Generation of Gravitational Waves from Inspiralling Compact Binaries:

3PN Luminosity, 2PN Linear Momentum Flux and Applications

by

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Certificate

This is to certify that the thesis entitled

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submitted by

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is his original work. This has not been published or submitted to any other University for any other Degree or Diploma.

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Declaration

I hereby declare that the work reported in this thesis is entirely original. This thesis is composed independently by me at Raman Research Institute under the supervision of Professor Bala R Iyer. I further declare that the subject matter presented in this thesis has not previously formed the basis for the award of any degree, diploma, membership, associateship, fellowship or any other similar title of any university or institution.

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To my Family:

Father, Mother,

Brothers, Sisters,

Wife, Children,

all Friends

and

to all Yemenis who love and support Physics and Science.

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Contents

Preface

The extreme weakness of the gravitational interaction implies that in general gravitational waves (GWs) are extremely weak. Only in astrophysical situations involving strong concentrations of mass-energy (strong gravity) or those involving relativistic velocities can one hope to find sources of strong GWs. Inspiralling compact binaries (ICBs) consisting of neutron stars (NSs) or black holes (BHs) are one of the most promising of such sources. However, even such sources represent a weak signal buried in the strong noise of the detector and is optimally treated by methods such as matched filtering (MF). The success of MF depends crucially on the availability of accurate templates which in turn require an accurate computation of GW phasing for ICBs.

Though the general prototypical ICBs would be in circular orbits towards their late inspiral, there do exist astrophysical scenarios producing binaries with eccentricities when they enter the sensitivity bandwidths of the laser interferometric GW detectors. The construction of templates for eccentric binaries is more involved than that for binaries in circular orbits. One of the theoretical inputs required to compute GW phasing of eccentric binaries is the energy flux from such ICBs moving in general (non-circular) orbits. In this thesis we first compute the total energy flux from ICBs moving in general orbits at the third post-Newtonian order beyond the leading quadrupolar approximation.

In addition to energy and angular momentum, GWs also carry away linear momentum from the binary system leading to the possibility of GW recoil of the center-of-mass. The second set of problems in this thesis is the computation of the second post-Newtonian order linear momentum flux for ICBs moving in quasi circular orbits. Employing this linear momentum flux the resultant recoil is first computed for inspiral up to the last stable circular orbit (ISCO). A more physical estimate including the plunge from the ISCO to the horizon is finally provided.

Though the first mandate of the GW detectors is the direct detection of GWs, the ultimate goal of these GW experiments is to inaugurate GW astronomy. The ultimate excitement is to unravel new astrophysics and also probe the fundamental physics of gravitation. The test of general relativity using gravitational wave phasing in ground-based GW interferometric detectors and more importantly, space-based LISA is the third and final theme investigated in this thesis. In what follows we provide a brief summary of each chapter.

In chapter 2 the instantaneous contributions to the 3PN gravitational wave luminosity from the inspiral phase of a binary system of compact objects moving in general orbits is computed using the Multipolar post-Minkowskian wave generation formalism. The new inputs for this calculation include the mass octupole and current quadrupole at 2PN for general orbits and the 3PN accurate mass quadrupole. Using the 3PN quasi-Keplerian representation of elliptical orbits obtained recently the flux is averaged over the binary's orbit. The expression for the instantaneous contributions averaged over an orbit is presented in different coordinate systems: Standard harmonic coordinates (with logs), modified harmonic coordinates (without logs) and ADM coordinates. Alternative ***gauge invariant*** expressions of the energy flux are also provided.

The far-zone flux of energy contains hereditary contributions that depend on the entire past history of the source. In chapter 3, using the Multipolar post-Minkowskian wave generation formalism, we have proposed and implemented a semi-analytical method to compute the hereditary contributions from the inspiral phase of a binary system of compact objects moving in quasi-elliptical orbits up to 3PN order. The method explicitly uses the 1PN quasi-Keplerian representation of elliptical orbits and crucially exploits the implicit double periodicity of the motion to average the fluxes over the binary's orbit up to 3PN order. Together with the instantaneous contributions evaluated in the previous chapter, it provides crucial inputs for the construction of ready-to-use templates for binaries moving on quasi-elliptic orbits, an interesting class of sources for the ground based gravitational wave detectors and especially space based detectors like LISA.

In chapter 4, the gravitational recoil of non-spinning black-hole binaries (in quasi-circular orbits) is calculated at the second post-Newtonian order (2PN) beyond the dominant effect, obtaining, for the first time, the 1.5PN correction term due to tails of waves and the next 2PN term. The maximum value of the net recoil experienced by the binary due to the inspiral phase up to the innermost stable circular orbit (ISCO) is of the order of $22\,\mathrm{km\,s^{-1}}$. The kick velocity accumulated during the plunge from the ISCO up to the horizon is estimated by integrating the momentum flux using the 2PN formula along a plunge geodesic of the Schwarzschild metric. The contribution of the plunge dominates over that of the inspiral. For a mass ratio $m_2/m_1 = 1/8$, a total recoil velocity (due to both adiabatic and plunge phases) of $100 \pm 20\,\mathrm{km\,s^{-1}}$ is estimated. For a ratio 0.38, the recoil is maximum and estimated to be $250 \pm 50\,\mathrm{km\,s^{-1}}$. In the limit of small mass ratio, $V/c \approx 0.043\,(1 \pm 20\%)\,(m_2/m_1)^2$. These estimates are consistent with, but span a substantially narrower range than, those of Favata et al (2004).

Observations of the supermassive binary black hole mergers in the Laser Interferometer Space Antenna (LISA) and stellar mass binary black holes in the European Gravitational-Wave Observatory (EGO) offer an unique opportunity to test the non-linear structure of general relativity since they will observe events with amplitude signal-to-noise ratio of several thousands and hundreds respectively. For a binary composed of two non-spinning black holes, the non-linear general relativistic effects depend only on the masses of the constituents. In chapter 5, we investigate the extent to which such observations afford high-precision tests of Einstein's gravity by exploring the possibility of a test to determine all the post-Newtonian coefficients in the gravitational wave-phasing. We show that LISA provides a unique opportunity to probe the non-linear structure of post-Newtonian theory both in the context of general relativity and its alternatives. However, mutual covariances between the various PN coefficients dilute the effectiveness of such a test.

In chapter 6, we propose a more powerful test in which the various post-Newtonian coefficients in the gravitational wave phasing are systematically measured by treating ***three*** of them as independent parameters and demanding their mutual consistency. LISA (EGO) will observe BBHs inspirals with a signal-to-noise ratio of more than 1000 (100) and thereby test the self-consistency of each of the nine post-Newtonian coefficients that have so-far been computed, by measuring the lower order coefficients to a relative accuracy of $\sim 10^{-5}$ (respectively, $\sim 10^{-4}$) and the higher order coefficients to a relative accuracy in the range 10^{-4}-0.1 (respectively, 10^{-3}-1).

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1. Luc Blanchet, Moh'd S S Qusailah, Clifford M Will, " Gravitational Recoil of Inspiralling Black-Hole Binaries to Second Post-Newtonian Order", Astrophys. J. 635 (2005) 508; astro-ph/0507692

2. K G Arun, B R Iyer, Moh'd S S Qusailah and B S Sathyaprakash, "Testing the post-Newtonian theory with Gravitational Wave observations", Class. Quant. Grav. 23 (2006) L37-L43; gr-qc/0604018

3. K G Arun, B R Iyer, Moh'd S. S. Qusailah and B S Sathyaprakash, "Probing the non-linear structure of general relativity with black hole binaries", Phys. Rev. D 74 (2006) 024006; gr-qc/0604067

4. K G Arun, L Blanchet, B R Iyer and Moh'd S S Qusailah, "Gravitational wave energy flux from inspiralling compact binaries in quasi-elliptical orbits: The instantaneous terms" (To be submitted)

5. K G Arun, L Blanchet, B R Iyer and Moh'd S S Qusailah, "Tail effects in the 3PN gravitational wave energy flux of inspiralling compact binaries" (To be submitted)