Sunyaev–Zeldovich Effect

Shadows in the Cosmic Radiation

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Soon after the discovery of the cosmic background radiation, Zeldovich and Sunyaev proposed that hot gas in galaxy clusters should cast a faint shadow because of the interaction between energetic electrons and the radiation photons. Sunayev–Zeldovich effect is now routinely observed, and it has become an important tool for studying the history of the universe.

1. Introduction

There is a background radiation that fills our universe, and it is thought to be a relic of the early, hot phase of the universe. We know that the universe is expanding, and therefore matter in it must have been denser in the past. It also follows from simple thermodynamics that the matter must have been hotter as well, since we know that matter cools upon adiabatic expansion. In addition to matter, there is likely to be some amount of radiation, because matter radiates photons. And the same principle applies to radiation too, which means that radiation must have been hotter in the past.

We also know that when matter and radiation are in thermal equilibrium, then the radiation acquires some special characteristics, that of a 'blackbody' radiation. The spectrum is then specified by a function that was first derived by Max Planck[1] and is known as the Planck spectrum. This radiation peaks at a certain wavelength that depends on the temperature (of matter and radiation) and nothing else. In fact, the properties of this radiation depend *only* on the temperature.

Because of the high density of matter in the universe in

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[1] See *Resonance*, Vol.13, No.2, 2008.

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the past, it is inevitable that matter and radiation interacted very strongly then, and came to a thermal equilibrium. One therefore expects the radiation in the early universe to have a Planck spectrum depending on the ambient temperature. And as the universe expanded, this radiation would have cooled too, which means that the peak wavelength would have shifted towards red, toward the long wavelength portion. It so happens that the universe has expanded to a degree where the radiation has now cooled to a temperature of ~ 2.7 Kelvin, and its peak wavelength corresponds to the microwave region (mm–cm in wavelength). The universe is therefore a giant microwave oven. This radiation, called the Cosmic Microwave Background radiation (CMBR), was discovered in 1965 by Arno Penzias and Robert Wilson.

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The matter in the universe kept interacting strongly with photons until atoms formed. Before this era, photons mainly interacted with free electrons (through what is known as Thomson scattering). But when atoms formed, bound electrons inside atoms could interact with photons of only a specific set of frequencies, and therefore the rate of interaction dramatically dropped at this point of time. This happened when the matter (and radiation) temperature of the universe dropped below ~ 3000 K, when the universe was roughly $300,000$ years old. After this era, the photons travelled freely (while decreasing in energy content because of the expansion of the universe), and this era is called the 'decoupling era'. These are the photons we now observe as the CMBR photons, the background radiation that pervades the universe.

As the CMBR photons travel through the universe, they may encounter matter which might affect their energy and path in ways that will have observable effects. One such case is that of CMBR photons streaming through a region of galaxy clusters. These clusters of galaxies contain, in addition to member galaxies, a large amount of hot gas. This gas often has temperatures above ten million degrees Kelvin. The energetic electrons in such a gas may encounter a background radiation photon and change its energy in a dramatic fashion. This is called the 'inverse Compton scattering', but to understand it we need to understand the basics of Compton scattering.

2. Compton Scattering

When an energetic photon interacts with a charged particle such as an electron, it gives it an impulse transfering momentum, losing energy in the process. This is known as the *Compton scattering.* One also encounters an opposite effect, of energetic particles transfering momentum to low energy photons, which is known as the *inverse Compton scattering.* These two processes can be thought of as manifestations of the same process viewed in two different frames of reference. If the observer is at rest with respect to the energetic electron, the inverse Compton scattering would appear as the normal Compton scattering, since to the observer the photon would appear to be highly energetic and being scattered by a stationary particle.

If ν and ν' are the frequencies of the photon before and after the scattering, m_0 is the rest mass and E is the energy of the charged particle (*Figure* 1), then conservation of energy would imply,

$$m_0c^2 + h\nu = E + h\nu' \,, \tag{1}$$

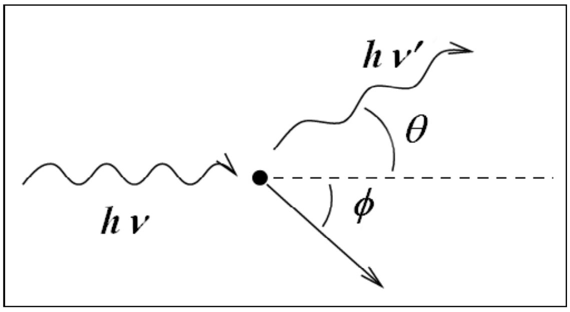


Figure 1. Geometry of scattering of a photon of energy hν by an electron initially at rest.

where $E = m\gamma c^2$, and $\gamma = 1/\sqrt{(1 - v^2/c^2)}$ is the Lorentz factor.

Conservation of momentum *along* the direction of the incident photon gives

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + m_0\gamma v\cos\phi\,, \qquad (2)$$

whereas momentum conservation in the transverse direction gives

$$0 = \frac{h\nu'}{c}\sin\theta - m_0\gamma v\sin\phi\,. \qquad (3)$$

These equations can be solved to give, in terms of the wavelengths ($\lambda = c/\nu$),

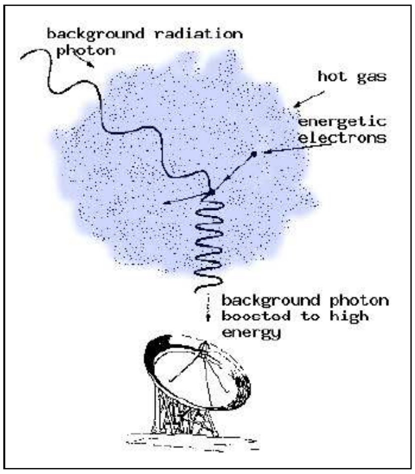
$$\lambda' - \lambda = 2\lambda_c\sin^2\frac{\theta}{2}\,, \qquad (4)$$

where $\lambda_c = h/m_0c$ is called the *Compton wavelength* of the particle. The scattering cross-section is essentially the Thomson scattering cross-section ($\sigma_T = \frac{8\pi}{3}\frac{e^2}{m_0c^2} \sim 6.65 \times 10^{-25}$ cm^2 for electrons) for low energy photons, for which $h\nu/m_0c^2 \ll 1$.

3. Inverse Compton Scattering

Next consider the case when the electron is not stationary and the photon (of energy $h\nu$) collides with it at an angle α to the trajectory of the electron (which has energy $\gamma m_e c^2$). If we view the process from the frame of reference of the electron, then we would find the photon frequency to be blue shifted to $h\nu' = \gamma h\nu(1 - v\cos(\alpha)/c)$, which gives $h\nu' \sim h\nu/2\gamma$ for $\alpha = 0$ and $h\nu' \sim 2\gamma h\nu$ for $\alpha = \pi$. That is, photons with $\alpha = 0$ appear to be of very low energy as compared to the electron and do not cause much of Compton recoil. For an average angle of $\alpha = \pi/2$, the photon appears to be blue shifted to $h\nu' \sim \gamma h\nu$. If we consider the low energy limit (Thomson scattering) limit of Compton interaction, then in the frame of reference of the electron,

Figure 2. Schematic diagram for Sunyaev–Zeldovich effect.



there is no appreciable frequency shift. So, in the frame of reference of the electron, the scattered photon has energy, $h\nu'' \sim h\nu' \sim \gamma h\nu$. When we transform to the stationary frame of reference again, we again find the photon blue shifted to $h\nu_{\mathrm{s}} \sim \gamma h\nu'' \sim \gamma^2 h\nu$.

This result has important implications in astrophysics. One often encounters electrons with large γ, which can scatter a low energy photon to very high energy, increasing its energy by a factor γ^2. For example, if $\gamma = 100$, the electron can scatter radio photons with $\nu = 10^9$ Hz to ultraviolet range with $\nu' = 10^{15}$ Hz.

3.1 *Sunyaev–Zeldovich Effect*

In the general case when high energy photons interact with electrons, the exchange of energy between them can lead to distinct changes in the photon spectrum and the energy content of the electrons. If this interaction happens mainly through Compton scattering, then the process in general is called *Comptonization.* Consider non-relativistic electrons with a high temperature interacting with photons with a Planckian spectrum. On one hand, the electron-recoil as a result of Compton scattering will tend to transfer energy from photons to electrons. This will be pronounced for high energy photons.

On the other hand, the high velocities of electrons will tend to transfer energy from electrons to photons. This will be important for high electron energy. If the electron temperature and the temperature of the Planckian spectrum is equal, then these two effects will balance.

In some astrophysical situations, when the electron temperature is much higher than the blackbody radiation temperature, then the spectrum of photons can be significantly changed from Planckian. Zeldovich and his student Rashid Sunyaev worked out this effect in detail in 1969, barely four years after the discovery of the CMBR, and it has now become a cornerstone of modern observational cosmology.

One can express the degree of interaction between photons and electrons by a parameter y, where

$$y = k_{\mathrm{B}} T_{\mathrm{e}} m_{\mathrm{e}} c^2 n_{\mathrm{e}} \sigma_{\mathrm{T}} L \,, \qquad (5)$$

where T_{e} is the temperature of the electrons, n_{e} is their number density and L is the size of the gaseous system. An example of the distortion of the CMBR spectrum from the Planckian is shown in *Figure* 3 for the case when $y = 0.2$.

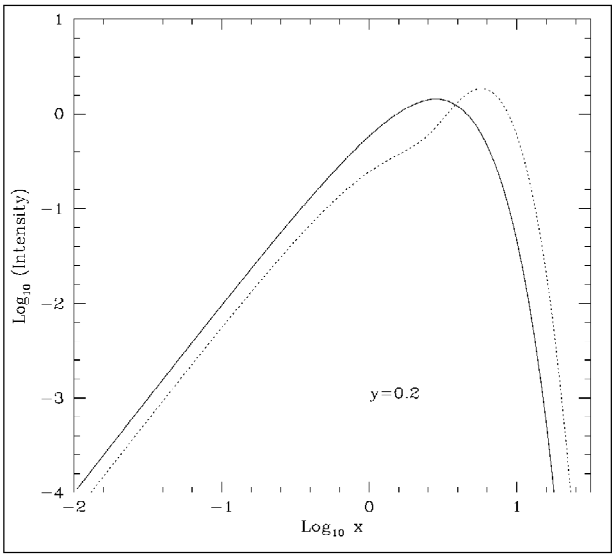


Figure 3. The effect of hot electrons on blackbody radiation is shown for y = 0.2, where x = hν/kT. The solid line shows the intensity for the original blackbody radiation in arbitrary units and the dashed line shows the effect of Comptonization.

In the Rayleigh–Jeans part of the spectrum, the intensity of radiation decreases, and the fractional decrement is roughly $-2y$. The number of photons in this part of the spectrum decreases essentially because some photons are boosted from this part to higher energies. Equivalently, one can say that the equivalent temperature of the radiation decreases in this part of the spectrum, and that $\frac{\Delta T}{T} \sim -2y$.

We can apply this to the case in which photons of the microwave background radiation of the universe interact with energetic electrons in a cluster on their way to radio telescopes on earth. In clusters of galaxies, the electron density is $n_e \sim 10^{-3}$ cm^{-3} with temperature of $\sim 5 \times 10^7$ K. The clusters have sizes of order $\sim 6 \times 10^{23}$ cm. The distortion in the Rayleigh–Jeans part of the spectrum of the microwave background then amounts to,

$$\frac{\Delta T}{T} \sim 10^{-5}\,. \tag{6}$$

This effect was first predicted by Sunyaev and Zeldovich in 1969, and is known as the Sunyaev–Zeldovich effect. This is a very small effect, because given the fact that $T \sim 2.7$ K, one has to measure a difference of a few micro Kelvin in the temperature in a portion of the sky, and it is a difficult experiment.

Astronomers have been able to detect this effect recently. *Figure* 4 shows the X-ray image of the Coma Cluster of Galaxies on the right, superimposed on the optical picture which shows the member galaxies. One can clearly

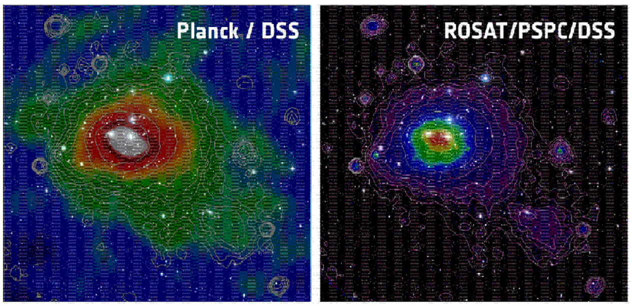


Figure 4. Detection of SZ effect toward Coma Cluster of Galaxies, shown in colour-coded image on the left, superimposed on the optical picture of the region. The image on the right-hand side is taken using X-rays.

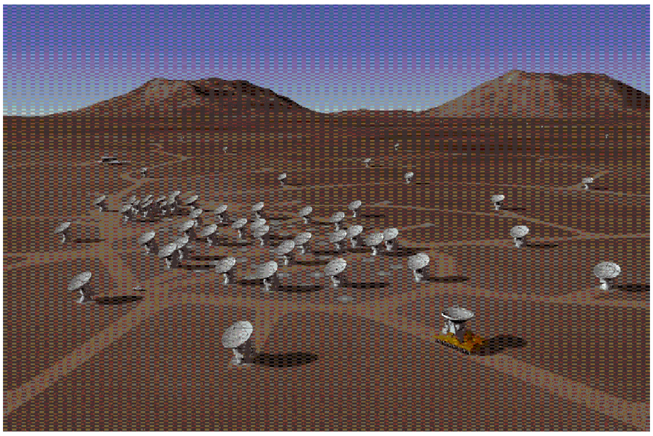


Figure 5. The proposed ALMA telescope with 6612-meter and 7-meter diameter radio telescopes which will observe the sky in millimeter (and submillimeter) wavelengths.

see the diffuse hot gas lurking between the galaxies and shining bright in X-rays (observed with ROSAT satellite). On the left, the same region is shown in microwave wavelengths, which shows a hole or a shadow in the background radiation, as observed recently by PLANCK satellite.

The exact shape of the distorted CMBR spectrum has also been detected and the observations bear out the detailed prediction made by Sunyaev and Zeldovich, that there is a decrement in the longer-wavelength side of CMBR radiation, whereas there is an increment in the shorter-wavelength side. Essentially, the photons from the low energy (longer-wavelength) side are boosted to the shorter-wavelength side, so that there is a shadow on the longer-wavelength side and an extra emission on the shorter-wavelength side.

Since this is a shadowing effect, the ability to detect these regions remains the same no matter how far or how near the regions are to the earth. This is in contrast to any emission from galaxy clusters, which is bound to be dimmed when viewed from far. This implies that whereas the X-ray emission from very distant galaxy clusters may not be visible, the shadowing of CMBR radiation due to Sunayev–Zeldovich effect can still be seen from very far. This makes it a very important tool to probe the distant (or, equivalently, the early) universe, and study the history of the universe and its constituents. Several big surveys are underway at this time to look for Sunyaev–Zeldovich effect from thousands of clusters. Recently a new telescope, called the Atacama Large Millimeter Array (ALMA), is being built in the Atacama desert of South America which will study this unique effect.

Suggested Reading

[1] **Y Rephaeli, Comptonization Of The Cosmic Microwave Background: The Sunyaev-Zeldovich Effect. *Annual Review of Astronomy and Astrophysics*, Vol.33, No.1, pp.541–580, 1995.**

[2] **Mark Birkinshaw, The Sunyaev Zel'dovich Effect, *Physics Reports*, Vol.310, No.97, pp.2–3, 1999.**

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