{45}}} \tag{7}$$

TABLE I

| Substance | $\lambda$ in Å | $\psi_0$ in degrees | $\psi_{45}$ in degrees | $\rho$ in degrees | V in minutes/cm. per oersted | $\delta$ in $\lambda$ | $\delta\lambda\ 10^8$ cm. |
|---|---|---|---|---|---|---|---|
| MgO .. | 5780 | 8·77 | 8·90 | 8·81 | 0·0377 | 0·0262 | 152 |
| | 5461 | 9·93 | 10·10 | 9·99 | 0·0427 | 0·0289 | 158 |
| | 4358 | 16·46 | 16·93 | 16·61 | 0·0712 | 0·0382 | 166 |
| | 4047 | 19·61 | 20·14 | 19·79 | 0·0848 | 0·0376 | 152 |
| | 3650 | 25·15 | 26·00 | 25·44 | 0·1089 | 0·0412 | 150 |
| | 3132 | 37·84 | 39·66 | 38·45 | 0·1646 | 0·0491 | 154 |
| $Pb(NO_3)_2$ .. | 5780 | 11·86 | 12·20 | 11·97 | 0·04907 | 0·0383 | 221 |
| | 5461 | 13·53 | 13·96 | 13·68 | 0·0561 | 0·0402 | 219 |
| | 4916 | 17·62 | 18·40 | 17·88 | 0·0733 | 0·0475 | 233 |
| | 4358 | 24·24 | 25·70 | 24·72 | 0·1013 | 0·0546 | 238 |
| | 4047 | 29·89 | 31·95 | 30·57 | 0·1253 | 0·0581 | 235 |
| | 3650 | 41·29 | 44·62 | 42·40 | 0·1738 | 0·0632 | 231 |

obtained from equations (3) and (5) and column (8) gives the value of $\delta\times\lambda$. The fact that $\delta\times\lambda$ is approximately constant (to about 10%) is a clear indication that this formula can be used with confidence. A 10% variation in a birefringence whose magnitude itself is about $\lambda/30$ to $\lambda/40$ would scarcely affect the magnetic rotation. There is no doubt that the rotation given in column (4) is of much greater accuracy as the variation in it is actually due to the effect of the dispersion of birefringence. It is indeed most gratifying to find that this method can actually detect the dispersion of birefringence when the magnitude of the birefringence at $\lambda$ 5780 Å is about $\lambda/30$ thus justifying the expectations[7, 8] that it could be used for the accurate determination of the dispersion of stress optic coefficients.

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