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Influence of mosaicity on the Bragg reflexion of polarized X-rays. By S. RAMASESHAN and G. N. RAMACHANDRAN,* Department of Physics, Indian Institute of Science, Bangalore 3, India

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When the incident X-rays are unpolarized, the integrated reflexions for a mosaic and a non-absorbing perfect crystal $(\varrho_M \text{ and } \varrho_P \text{ respectively})$ are given by

$$\varrho_{M} = \frac{1}{2\mu} \frac{l^{2}}{m^{2}c^{4}} N^{2} |F_{H}|^{2} \frac{\lambda^{3}}{\sin 2\theta} \frac{(1 + \cos^{2} 2\theta)}{2}$$
$$= R_{M} (1 + \cos^{2} 2\theta), \quad (1a)$$
$$\varrho_{P} = \frac{8}{3\pi} \frac{l}{mc^{2}} N |F_{H}| \frac{\lambda^{2}}{\sin 2\theta} \frac{(1 + |\cos 2\theta|)}{2}$$

$$= R_P(1 + |\cos 2\theta|)$$
 (1b)

(cf. Internationale Tabellen, 1935, vol. 2, pp. 562, 564). Here θ is the Bragg angle and the formulae refer to symmetrical 'surface' reflexions. If, however, the incident X-rays are completely polarized and the electric vector makes an angle φ with the normal to the plane of reflexion, then the integrated reflexions in the two enses are:

$$\varrho_{M\varphi} = R_M \left[\cos^2 \varphi + \sin^2 \varphi \cos^2 2\theta \right], \qquad (2a)$$

$$\varrho_{P\varphi} = R_P \left[\cos^2 \varphi + \sin^2 \varphi \right] \cos 2\theta'_{\downarrow} \,. \tag{2b}$$

Rewriting these equations in the form

 $r_M(\varphi) = \varrho_{M\varphi}/\varrho_{M\alpha} = \cos^2 \varphi + \sin^2 \varphi \cos^2 2\theta , \quad (3a)$

$$r_P(\varphi) = \varrho_{P\varphi}/\varrho_{P\sigma} = \cos^2 \varphi + \sin^2 \varphi |\cos 2\theta| , \qquad (3b)$$

where the symbol σ refers to the case when $\varphi = 0$, it is seen that, except for values of $\theta = 0^{\circ}$, 45° or 90°, the two functions are different, the difference being a maximum for $\theta = 30^{\circ}$ and 60°. Thus, it is seen that mosaic and perfect crystals differ in their behaviour with respect to polarized X-rays.

An experimental investigation was made to demonstrate this effect and to find whether it is possible to use it to assess the degree of perfection of a crystal. X-rays from the copper target of a Coolidge tube were allowed to fall on the ground (110) face of a large KCl crystal, which served as the monochromator. The 440 reflexion (Bragg angle 43° 58') was used to give a monochromatic polarized beam. † This beam was collimated through a tube 1 mm. in diameter and 4 cm. long and was allowed to fall on the experimental crystal set at the appropriate Bragg angle. Different values of φ were obtained by rotating the goniometer, on which the crystal was mounted, about an axis coincident with the X-ray beam. The X-rays reflected by the crystal were detected by a Geiger counter. The integrated reflexion was obtained by plotting the rocking curve and finding its area. The

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† The beam will be completely polarized only if the Bragg angle of the monochromator is exactly 45°. As the Bragg angle of KCl 440 is 43° 58°, there is less than 1% of unpolarized X-rays in the incident beam. It can be shown that the maximum error arising from this in the measurement of r(q) is much less than the experimental errors. voltage and current in the X-ray tube were maintained constant manually. All measurements were relative and were estimated to be accurate to about 5%.

Crystals of sodium nitrate slowly grown from a saturated solution were used in these experiments. The 633 and 422 reflexions were first studied using a natural (211) face and later after grinding the surface. The results are given in Fig. 1 in which the curves are the theoretical



Fig. 1. Variation of integrated reflexion with azimuth of polarization. The curves marked r_P and r_Y respectively are the theoretical curves for a perfect non-absorbing and for a mosaic crystal. The experimental data are indicated by the points, circles representing those obtained with a natural surface and the crosses those with a ground surface. The upper two curves and the points between them are for the 422 reflexion ($2\theta = 60^{\circ}$ 58') while the lower set is for the 633 reflexion ($2\theta = 99^{\circ}$ 12') of NaNO₂.

curves given by equations (3a) and (3b). It may be remarked that the value of 2 θ for the 422 reflexion is 60° 58', which is very close to the value for which $r_P(\varphi) - r_M(\varphi)$ is a maximum.

In both the reflexions, one notices that the experimental points for the gound crystal are very near the theoretical curves for the mosaic crystal. However, they lie consistently above the curve, showing that the grinding has not made the crystal ideally mosaic. The data for the reflexions from the unground crystal are considerably different from those for the ground one. Thus the reflexion coefficient for perpendicular polarization ($\varphi = 90^{\circ}$) relative to that for parallel polarization is higher for the unground crystal than for the ground one. Therefore, if unpolarized incident X-rays are used the degree of polarization of the Bragg reflected X-rays would be higher for the ground than for the unground crystal. As far as the authors are aware, this is the first experimental demonstration of the fact that the polarization characteristic of a Bragg reflexion depends on the perfection of the crystal. This could, therefore, be used for assessing the degree of perfection of crystals. For instance, if $r_{C}(\varphi)$ is the experimental value for a crystal, one may define the degree of perfection as

$$\{r_{\mathcal{C}}(\varphi)-r_{\mathcal{M}}(\varphi)\}/\{r_{\mathcal{P}}(\varphi)-r_{\mathcal{M}}(\varphi)\}.$$

This ratio is found to be approximately constant for different values of φ for NaNO₃ crystals grown from solution and has a mean value of about 0.5. An important advantage of this method of finding the degree of perfection of crystals is that the difficulty of determining absolute integrated reflexions is not present, as it requires only relative measurements for different azimuths.

It must be mentioned that the theoretical curve for $r_P(\varphi)$ relates to a perfect non-absorbing crystal. Preliminary theoretical studies show that, when there is a finite absorption, the curve for $r_P(\varphi)$ departs from this, approaching that for $r_M(\varphi)$ for increasing absorptions and coinciding with the latter for large absorption coefficients. This is in agreement with the theoretical result obtained by Hirsch & Ramachandran (1950) that perfect and mosaic crystals give the same integrated reflexions when absorption is large. When the effect of finite absorption is taken into account, it is seen that the degree of perfection of the grown crystal of NaNO₃ should be higher than the value 0.5 given above. Further investigations are in progress.

Reference

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