Chapter 1

Introduction

1.1 Introduction

The standard model of cosmology is now well established in the broad features (see e.g. Padmanabhan 2003, Dodelson 2002, Peebles 1993). The currently popular ACDM model is consistent with various existing observations (see e.g. Spergel et al. 2003, Hu & Dodelson 2002, Dodelson 2002). However there is still some place for the presence of certain sub-dominant components, primordial magnetic fields being one of them. Magnetic field is ubquitous in the the universe. The presence of magnetic fields on variety of scales from solar system planets to galaxies is well established. There is now compelling evidence for the existence of magnetic fields on even large scales like cluster of galaxies. Accurate quantitative estimates of the field strengths as well as tracing the field structure in galaxies and clusters of galaxies is still an ongoing effort which is progressing with improving observations (See Han & Wielebenski (2002) for a recent summary). Hence, it is quite pertinent that detailed theoretical analysis of various possible scenarios related to the origin, the evolution and the dynamical effect of large-scale magnetic fields be studied. In this thesis we have attempted to make a detailed study of some of these aspects.

The origin of magnetic fields with strength of the order of a microGauss observed in large-scale systems like galaxies, cluster of galaxies etc is still an unsolved problem (for a summary see e.g. Widrow 2002). There are two main scenarios in which this problem is being addressed. According to one scenario, the observed fields in galaxies could have resulted from an exponential amplification of a small seed field which might have been generated in the very early Universe. In this approach, also known as the dynamo mechanism, the amplification occurs primarily during the later epochs when the evolution of the fluid enters the non-linear regime. The other alternative is to avoid the need for dynamo by postulating the presence of significant large-scale magnetic fields already frozen in the primordial plasma before the commencement of structure formation (Hoyle 1958, Piddington 1964, 1972). Adiabatic compression of the magnetic field lines during the non-linear collapse of a structure can then amplify fields of order $10^{-9}G$ to the present observable values. Such fields could have originated during the inflationary era in the very early universe (e.g. Ratra 1992, Widrow & Turner 1988). In chapters 2 and 4 of this thesis, we investigate the effects of such large-scale magnetic fields on the two main cosmological observables viz. the large-scale matter distribution at the current epoch and the CMBR.

The cosmic microwave background radiation (CMBR) is a very important probe of the conditions in the early Universe. The anisotropies in the CMBR are a direct imprint of the spatial fluctuations in the primordial plasma at the last scattering surface. These anisotropies occur due to variations in the specific intensity distribution (temperature anisotropies) as well as in the polarization distribution (Hu & White 1997). The presence of a magnetic field in

the early Universe affects the evolution of metric perturbations, and as a result, produces temperature and polarization anisotropies in the CMBR (Kahniashvili et al. 2000). It is interesting to note that a field strength of the order of a micro-Gauss has an energy density equal to the energy density in CMBR at the current epoch. This implies that a field strength of the order of a nano-Gauss can induce fluctuations in the CMBR at the level of one part in 10^{-5} which is also the level at which these are observed. With improving precision data coming from CMBR experiments like WMAP (Hinshaw et al. 2003) and the upcoming future CMBR mission like Planck it might be possible to put better constraints on the magnetic field strength.

In addition to its effects on the CMBR, magnetic fields can also potentially influence the structure formation process. This can happen in two ways. A fully homogenous magnetic field can affect the background cosmology because it breaks isotropy and hence cannot exist in a strictly Lemaitre-Friedmann-Robertson-Walker (LFRW) Universe. Although such a magnetic field cannot originate from any causal process it still can be postulated as an initial condition. A more realistic scenario however is the case of a completely tangled field. A tangled field of sufficient strength if present can influence the dynamics of the fluctuating component of the plasma. This analysis was pioneered by Wassermann (1978), who showed that a tangled magnetic field can source density fluctuations which can subsequently amplify by gravitational collapse. This suggests an alternative scenario in which the presence of primordial magnetic fields would have entirely caused large-scale structure formation. In chapter 1 of this thesis we discuss this aspect in detail. In particular, we discuss the signature that a tangled magnetic field will generate in the distribution of matter in redshift space. We find that there could be potentially interesting observational signatures of such an effect which could be detected with ongoing and future large-scale galaxy redshift surveys.

The question of the origin of cosmic magnetic fields is still an unsolved problem and historically this was the prime motivation to study them. Various mechanisms have been proposed for the generation of magnetic fields. Some of these are briefly reviewed in the final section of the current chapter. In chapter 2 of this thesis we discuss one such mechanism which operates in the pre-recombination plasma within the framework of the standard cosmological model.

1.2 Observational evidence for magnetic fi elds

Various direct/indirect methods exist for detecting the presence of magnetic fields in galaxies, clusters, IGM etc. Each of these methods however yield an estimate only for one of the components either perpendicular (synchrotron radiation) or parallel (Faraday rotation) relative to the line-of-sight. In this section, a brief discussion of each of these methods and the corresponding estimates of field strengths is given.

1.2.1 Faraday rotation of distant radio sources

When a plane polarized radiation propagates through an ionized medium in which magnetic fields are present, the plane of polarization gets rotated by an angle (Jackson 1999) which is given as:

$$\Delta \psi = \mathbf{R} \mathbf{M} \lambda^2 \tag{1.1}$$

where $\Delta \psi$ is the angle of rotation and *RM* is the rotation measure given by,

$$RM = 0.81 \int_0^L n_e B_{\parallel} dl \ rad \, m^{-2}$$
(1.2)

The rotation measure *RM* thus depends on $n_e(cm^{-3})$, the free-electron density, $B_{\parallel}(\mu G)$ the line-of-sight magnetic field and L(parsec) the path-length traversed. RM observations are made at three or more wavelengths in order to remove the $n\pi$ ambiguity in the measurement of $\Delta \psi$ (Ruzmaikin Shukurov & Sokoloff 1988). The observed rotation measure is then the sum of all contributions along the line-of-sight between the source and the observer. This method has been widely used to obtain estimates of the magnetic field strength in various large-scale systems as enumerated below:

(1) Galactic magnetic field: Observations of pulsars in the Milky Way have been used extensively to model the Galactic magnetic field using the above effect. Pulsars are astrophysical objects emitting regular pulses of electromagnetic radiation with periods ranging between few milliseconds to few seconds. From pulsars, the column density of electrons can be obtained in the form of the dispersion measure $DM \propto \int n_e dl$. Hence by comparing the ratio RM/DM, estimates of the field strength can be made (Beck 2001). The earliest significant observations was presented by Manchester (1972,1974) which revealed a local uniform magnetic field of about 2.2 μG lying in the Galactic plane. Several detailed measurements tracing the field structure have been performed since then (See Han et al. 2002).

(2) Cluster magnetic fields: Estimate of field strength in galaxy clusters are made using Faraday effect by identifying a radio source inside or behind the cluster. In some clusters an independent determination of the electron number density is possible by studying the X-ray emission from the hot gas forming the intra-cluster medium. Kim et al. (1990) determined the RM for 18 sources behind the Coma cluster and estimated an intra-cluster field strength of $B \sim 2.5(L/10 \text{ kpc})^{-1/2} \mu G$ where L is the typical field reversal scale. In many cases where very few radio sources are available in the background, a statistical study of a sample of clusters are made. Kim,Tribble & Kronberg (1991) used RM data from 50 clusters and plotted it as function of their impact parameters from the cluster centers. They found that

Chapter 1. Introduction

the dispersion rises gradually with decreasing impact parameters revealing the presence of magnetic fields suggesting strong magnetic fields of the order of a micro-gauss on scales of order 10kpc. In addition to the cluster medium, the contribution from the Galaxy too can be significant. However for observations limited to high Galactic latitudes, this contribution is very small and can be neglected. Recent studies of cluster magnetic fields using this method give estimates of a few to 10μ G coherent over scales of 10-20 kpc.

(3) High-redshift galaxies: Evidence of magnetic fields in concentrations at high or even intermediate redshifts is significant because it poses a big problem for the dynamo mechanism. At present, there are a few measurements which seem to indicate that significant magnetic fields could be present in systems at high redshifts. The analysis of the RM map of the radio jet associated with a quasar PKS 1229-121 showed that the RM changes sign along the "ridge line" of the jet in an oscillatory manner (Kronberg, Perry & Zukowski 1992). The spectrum from the quasar is found to have a prominent absorption feature which is thought to be due to an intervening object at a redshift z = 0.395. The authors concluded that the intervening object could be a spiral galaxy and estimated the magnetic field to be in the range $1 - 4\mu$ G. Athreya et al. (1998) found significant rotation measures in a sample of 15 high redshift radio galaxies ($z \approx 2$) which they studied in radio emission at multiple frequencies and estimated that they could possess fields of the order of a micro-Gauss coherent over several kpc.

(4) Cosmological intergalactic magnetic field: There have been various attempts to detect the presence of a cosmological magnetic field in the intergalactic medium (Widrow 2002). However no positive detection has resulted yet and only model dependant upper limits exist. The main difficulty is that the medium is rarefied and hence its difficult to estimate observationally the electron number density as well as the coherence scale. However some interesting upper limits can be derived using theoretical estimates of the ionization fraction and assuming reasonable values of the coherence length. In this case, the formula for the *RM* is generalized to take into account the expansion of the Universe:

$$RM = 8.1 \operatorname{rad} \mathrm{m}^{-2} \left(\frac{n_e(t_0)}{10^{-5} cm^{-3}} \right) \left(\frac{H_0^{-1}}{1Mpc} \right) \int_0^{z_s} dz \frac{(1+z)^2 \mathbf{B} \cdot \hat{n}}{(\Omega_m (1+z)^3 + \Omega_\Lambda)^{1/2}}$$
(1.3)

The simplest case of a magnetic field uniform across the Hubble volume was considered by various authors. In this case, the RM distribution across the sky will have a dipole component (Sofue, Fujimoto, & Kawabata 1968) A sample of 309 galaxies and quasars was tested by Vallee (1990) for signatures of the dipole and he obtained an upper limit to the RM of about 2 rad m^{-2} which translates to an upper limit of $6 \times 10^{-12}G$ for the uniform component of the field.

In cases where the magnetic field varies on smaller scales the RM contribution will be more complicated than a simple dipole. In such a case, the average rotation measure will be zero and the variance of the RM distribution σ_{RM}^2 is the lowest non vanishing statistical measure. Kronberg & Perry (1982) considered the variation of the variance with the redshift in a simple model where they assumed that clouds of uniform electron density and magnetic field randomly populate the entire Universe. They made some assumptions about the number density and size of the clouds. Their model was motivated by observations of the Ly α forest which implied the existence of a large number of neutral hydrogen clouds at cosmological distances. They however obtained very weak bounds of magnetic field strength of the order of $0.1\mu G$. This analysis was refined by Blasi,Burles & Olinto (1999) wherein they assumed more plausible model parameters for the clouds and concluded that a detectable variance is possible for magnetic fields as low as $6 \times 10^{-9}G$.

A much better way to probe cosmological magnetic fields would be to look for the correlations in the RM of the sources. This is because the correlation method can provide information about the power spectrum of the magnetic fields. This approach was first suggested by Kolatt (1998) and subsequently applied for the case of cosmologically tangled magnetic fields by Sethi (2003).

1.2.2 Faraday rotation of CMB

The same mechanism which causes Faraday rotation in the radio emission of quasars can also affect the CMBR since it is linearly polarized (see e.g. Dolgov 2005). At early epochs, before the beginning of recombination, photon-baryon plasma is in thermal equilibrium with tight-coupling between the various species. Hence, only the monopole and the dipole are the non-vanishing multipole moments characteristing the photon distribution. However, as the plasma recombines, the mean-free path of photons starts increasing rapidly generating a quadrupole moment which acts as the source of polarization. If large-scale magnetic fields are present at this epoch, they will induce Faraday rotation of the polarisation plane. This effect was worked out in detail by Kosowsky and Loeb (1996). A rough estimate of the rotation angle can be made as follows: The rate of variation of the rotation angle ϕ for radiation of frequency v passing through a plasma, containing free electrons with number density n_e , in the presence of a magnetic field **B** is given as:

$$\frac{d\phi}{dt} = \frac{e^3 n_e}{2\pi m_e^2 v^2} (\mathbf{B} \circ \hat{n}) \tag{1.4}$$

Here, \hat{n} is the propagatin direction of the radiation. Using the fact that B/v^2 is time-independent and $\int n_e dt \sim 1/\sigma_T$, σ_T being the Thomson cross section, the rms of the rotation angle can be estimated as:

$$\langle \phi^2 \rangle^{1/2} \simeq 1.6^o \left(\frac{B_0}{10^{-9}G}\right) \left(\frac{30GHz}{v}\right)^2$$

As seen above, the rotation measure is inversely proportional to the observed frequency and hence only low frequency CMBR observations have the potential to probe the above effect.

There are however various other interesting consequences on the CMB anisotropies as a result of the above Faraday effect. In the standard cosmological scenario, there is no cross correlation between the E and B modes of polarization since such an effect would be parity violating (see for eg Hu and White 1997). However, it was first shown by Scannapieco and Ferreira (1997) that a homogenous nano-gauss field at recombination may induce an observable parity-odd cross-correlation between the temperature and polarisation modes. More recently (Kosowsky et al. 2005) the effect of Faraday rotation on the CMBR polarization power spectrum was worked out in detail for the case of a stochastic magnetic field. They found that the B-mode spectrum induced by Faraday rotation peaks at arc-minute angular scales.

1.2.3 Synchrotron emission

Relativistic electrons in the presence of a magnetic field accelerate and emit polarized radiation (For an elementary derivation see Jackson 1999). This phenomenon is known as synchrotron emission. This is the most direct way of inferring the presence of a magnetic field. The total synchrotron emission from a source provides an estimate of the field strength and the degree of polarization gives an indication of its structure. The net synchrotron emissivity however in addition to the magnetic field energy also depends on the distribution of relativistic electrons n_e . The intensity of emission roughly goes as $n_e B^2$ (see for eg Widrow 2002). Hence an additional assumption/input is required to infer the field value. This is the main drawback of this method as far as quantifying the field strength is concerned. One assumption that is often made is that the energy density in the magnetic field is in equipartition with the plasma energy density. The average equipartition field strengths in galaxi es ranges from 4 μ G in M33 upto 19 μ G in NGC2276 (Buczilowski and Beck 1991 ; Hummel and Beck 1995).

1.3 The alternative: dynamo or primordial

The current observational status which was briefly described above can then be summarised as:

• Magnetic fields with strength of the order of few micro-Gauss are inferred in galaxies whenever the relevant observations are made.

• Microgauss magnetic fields have been observed in the intracluster medium of various rich clusters with coherence lengths comparable to the scale of the clusters.

• There is compelling evidence for galactic scale magnetic fields at high redshifts as well. Magnetic fields may also exist in damped Lyman alpha systems at high redshifts.

• There are no detections as yet of fully cosmological fields (i.e fields associated with the inter-galactic medium) but model dependant constraints are consistent with their having strength of the order of a nano-Gauss.

In trying to explain the above mentioned observations there are two main complementary approaches which basically follow from the time evolution equation of the magnetic field:

$$\frac{\partial (a^2 \mathbf{B})}{\partial t} = \nabla \times \mathbf{v} \times (a^2 \mathbf{B}) + \frac{1}{4\pi\sigma} \nabla^2 \mathbf{B}$$
(1.5)

Here, σ is the electrical conductivity and v is the fluid velocity and a(t) is the scale-factor of the Universe. When the first term on the RHS dominates over the second term which accounts for diffusion, then amplification of the field can occur through the conversion of the kinetic energy of the fluid to magnetic energy. The main ingredients required for such a process to occur are hydrodynamic turbulence and differential rotation. Turbulent motion in the plasma can distort and stretch magnetic field lines which can consequently result in an increase of the field strength. It can be shown (Subramanian & Brandenburg 2004), that generically exponential amplification of the field can take place. The amplification ends when equipartition between the kinetic energy density of the small scale turbulent motion and the magnetic energy density is reached. The time required to reach saturation starting from a seed field as low as $10^{-20}G$ may be 10^8-10^9 years in a Universe dominated by cold dark matter (CDM) with no cosmological constant. This limit on the seed field can however be relaxed to as low as $10^{-30}G$ in the case of a cosmoogical constant dominated Universe (Davis et al. 1999) as is indicated by recent observations of type -IA supernovae (Perlmutter et al. 1999; Riess et al. 2004; Schmidt et al. 1998) and CMB anisotropy measurements (Spergel 2003).

The effectiveness of the dynamo mechanism however has been questioned by several authors lately. One of the arguments raised is that the amplification of small scale fields is neglected in the approach. This could be important because the small scale fields saturate earlier and can thus stop the dynamo process even before a coherent field may develop on larger scales. (See for instance Kulsrud et al. 1997).

The main alternative to the dynamo mechanism is the primordial field hypothesis. In this scenario, it is assumed that there is no regeneration or amplification of the magnetic field due to the back-reaction term in Eq. (1.5) because this term in the absence of an external source like turbulence is of a higher order in perturbation theory and hence contributes negligibly on large scales. It thus follows that the time evolution of a primordial magnetic field on large scales follows a flux frozen evolution given by:

$$B(t) = \frac{B_0}{a^2(t)}$$
(1.6)

Here, B(t) is the magnetic field strength at a given spatial location at time *t* whereas B_0 is the field strength at the same spatial location but evaluated at the current epoch where the scale-factor today is normalized to unity. The above relation can also be used to deduce the variation of the field strength in an isotropic collapse of matter in the following manner.

In an isotropic collapse of a magnetized gas cloud of size *L*, the magnetic field varies with the size as $B \propto L^{-2}$ whereas the density ρ of the cloud varies as $\rho \propto L^{-3}$. These relations also imply that the magnetic field strength varies with density as $B \propto \rho^{2/3}$. Using this relation we can deduce the primordial field strength B_i required in a proto-galactic cloud of density ρ_{IGM} to produce the observed field B_f in galaxies with density ρ_{gal} as :

$$B_i = B_f \left(\frac{\rho_{\rm IGM}}{\rho_{\rm gal}}\right)^{2/3} \tag{1.7}$$

Assuming roughly a ratio of $\rho_{IGM}/\rho_{gal} \sim 10^{-6}$ today, and $B_f \sim 10^{-6}G$, we can estimate

$$B_i \sim 10^{-9}G$$
 (1.8)

Thus we see that a primordial field strength of the order of a nano-Gauss is required in a primordial scenario to explain the observed fields in galaxies. This value as we will see in later sections is consistent with all the observational limits on the intergalactic field, big bang nucleosynthesis constraints and also the CMBR observations.

1.4 Standard cosmology and primordial magnetic fields

In the previous sections we discussed the primordial field scenario and the time evolution of large-scale fields. Primordial magnetic fields of sufficient strength can have interesting effects on various cosmological processes. They can affect the background cosmological model by affecting the space-time geometry, they can also affect the fluctuating plasma both in the pre-recombination as well as post-recombination era. Effects of primordial magnetic fields in the pre-recombination era can produce distinct detectable signals in the CMBR whereas effects in the post-recombination era can get manifested in the large-scale matter distribution at the present epoch. In addition to these, magnetic fields if they existed at the epoch of nucleosynthesis can also have a strong effect on that process. Below we review each of these detectable effects of the primordial fields.

1.4.1 Standard background model

The background model in cosmology (Peebles 1993) rests on the following assumption also known as the cosmological principle—which states that "The spatial section of the

Chapter 1. Introduction

Universe on very large scales is homogenous and isotropic". The mathematical formulation of this principle within relativity leads to the following general (LFRW) form for the background space-time metric:

$$ds^{2} = dt^{2} - a^{2}(t) \left(\frac{dr^{2}}{1 - kr^{2}} - r^{2}d\Omega^{2} \right)$$

Here $k = 0, \pm 1$. As the universe expands, the physical distance between any two fixed points scales with time as the scale-factor a(t) Consistency with the cosmological principle also requires that the matter stress energy tensor have the form that of a perfect fluid/mixture of fluids given as: $T_{\mu\nu} = \sum P_i g_{\mu\nu} + (\rho_i + P_i)u_{\mu}u_{\nu}$ where ρ_i is the fluid energy density, $P(\rho)$ is the isotropic pressure and u_{μ} is the fluid four-velocity. The LFRW metric provides the kinematical framework for cosmological models. The time evolution of the scale-factor a(t) is given by solving the dynamical equations of General Relativity. This leads to the Friedmann-Lemaitre equation:

$$H^{2} = \frac{8\pi G}{3}\rho + \frac{1}{3}\Lambda - \frac{k}{a^{2}}$$
(1.9)

Here $H = \dot{a}/a$ is called the Hubble-parameter and ρ is the total matter density. The various species present in the cosmological models are assumed to satisfy the equation of state of the form $P = w\rho$ where w is a constant and P is the thermodynamic pressure. This includes photons/relativistic matter w = 1/3, non-relativistic baryonic/dark matter w = 0, cosmological constant w = -1. The contribution of each of the species is specified by the dimensionless quantity $\Omega = \rho/\rho_c$ where $\rho_c = 3H^2/8\pi G$ is the critical density corresponding to a flat universe k = 0. Observations currently favour the scenario in which the matter composition at the present epoch is: dark matter $\Omega_m = 0.27$, cosmological constant $\Omega_{\Lambda} = 0.73$ (Spergel et al. 2003; Perlmutter et al. 1999; Riess et al. 2004), baryonic matter $\Omega_b h^2 = 0.044$ (Tytler et al. 2000), relativistic species $\Omega_r h^2 = 4 \times 10^{-5}$, with $h \simeq 0.7$ (Freedman et al. 2000).

1.4.2 Effect of a homogenous magnetic fi eld

The standard cosmological model assumes homogeneity and isotropy in the background space-time. A homogenous magnetic field has non-vanishing anisotropic pressure and hence cannot exist in a strictly LFRW universe. However, a universe in which there is a small deviation from FRW geometry in the form of a global space-time anisotropy can accomodate such a field. A large-scale homogenous magnetic field will modify the background space-time since it will break the isotropy. Cosmological models with a homogenous magnetic field have been considered by many authors (see e.g. Thorne(1967)) Below we discuss the analysis performed by Zeldovich & Novikov (1983). More specifically the background space-time which can accomodate a uniform large-scale magnetic field will be a homogenous

anisotropic space-time which has the following general form:

$$ds^{2} = dt^{2} - a^{2}(t)(dx^{2} + dy^{2}) - b^{2}(t)dz^{2}$$
(1.10)

In the above, it is assumed that the uniform magnetic field is present along the z-axis and a(t) and b(t) are the two scale factors which give the expansion rates along the transverse (x and y) and longitudinal(z) directions respectively. Thus the effect of a uniform magnetic field is to change the relative expansion rates in the various directions. The evolution of the two scale factors is dictated by the Einstein equations. The CMBR temperature in such a background will be direction dependant. Specifically the temperatures measured in the transverse and longitudinal directions will be:

$$T_x = T_y = T_{rec} \frac{a}{a_0} \tag{1.11}$$

$$T_z = T_{rec} \frac{b}{b_0} \tag{1.12}$$

Here, T_{rec} is the temperature existing at recombination. The maximal temperature anisotropy is then given as:

$$\frac{\Delta T}{T} = \frac{T_x - T_z}{T_r ec} \left(\frac{a}{a_0} - \frac{b}{b_0} \right) \tag{1.13}$$

By using the above expression and determining the evolution of the scale factors from Einstein equations, Zeldovich & Novikov were able to estimate that a field strength of $10^{-9} - 10^{-10}G$ today would produce a temperature anisotropy $\delta T/T \sim 10^{-6}$.

The above analysis was updated by Barrow et al. (1997) on the basis of the COBE anisotropy measurements (Bennett et al. 1996). Barrow et.al performed a statistical analysis appropriate for the non-Gaussian nature of the global anisotropy pattern on the COBE 4-year data set and thus derived an upper limit for the magnetic field strength of $B_0 < 6.8 \times 10^{-9} (\Omega_0 h^2)^{1/2}$, where Ω_0 is the cosmological density parameter.

1.4.3 Effect of magnetic fi elds on fluctuations

As discussed in the previous sections, a large-scale homogenous magnetic field is strongly constrained because of the observed homogeneity and isotropy of the background. A uniform field is very difficult to generate from physical processes and it can be postulated only abinitio i.e arising in the initial conditions. A more realistic scenario is to consider magnetic fields which are fully inhomogenous/tangled. However, tangled magnetic fields of similar strength if they exist can induce additional fluctuations in the matter as well as the metric components over and above the background.

The evolution of the linear perturbations can be studied within the framework of linear perturbation theory in General Relativity. This approach was pioneered by Lifshitz (1946)

Chapter 1. Introduction

in which he formulated the problem in the synchronous gauge. Much later, a gauge invariant presription of studying the evolution of perturbations was given by Bardeen (1980). According to this prescription, the perturbations in the space-time metric and the matter energy momentum tensor can be decomposed into three types i.e scalar, vector and tensor depending on the transformation properties in the spatial hypersurface. Physically, these fluctuations correspond to density perturbations (scalar), vorticity perturbations (vector) and gravitational waves (tensor). Bardeen showed that the evolution of each of these fluctuations is decoupled from each other and hence they can be studied separately. In the standard cosmologically scenario, scalar fluctuations are the most interesting because they lead to all the main interesting features reflected in the CMBR anisotopies (see e.g. Hu and Sugiyama 1995) and also because their time evolution exhibits gravitational instability which is useful in explaining the structure formation in the Universe (Peebles 1980). The vector and tensor modes of fluctuations are less interesting from the point of view of structure formation mainly because they do not exhibit gravitational instability. Moreover, in the absence of a continual source, the vector modes decay with time on all scales (see e.g. Bardeen 1980; Hu and White 1997) whereas the gravitational waves decay once they enter the horizon (see e.g. Pritchard and Kamionkowski 2004). However in the presence of primordial magnetic fields, each of these modes can be affected in a definite manner and may lead to observational signatures which could either detect/rule out the presence of the magnetic fields. We will briefly review these effects below:

1.4.3.1 Scalar fluctuations

Scalar perturbations in the metric are sourced by the corresponding scalar components of the energy momentum tensor which physically correspond to density and longitudinal velocity perturbations in matter (for a review see e.g. Mukhanov, Feldman & Brandenburger 1992). In the standard scenario scalar fluctuations are posulated as natural initial condition mainly because it can be shown that the evolution of scalar fluctuations exhibits gravitational instability and hence is useful in explaining the growth and formation of structures in the Universe.

In the standard cosmological scenario, primordial density fluctuations produce three main effects in the photon-baryon fluid depending on the scale of the fluctuation (See Hu & Dodelson 2002). On superhorizon scales, the dominant effect is the gravitational redshift of photons as they experience the effect of gravitational potential produced by density fluctuations (Sachs Wolfe effect). For modes which are sub-horizon, acoustic oscillations are induced in the photon baryon fluid which are manifested as the acoustic peaks in the CMBR temperature power spectrum. On scales smaller than the Silk damping scale, the finite heat conductivity and viscosity of the medium lead to an exponential damping of the perturbations.

The presence of a large scale magnetic field can modify each of the above effects and hence in principle constraints can be derived on the field strength at the last scattering surface by studying these effects.

Adams et al. (1996) studied the effect of primordial magnetic fields on scales smaller than the Hubble radius at the last scattering surface which corresponds to angular scales less than a degree. They found that the in the presence of magnetic fields the effective baryon sound velocity is modified depending on the magnetic field strength and the orientation of the field direction with respect to the wave vector of the mode. By performing an all sky average summing also over the the orientation angle of the magnetic field, they determined the effect on the CMBR temperature power spectrum by a simple modification of the CMBFAST (Seljak & Zaldarriaga 1996) numerical code for a magnetic field strength of $2 \times 10^{-7}G$. They also predicted that a field strength of $5 \times 10^{-8}G$ should be detectable in MAP and PLANCK experiments.

Recently Giovannini (2004) performed a systematic treatment of the linear theory of scalar fluctuations for the case of stochastic magnetic field. They addressed the problem of initial conditions for the super-horizon modes of fluctuations for the coupled photon baryon plasma including the contribution of neutrinos as well. One of the results they deduced was that the conventional adiabatic mode that is discussed in the standard scenario gets modified and hence should be taken into account in accurate calculations of CMBR anisotropies which was neglected in earlier works.

In general, though, it is believed as a result of the above calculations that constraints on magnetic field strengths from scalar fluctuations will be weak (Kahiniashvili et al. 2000) and much better and stronger constraints will be obtained from vector or tensor modes although a complete calculation including the appropriate initial conditions for scalar modes is still lacking.

1.4.3.2 Vector modes

Vector fluctuations affect the transverse part of the spatial metric and correspond to rotational velocity perturbations in the fluid (see e.g. Giovannini 2005). It can be shown that the contribution to the CMBR temperature anisotropy is due to the vorticity of the fluid at decoupling. In the absence of any source, the vorticity simply decays in time and hence is unimportant in the standard model. In the presence of magnetic fields however various effects can arise which could be potentially detectable in the CMBR spectrum. We will enumerate some of these effects.

Subramanian and Barrow (1998) evaluated the effect of a tangled magnetic field on the vector perturbation of the cosmological fluid. These correspond to rotational velocity perturbations or Alfven wave modes in the photon baryon plasma. In the standard cosmological scenario, in which only scalar perturbations are present, the finite heat conductivity and viscosity of the medium due to finite photon mean free path causes dissipation of scalar modes below the Silk damping scale which reflects in the CMBR anisotropy as an exponential cutoff on arc minute scales. However, the Alfven wave modes survive Silk damping on much smaller scales than the scalar modes. They estimated that a magnetic field strength amplitude of $3 \times 10^{-9}G$ will induce temperature anisotropies in the CMBR at the level of $10\mu K$ at and below arc minute scales and hence could potentially be detected by future CMBR experiments.

Durrer et al. (1998) also independantly studied the possible effects of Alfven waves on the CMBR anisotropies. They however did not consider fully tangled fields but made split of the magnetic field into a large background uniform component and a small tangled component. The background component simply redshifts with the expansion of the Universe while the small scale tangled component can be thought of as arising due to the back-reaction of the fluid motion on the large scale component. Durrer et al. argued that in such a case the Alfven waves may produce an interesting signature on the statistical properties of the CMBR. More specifically they showed that in addition to the temperature auto-correlations for a given multipole (*l*) of the CMBR, the vorticity field also induces transitions $l \rightarrow l \pm 1$ and hence a correlation between different multipole amplitudes. On the basis of these considerations, they used the 4 year COBE data to obtain a limit on the homogenous magnetic field of the order of $(2-7) \times 10^{-9}G$ for the range of spectral indices -7 < n < -3 of the small scale tangled field.

In addition to its effect on the CMBR temperature anisotropies, the presence of a tangled magnetic field can also induce polarisation anisotropies. The polarisation of CMBR is induced during the recombination process as the mean free path of photons increases to the horizon size. In general polarization can be described in terms of 'E type (curl free)' and 'B type (divergence free)' modes which correspond to some particular linear combinations of the familiar Stokes parameters Q and U (Zaldarriaga & Seljak 1997). According to this decomposition it follows readily from parity properties that scalar fluctuations generate only the E mode of polarisation whereas vector and tensor perturbations can generate B modes as well. Thus the standard cosmological model which contains only scalar fluctuations predicts vanishing B mode polarization. Hence, the detection of B mode polarization can be used to either confirm or constrain the magnetic field strength.

Semi-analytical calculations of the CMBR polarization anisotropies induced by stochastic magnetic fields were performed by Subramanian, Seshadri & Barrow (2003) for small angular scales corresponding to l > 1000. They deduced that for a scale-invariant spectrum of magnetic tangles with a field strength amplitude of $3 \times 10^{-9}G$, the induced B-type polarization anisotropy is roughly $0.3-0.4\mu K$ for angular scales 1000 < l < 5000. Similar calculations were also carried out by Mack, Kahniashvili & Kosowsky (2001) in which they derived analytic expressions for the expected polarization anistropies from vector modes.

1.4.3.3 Tensor fluctuations and magnetic fields

Tensor fluctuations in the photon baryon plasma correspond to gravitational waves. The source of these fluctuations is the tensor component of the anisotropic stress in matter. In the absence of any continual source, the behaviour of gravitational waves is such that their amplitude is constant on superhorizon scales whereas on crossing the horizon they experience strong damping. As a result, the observable effect of such fluctuations in the CMBR temperature power spectrum can only manifest on very large scales or at multipoles l < 100 (Pritchard & Kamionkowski 2004)

The tensor perturbations induced by a stochastic magnetic field were first studied by Durrer, Ferreira & Kahniashvili (2000). They computed the temperature angular power spectrum of CMBR for various magnetic spectral indices and compared it with observations thereby constraining the field amplitude for different spectral indices. In particular for a scale-invariant spectrum they derived a limit of the order of a nano-Gauss. The bound weakens as one goes to steeper magnetic power spectrum.

Recently, Lewis (2004) pointed out an important mechanism that of neutrino anisotropic stress compensation for the case of magnetised tensor modes which was neglected in previous calculations. According to this mechanism, the anisotropic stress due to neutrinos after they decouple from the plasma at epochs T > 1 MeV, gets cancelled by the magnetic stress at super-horizon scales. As a result of this the tensor modes evolve in a source free manner and produce CMB signatures exactly in the same manner as in inflationary models. Lewis further calculated numerically the temperature anisotropy and found the contribution to be of the order of few μK for a magnetic field amplitude of 3nG and nearly scale-invariant spectrum with spectral index n = -2.9.

Magnetic field induced tensor fluctuations also produce CMBR polarization anisotropies. As discussed above, the pattern of anisotropies is exactly identical to the signal produced by gravitational waves generated in the standard inflationary models (Lewis 2004). The B-mode signal for instance peaks at the angular scale $l \approx 100$ which corresponds to the size of the horizon at recombination.

1.4.4 Effect on nucleosynthesis

Big-bang nucleosynthesis (BBN) provides the framework for explaining the origin of the observed abundances of the light nuclei (Peebles 1993) According to the predictions of standard BBN, the He abundance is fixed at the epoch at which weak interactions go out of equilibrium. At temperatures T > 1 MeV, corresponding to a time 1 sec after the big-bang, weak interactions were in thermal equilibrium. This fixes the ratio of the neutron to proton number density as $n/p = e^{-Q/T}$, where Q = 1.29 MeV is the neutron proton mass difference. This ratio becomes frozen when the weak reactions go out of equilibrium. The temperature at which the above ratio is frozen is determined by the balance between the weak interaction reaction rate and the Hubble expansion rate. The Hubble rate in turn depends on the energy-density of the Universe.

Magnetic fields, if they existed at the BBN epoch can modify the predicted He abundance because of two primary reasons:(1) The additional energy density of the magnetic field $\rho_B = B^2/4\pi$ can affect the expansion rate of the universe. (2) In the presence of a magnetic field, the electron energy levels become quantized which in turn affects the phase space distribution of the electrons finally affecting the weak interaction rates. It has been shown that the first effect is the most dominant effect (Grasso & Rubinstein 1996). The magnitude of the field strength computed in this manner is $B(T = 10^9K) = 10^{11}G$. Assuming adiabatic evolution of the field, the corresponding value for the field strength today is $B_0 = 7 \times 10^{-7}G$. There are a few things to be kept in mind while interpreting the above limit on the field strength at the current epoch as a limit on the protogalactic magnetic field. This is because the maximum coherence scale of the field at the BBN epoch is set by the horizon size at that epoch which is roughly of the order of 100 pc whereas the comoving size of a protogalaxy is of the order of 1 Mpc. Therefore, if cosmic magnetic fields are tangled on scales smaller than the protogalactic size, the proto-galactic magnetic field has to be evaluated as a proper average of smaller flux tangles.

1.4.5 Magnetic fi elds and structure formation

In the previous section we had summarised briefly the various effects primordial magnetic fields will induce in the pre-recombination plasma and how each of the effects gets manifested at different angular scales in the CMBR power spectrum. Following recombination however, the photons get decoupled from the baryon plasma due to which the effect of photon pressure vanishes. The main forces affecting the baryonic fluid will be the Lorentz force due to the magnetic field and the ordinary Newtonian gravitational force. The post recombination evolution of the plasma will thus be quite different and it would be interesting to know what role the magnetic fields will play in the structure formation process. The earliest attempt in this direction was by Wassermann (1978) which was subsequently extended for more general field configurations by Kim,Olinto & Rosner (1996).

Large scale magnetic fields modify the standard equations of linear density perturbations by adding the effect of the Lorentz force. Due to the Lorentz force, it can be shown that an inhomogenous magnetic field becomes itself a source of density, velocity and gravitational perturbations in the electrically conducting fluid. It can be estimated (Peebles 1980) that the magnetic field needed to produce a density contrast of unity as required to induce structure formation on a scale *L*, is

$$B(L) \sim 10^{-9} G\left(\frac{L}{1 \,\mathrm{Mpc}}\right) \Omega h^2 \tag{1.14}$$

It is thus a curious coincidence that the primordial magnetic field required to explain galactic fields without dynamo mechanism would also play a dynamical role in the process of galaxy formaion. Kim et al. extended the work of Wassermann by determining the power spectrum of density perturbations due to a primordial tangled magnetic field. They showed that at the present time rms magnetic field of $10^{-10}G$ may have produced perturbations on galactic scale which should have entered the non-linear growth stage at redshifts of $z \approx 6$ which is also compatible with observations. Although Kim et al. showed that magnetic field alone cannot be responsible for structure formation, it might be possible that they introduced a bias for formation of galaxies in the CDM scenario.

Recently further effects of primordial magnetic fields on large scale structure were worked out in detail. Sethi (2003) calculated the two point correlation function in redshift space and found that the magnetic field contribution to the redshift space clustering is comparable to the gravity driven clustering for field strength of $5 \times 10^{-8}G$. Using 2dF galaxy survey data they ruled out field strength greater than 3×10^{-8} . Future surveys like SDSS can be used to constrain even smaller field strengths of the order of $10^{-8}G$. Such analyses are important because if a field strength of order 10^{-8} is detected in the data today, then comparison with constraints from CMBR which are at the level of a nano-Gauss, would imply that these fields could have been generated only in the post-recombination era.

A systematic study of the effects of magnetic fields on structure formation was recently undertaken by Battaner et al. (1997) and Florido & Battaner (1997). They concluded that magnetic field with nano-Gauss strengths in the pre-recombination era can produce significant anisotropic density inhomogenities in the metric as well as in the photon baryon plasma. In particular Battaner et al showed that magnetic fields tend to organize themselves and the ambient plasma into filamentary structures. This prediction seems to be confirmed by recent observations of magnetic fields in galaxy clusters (Eilek 1999). Battaner et al suggest that such a behaviour may pervade the entire Universe and be responsible for the cosmic webs observed for instance in the local supercluster (Einasto 1997).

Very recently King & Coles (2005) studied the nature of amplification of magnetic fields when the collapse of gravitational perturbations occurs in an anisotropic manner. In an isotropic collapse, the magnetic field *B* varies with the plasma density ρ according to $B \propto \rho^{2/3}$. Using Zeldovich approximation, King and Coles investigated the range of amolifications which are possible in realistic gravitational collapses assuming Gaussian initial conditions.

1.5 Origin of magnetic fi elds

In earlier sections we have focussed on various aspects related to the effects of large-scale magnetic fields on radiation and matter as well as briefly reviewed observational signatures of such fields. However one of the most important issues and still an unresolved one is related to the origin of such fields even if they are known to exist. This topic has been studied by various people over many years and is still an open problem. We broadly categorize the various mechanisms in three classes and review them below:

1.5.1 Early universe high energy scenarios

Almost all of the field-generation mechanisms proposed in the pre-radiation post inflation era rely on particle physics processes to produce a seed field. Most of the models make use of processes occuring at the different out-of-equibrium epochs in the early universe such as QCD transition or the electro-weak transition. At high temperatures, quarks and gluons exist as free particles in the plasma. These particles get bound into mesons and baryons at temperatures $T \sim 150$ MeV. The nature of the phase-transition is not known. If its a second-order transition, then it will occur adiabatically whereas if its first order then its much more dramatic resulting in processes like shocks, turbulent motions etc which may generate significant magnetic fields through battery and/or dynamo mechanisms. The same holds true for the electroweak phase transition which occurs at $T \sim 100$ GeV

Detailed calculations of magnetic field generation during the electroweak and QCD transitions have been performed by various groups assuming that the transitions are of first order. For instance Quashnock, Loeb & Spergel (1989) demonstrated that a Biermann battery will operate during the QCD phase transition as a result of which magnetic fields will be genrated. They estimated a field strength of 5 G at the time of the phase transition for a coherence length of roughly 100 cm. This translates to a galactic scale field strength at recombination of $B = 6 \times 10^{-32}G$ assuming the scale dependance $B \propto L^{-3/2}$ as suggested by Hogan (1983). Magnetic fields can also arise in second order phase transitions. Vachaspati(1991) proposed a mechanism which generates magnetic fields from the breakdown of electroweak-symmetry and estimated field strengths of 10^{-23} G at decoupling.

It is important to note that all of the early-universe processes of magnetic field generation are causal and hence the field coherence length at the time of generation cannot exceed the size of the horizon at those epochs. This scale is however very small. For instance, the horizon at the QCD transition is 1 pc compared to the comoving proto-galactic scale of 1 Mpc.

1.5.2 Inflationary scenarios

Inflationary models are based on the idea that the scale factor grows exponentially with time $a(t) = \exp(Ht)$ (Liddle & Lyth 1995) in the very early universe ($T \simeq 10^{15}$ GeV). This occurs because the vacuum energy density which sources the geometry is constant. The exponential growth of the scale-factor has its advantages as well as disadvantages as far as generation of magnetic fields is concerned. The most important advantage is that the coherence scale of the field is no more limited by the size of the horizon. This can be seen as follows.

During inflation, the horizon is approximately constant whereas physical length scales evolve as $L_{phy} \propto a$ i.e proportional to the scale-factor which itself evolves exponentially (Guth 1981 ; Linde 1982). Hence every mode which would have been sub-horizon initially crosses outside the horizon during inflation. Once inflation ceases and the radiation era begins, these modes cross back inside the horizon since the horizon now grows as a^2 or as $a^{3/2}$ in the matter-dominated era. Thus any mechanism which generates magnetic field in the inflationary epoch does not face problems as far as causality restrictions are concerned. However, this exponential expansion also results in a very weak field since the expansion dilutes the strength considerably. Hence some additional mechanism must be introduced to overcome this problem.

Turner & Widrow (1988) proposed that the breakdown of conformal invariance of electromagnetism can solve such a problem. In particular, this results in the photon acquiring a mass of the order of 10^{-33} eV which however is undetectable. They were able to obtain $B_0 \sim 5 \times 10^{-10}$ G at scales of about 1 Mpc. Ratra (1992) considered the coupling of the scalar field responsible for inflation (inflaton) and the electromagnetic field, thereby breaking conformal invariance. They obtained field strengths as large as 10^{-9} G at scales of about 5 Mpc which is a very promising result. Dolgov (1993) proposed the breaking of conformal invariance by invoking a field theory mechanism called "phase anomaly".

1.5.3 Field generation in the pre-recombination era

All of the above scenarios of the early-universe generation of magnetic fields involve some kind of speculative physics and hence it would be of great importance if any field-generation mechanism within the standard cosmological framework could be found. The earliest such attempt was made by Harrison (1973). He proposed that vortical motions in the plasma can produce magnetic fields.

The mechanism is based on the idea that the electron and ion vorticities should decrease differently in the expanding Universe prior to recombination. This is because Thomson scattering with photons is more effective for electrons than protons and hence a differential rotation can occur between electrons and protons. In particular the angular velocities behave as $\omega \propto a^{-1}$ for electrons and $\omega \propto a^{-2}$ for the protons. This in turn can produce a magnetic field through the relation $B = -(mc/e)\Omega$ where *m* and *e* are the proton mass and charge and Ω is the vorticity. Under the assumption that a primordial turbulence was present at recombination, the field strength from this mechanism was estimated to be 10^{-8} G on scales of 1Mpc.

Its important to note that in this mechanism the main assumption made is that vorticity exists in the plasma as an initial condition and there is no process which sources it continuously. This is the main problem with this scenario since primordial vorticity decays with time as noted by Rees (1987). Although a very appealing and natural mechanism the question of the origin of such a primordial vorticity remains unanswered.

It is interesting to note however that even within standard cosmological framework, vorticity can be generated if the cosmological perturbation theory is studied in second order or the next to linear order. This is the main subject matter of Chapter 2 of this thesis in which we find that magnetic fields are generated in a natural way in the standard cosmological scenario if we evaluate the electron-photon collision rate by including terms upto second order in perturbation theory. We estimate a field strength to an order of magnitude as 10^{-30} G on scales of 1 Mpc. Similar analysis were also carried out by Matarrese et al. (2005) and Takahashi et al. (2005). Matarrese et.al considered the effect arising from the second order vector metric perturbations only without accounting for the second order collision term whereas we did not consider the metric perturbations. A complete formulation of the problem in second order perturbations theory is still a challenging task and not been attempted yet. The generated field is obviously very small for a primordial scenario but it still may just be sufficient as a seed field for a dynamo mechanism.

1.6 Layout of the thesis

Chapter 2: In this chapter, we work out in detail the effect of a primordial stochastic magnetic field on the matter fluid in the post-recombination era. In particular, we derive the real-space power spectrum of matter induced due to an assumed initial power law spectrum of magnetic field. In addition, we also study the features reflected in the reshift space power spectrum and compare it with the standard features in the no-magnetic field case and suggest possible obervational consequences.

Chapter 3: In this chapter, we study a mechanism of magnetic field generation in the cosmological plasma in the pre-recombination era. Field generation occurs naturally at the second order in perturbation theory and there are no additional ad-hoc assumptions made We make an estimate of the strength of the magnetic field as well as the power spectrum of the field. **Chapter 4:** In this chapter, we work out in detail the various signatures of a primordial

stochastic magnetic field on the CMB. We focus on all the three kinds i.e scalar, vector and tensor modes of CMB fluctuations induced by the field. In particular, we work out in a semianalytic manner the TE polarisation signal and put constraints on large-scale field strengths by comparing it with the WMAP data.

Bibliography

- Adams J., Danielsson U.H., Grasso D., Rubinstein H., 1996, Phys. Lett. B, 388, 253
- Athreya R.M. et al., 1998, A & A, 329, 809
- Bardeen J.M., 1980, Phys. Rev. D, 22, 1882
- Barrow J.D., Ferreira P.G., & Silk J., 1997, Phys. Rev. Lett., 78, 3610
- Battaner E., Florido E., Jimenez-Vicente J., 1997, A & A, 326, 13
- Beck R., 2001, SSRv, 99, 243
- Bennet C.L. et al., 1996, ApJ Lett., 464, L1
- Blasi P., Burles S., Olinto A.v., 1999, ApJ, 514, 79L
- Brandenburg A., Subramanian K., 2005, Physics Reports, 417, 1
- Buczilowski U.R., Beck R., 1991, A&A, 241, 47
- Davis A.C., Lilley M., Torqvist O., 1999, Phys Rev D, 60, 021301
- Dolgov A.D., 1993, Phys. Rev. D., 44, 2499
- Dolgov A.D., 2005, astro-ph/0503447
- Durrer R., Kahniashvili T., & Yates A., 1998, Phys. Rev. D, 58, 123004
- Durrer R., Ferreira P.G., Kahniashvili T., 2000, Phys. Rev. D., 61, 043001
- Eilek J., 1999, Diffuse thermal and relativistic plasma in galaxy clusters. Edited by Hans Bohringer, Luigina Feretti, Peter Schuecker. Garching, Germany : Max-Planck-Institut fur extraterrestrische physik, p71
- Einasto J. et al., 1997, Nature, 385, 139
- Florido E., Battaner E., 1997, A & A, 327, 1

- Giovannini M., 2004, Phys. Rev. D, 70, 123507
- Giovannini M., 2005, IGMP D., 14, 363
- Grasso D., & Rubinstein H., 1996, Phys.Lett. B., 379, 73
- Guth A.H., 1981, Phys. Rev. D., 23, 347
- Han Jin-Lin, & Wielebinski R., 2002, Chin. J. Astron. Astrophys., 2, 293
- Han J.L. et al., 2002, ApJ, 570, L17-L20
- Harrison E.R., 1973, Phys. Rev. Lett., 30, 18
- Hinshaw G. et al., 2003, ApJ Supplement Series, 148, 135
- Hogan C.J., 1983, Phys. Rev. Lett., 51, 1488
- Hoyle F., 1958, *La Structure et l'evolution de l'univers: XI Conseil de Physique, Solvay*, Bruxelles, edited by R.Stoops(Institut International de Physique Solvay, Bruxelles), p. 53.
- Hu W., & Dodelson S., 2002, Ann. Rev. of Astronomy & Astrophysics, 40, 171
- Hu W., & Sugiyama N., 1995, ApJ, 444, 489
- Hu W., White M., New Astronomy, 2, 323
- Hummel E. & Beck R., 1995, A&A, 303, 691
- Kahniashvili T., Kosowsky A., Mack A., Durrer R., 2000, AIP Conference Proceedings, 555, 451
- Kim K.-T., Kronberg P.P., Dewdney P.E., Landecker T.L., 1990, ApJ, 355, 29
- Kim, E.-J., Olinto A.V., Rosner R., 1996, ApJ 468, 28
- Kim, K.-T., Kronberg P.P., Tribble P.C., 1991, ApJ, 379, 80
- King E.J., Coles P., 2005, astro-ph 0508370
- Kolatt T., 1998, ApJ, 495, 564
- Kosowsky A., Loeb A., 1996, ApJ, 469, 1
- Kosowsky A. et al., 2005, Phys Rev D, 71, 043006
- Kronberg P.P., Perry J.P., 1982, ApJ, 263, 518

Kronberg P.P., Perry J.P., Zukowski E.L.H., 1992, ApJ, 387, 528

- Kulsrud R. et al., 1997, Physics Reports, 283, 213
- Lewis A., 2004, Phys. Rev. D., 70, 43011
- Lifshitz E.M., 1946, Zh. Eksp. Teor. Fiz., 16, 587
- Linde A.D., 1982, Phys. Lett., 108B, 389
- Mack A., Kahniashvili T., Kosowsky A., 2002, Phys. Rev. D., 65, 123004
- Manchester R.N., 1972, ApJ, 172, 43
- Manchester R.N., 1974, ApJ, 188, 637
- Matarrese S., Mollerach S., Notari A., Riotto A., 2005, Phys. Rev. D., 71, 043502
- Mukhanov F.V., Feldman H.A., Brandenberger R.H., 1992, Physics Reports, 215, 203
- Parker E.N., 1979, Cosmical magnetic fields: Their origin and their activity, Oxford, Clarendon Press; New York, Oxford University Press
- Peebles P.J.E., 1980, The Large-Scale Structure of the Universe, Princeton Series in Physics, Princeton, NJ, Princeon University Press
- Peebles P. J. E., 1993, Principles of physical cosmology, Princeton Series in Physics, Princeton, NJ, Princeton University Press
- Perlmutter S. et al., 1999, ApJ, 517, 565
- Piddington J.H., 1964, MNRAS, 128, 345
- Piddington J.H., 1972, Cosm. Electrodyn. 3, 60.
- Pritchard J.R., Kamionkowski M., 2004, Annals of Physics, 318, 2
- Quashnock J., Loeb A., Spergel D., 1989, ApJ Lett., 344, L49
- Ratra B., 1992, ApJ Lett., 391, L1
- Rees M.J., 1987, QJRAS, 28, 197
- Riess A.G. et al., 2004, ApJ, 607, 665
- Ruzmaikin A.A., Shukurov A.M., & Sokoloff D.D., 1988, Magnetic Fields of Galaxies, Kluwer Academic Publishers, London

Scannapieco E.S., Ferreira P.G., 1997, Phys. Rev. D., 56, R7493

- Seljak U., Zaldarriaga M., 1996, ApJ, 469, 437
- Sethi S.K., 2003, MNRAS, 342, 962
- Smoot G.F. et al., 2002, ApJ Lett., 396, L1-L5
- Sofue Y.M., Fujimoto M., Kawabata K., 1968, PASJ, 20, 388
- Spergel D.N. et al., 2003, ApJS, 148, 175
- Subramanian K., & Barrow J.D., 1998, Phys. Rev. Lett., 81, 3575
- Subramanian K., Seshadri T.R., Barrow J.D., 2003, MNRAS, 344, L31-L35
- Takahashi K. et al., 2005, Phys. Rev. Lett., 95, 121301
- Thorne K.S., 1967, ApJ., 148, 51
- Turner M.S., Widrow L.M., 1988, Phys. Rev. D., 37, 2743
- Tytler D., 2000, Physics Reports, 333, 409
- Vachaspati T., 1991, Phys. Lett. B., 265, 258
- Vallee J.P., 1990, ApJ, 360, 1
- Wasserman I., 1978, ApJ, 224, 337
- Welter G. L., Perry J. J. & Kronberg P. P., 1984, ApJ, 279, 19
- Vainshtein S.I., & Rosner R., 1991, ApJ, 376, 199
- Widrow L.M., Rev. Mod. Phys., 74, 775
- Zaldarriaga M., & Seljak U., 1997, Phys. Rev. D., 55, 1830