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Scanning Matrix Displays With Wavelets

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***Abstract*—Wavelets-based techniques for scanning the root-mean-squared (RMS) responding passive matrix displays are reviewed. Principle of line-by-line and multi-line implementation of these techniques is discussed. Analysis of the wavelets based techniques that are useful to display a large number of gray shades without flicker is presented in this paper.**

***Index Terms*—Liquid crystal displays (LCDs), matrix addressing, multiplexing, scanning, wavelets.**

I. INTRODUCTION

PASSIVE matrix displays (PMD) are extensively used in portable devices (cellular phones, MP3 players, PDA etc.) because of low cost. The main objective of this paper is to present a generalized analysis of wavelets-based addressing techniques for PMDs. A matrix display is a two-dimensional array of picture elements (pixels). Each pixel can be uniquely selected with a scanning line (row) and a data line (column) because all the pixels in each row are connected together to form row address lines and similarly all the pixels in each column are interconnected to form column address lines. Hence, (N.M) pixels that are located at the intersection of N rows and M columns can be driven with just (N + M) drivers by multiplexing the data through one set of address lines. Lawrence *et al.* proposed a technique to combine the addressing of PMD and decoding of images that are coded with wavelets [1]. A wavelets-based technique [2] was proposed to display gray shades in SID'05. A large number of gray shades can be displayed by using wavelets [3]–[6], and the hardware complexity of the drivers depends on the number of wavelets that are used to select the rows simultaneously in the matrix display [4]. Drivers that are capable of applying 1-of-8 voltages to rows and columns of the matrix displays are adequate to display 128 gray shades. Number of time intervals to complete a cycle can be reduced by a proper choice of wavelets [6]. A wavelet based line-by-line addressing technique to display a large number of gray shades was also proposed [7] recently. Although multiplexing involves orthogonal transform of the image; we have shown that it is not necessary to transform the image and we have demonstrated that the data bits can be used to select data voltages for the column waveform without computing the dot products of the orthogonal transform [6], [7]. Principle of wavelets based multiplexing is discussed first.

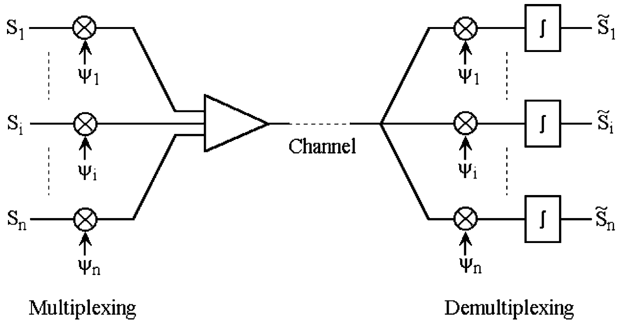

Fig. 1. Schematic diagram showing the multiplexing and de-multiplexing of signals.

II. WAVELETS FOR MATRIX LCD

Each column in a matrix display is equivalent to a communication system as shown in Fig. 1. The process of multiplexing and de-multiplexing in RMS responding matrix displays is shown schematically Fig. 2. The intrinsic nonlinear electro-optic response of pixels in liquid crystal displays is used to decode the state of pixels in that column (equivalent to N receivers). Data waveforms carry multiplexed information and the scanning waveforms are used to decode the multiplexed signal. Amplitudes of scanning and data waveforms are chosen to achieve the maximum contrast in the display.

A. Multiplexing

Multiplexing is useful to reduce the number of drivers and interconnections in matrix displays. It is similar to multiplexing signals (S_i) with orthogonal carriers in communication systems shown in Fig. 1. Data corresponding to state of a pixel $(d_{i,j})$ in an address line is multiplied with a wavelet $\Psi_i(t)$ (carrier) and the multiplexed signal is obtained by adding all such products (see Fig. 2) as shown in the following equation:

$$Mux_j = \sum_{i=1}^{N} d_{i,j} \cdot \Psi_i(t) \tag{1}$$

The orthogonal wavelets chosen to avoid crosstalk satisfy

$$\int_{\text{period}} \Psi_i(t) \cdot \Psi_k(t) dt = \begin{cases} 0 & \forall i \neq k \\ e_i & \forall i = k \end{cases} \tag{2}$$

wherein, e_i is proportional to energy of the wavelet 'i'.

B. De-Multiplexing

Scanning waveforms that are applied to the address lines (rows) are equivalent to replica of carriers for de-multiplexing in communication systems. Scanning waveforms $(\Psi_i(t) \cdot V_r)$ are proportional to the wavelets $\Psi_i(t)$ and the amplitude is controlled with the amplification factor V_r. A waveform that is proportional to the multiplexed signal is applied to the column. $V_c \sum_{k=1}^{N} d_{k,j} \cdot \Psi_k(t)$. Here again, V_c is the gain factor that

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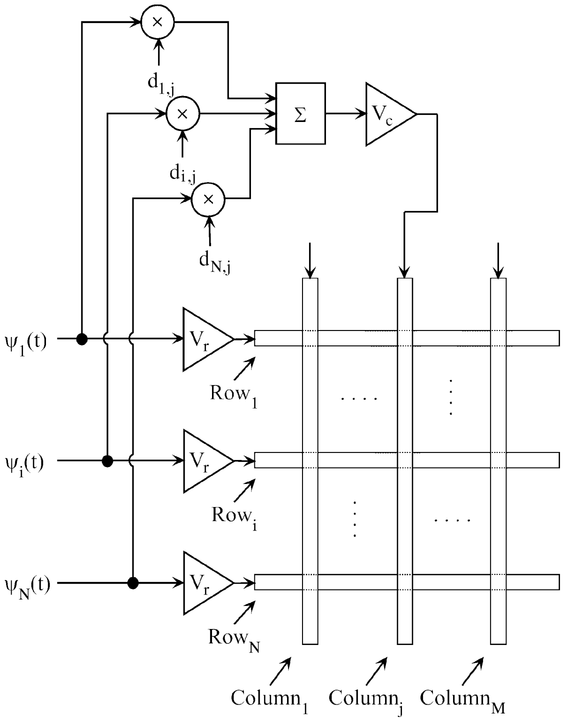

Fig. 2. Each column in the matrix display is equivalent to a communication system. Information on state of the pixels is multiplexed through the column waveforms and the intrinsic non-linear characteristic of the passive matrix LCD is used for decoding the multiplexed signal by applying waveforms that are proportional to the wavelets to the rows of the matrix display.

is used to control the amplitude of column (data) waveforms. The waveform across a pixel at the intersection of "row-i" and "column-j" is given in the following expression.

$$\Psi_i(t)\cdot V_r - V_c\sum_{k=1}^{N} d_{k,j}\cdot\Psi_k(t) \tag{3}$$

Liquid crystal displays respond to the RMS voltage of the electric field and the expression of RMS voltage across a pixel is given in

$$V_{i,j}(\mathrm{RMS})=\sqrt{\frac{1}{T}\int_0^T\left(V_r\cdot\Psi_i(t)-V_c\sum_{k=1}^{N}d_{k,j}\cdot\Psi_k(t)\right)^2 dt}. \tag{4}$$

Energy delivered to the pixel during T is proportional to the following expression:

$$\int_0^T\left(V_r\cdot\Psi_i(t)-V_c\sum_{k=1}^{N}d_{k,j}\cdot\Psi_k(t)\right)^2\cdot dt=E_{i,j} \tag{5}$$

and

$$\begin{aligned}E_{i,j}=V_r^2\int_0^T\Psi_i^2(t)dt-2\cdot d_{k,j}\cdot V_r\cdot V_c\sum_{k=1}^{N}\int_0^T\Psi_i(t)\cdot\Psi_k(t)dt\\+V_c^2\int_0^T\left(\sum_{k=1}^{N}d_{k,j}\cdot\Psi_k(t)\right)^2 dt.\end{aligned} \tag{6}$$

The order of summation and integration is changed second term in (6). This term is useful for de-multiplexing as shown in Figs. 1 and 2. The first and last terms are proportional to energy of the scanning (row) and data (column) waveforms respectively. They are not necessary for de-multiplexing; but it is not possible to suppress them because the middle term will also collapse even if one of these two terms is forced to zero. De-multiplexing can still be achieved if the first and last terms are constants and are independent of the state of the pixels in the column. Equation (6) can be simplified, as shown in (7) because the orthogonal wavelets satisfy the conditions in (2)

$$E_{i,j}=e_i\left(V_r^2-2\cdot d_{i,j}\cdot V_r\cdot V_c\right)+V_c^2\sum_{k=1}^{N}e_k d_{k,j}^2. \tag{7}$$

The first term can be a constant if amplitude of the scanning waveforms i.e. V_r is a constant. The last term will be a constant if data assigned to the state of pixels has the same amplitude and sign of the data can be either positive or negative.. The RMS voltage across the pixel is

$$V_{i,j}(\mathrm{RMS})=\sqrt{(E_{i,j})/(T)}. \tag{8}$$

In case the data assigned to ON and OFF states are '-1' and '$+1$' respectively (it is similar to assigning a voltage that is in phase with select pulse for OFF pixels and out of phase with the select pulse for ON pixels in the conventional line-by-line addressing technique [8]); then the RMS voltage across an ON and OFF pixels are as shown, respectively, in

$$V_{\mathrm{ON}}(\mathrm{RMS})=\sqrt{e_i\cdot\left(V_r^2+2V_rV_c+NV_c^2\right)/(T)} \tag{9}$$

$$V_{\mathrm{OFF}}(\mathrm{RMS})=\sqrt{e_i\cdot\left(V_r^2-2V_rV_c+NV_c^2\right)/(T)}. \tag{10}$$

Typical electro-optic response of twisted nematic (TN) LCD is shown in Fig. 3. Selection ratio; defined as the ratio of RMS voltage across ON pixel to that of the OFF pixel given by

$$\mathrm{SR}=\sqrt{\left(V_r^2+2V_rV_c+NV_c^2\right)/\left(V_r^2-2V_rV_c+NV_c^2\right)}. \tag{11}$$

It is a maximum when $V_r=\sqrt{N}V_c$ and the maximum selection ratio is given in

$$\mathrm{SR}=\sqrt{(\sqrt{N}+1)/(\sqrt{N}-1)}\quad\forall V_r=\sqrt{N}\cdot V_c. \tag{12}$$

This is the maximum selection ratio that can be achieved by any addressing technique for driving passive matrix LCD and a good contrast in the display is achieved by ensuring that the either $\mathrm{V_{OFF}}$ is near threshold voltage or $\mathrm{V_{ON}}$ is near the saturation of the electro-optic characteristics of the LCD [8]. Addressing waveforms when Haar wavelets are used to drive the pixels to either ON or OFF state (bilevel) are shown in Fig. 4. It is not advantageous to use wavelets to display bilevel images and that too by selecting all the rows. It has been presented here to introduce the technique. However, wavelets are useful to display large number of gray shades with less number of time intervals as compared to the conventional pulse width modulation [9] and frame rate control.

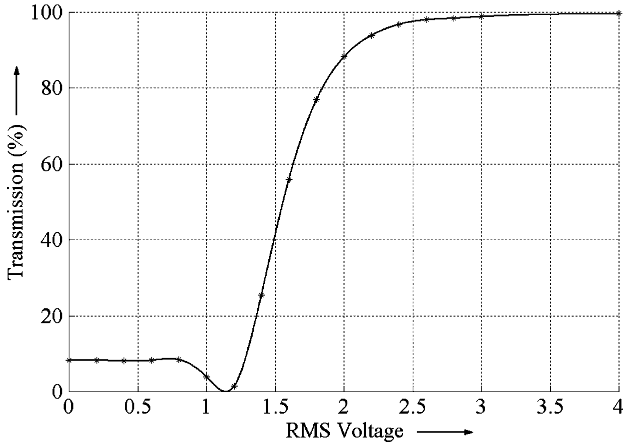

Fig. 3. Light transmission through the pixel depends on the RMS voltage across it in a liquid crystal display. Threshold voltage is the voltage at which the light transmission changes from unexcited state to 10% of the maximum change in transmission. Similarly the saturation voltage is the voltage at which the light transmission is 90% of the maximum change in light transmission through the pixel.

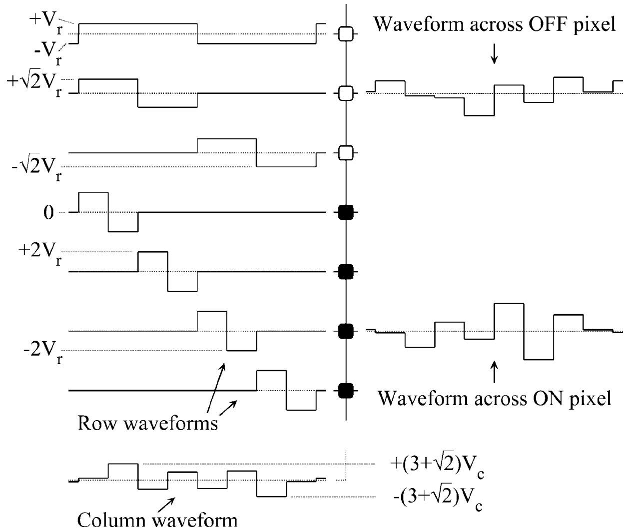

Fig. 4. Typical waveforms when the address lines ($N = 7$) are selected with the Haar wavelets.

III. GRAY SHADES WITH WAVELETS

Light transmission through pixels in RMS responding LCDs depends on energy delivered to pixels during a cycle. Energy that is proportional to weight of each bit of the gray shade can be delivered sequentially by varying both the amplitude and the duration of the wavelets. This is in contrast to pulse width modulation [9] and frame rate control wherein the amplitude of data voltage is constant and polarity of the data voltage (with reference to select pulse) is varied according to the gray during several time intervals in a cycle. On the other hand, just the amplitude of the data voltages is varied depending on the gray shade in amplitude modulation [10]. A set of wavelets is chosen such that each wavelet Ψ_i satisfy the conditions in (13) and (14); where in 2^i is the weight of the "bit-i"

$$E_i = \int_{\text{period}} \Psi_i^2 dt = 2^i \tag{13}$$

$$(E_{i+1})/(E_i) = \int_{\text{period of (i+1)}} \Psi_{i+1}^2 dt \Big/ \int_{\text{period of i}} \Psi_i^2 dt = 2. \tag{14}$$

Energy is proportional to $v^2 t$; hence either the duration of a wavelet can be doubled or the amplitude of a wavelet may be increased by a factor $\sqrt{2}$ to double the energy of a wavelet. Each address line is selected sequentially with all the wavelets to refresh the display. Data waveforms that are applied to the columns will have the same shape as the wavelet that selects an address line at a given instant of time. However, the polarity of the data waveform will be same as that of the select wavelet when the corresponding gray shade bit is "logic-0" and the polarity of the data waveform will be out of phase (opposite sign) when the data bit is "logic-1". Period of an address cycle will be N times the sum of periods of all the wavelets, as shown in (15), because each row will be selected with all the wavelets once to complete a cycle.

$$\text{Period}_{\text{address cycle}} = N \cdot \sum_{all\ i} \text{Period of wavelet}_i. \tag{15}$$

For example, three wavelets that are derived from the simple Haar wavelets and correspond to most significant bit (MSB) to least significant bit (LSB) of eight gray shades are shown in the following expressions. These wavelets satisfy the conditions in (13) and (14).

$$\langle +V_r,\ +V_r,\ -V_r,\ -V_r \rangle \tag{16}$$

$$\langle +V_r,\ -V_r,\ 0,\ 0 \rangle \tag{17}$$

$$\langle 0,\quad 0,\quad V_r/\sqrt{2},\quad V_r/\sqrt{2} \rangle. \tag{18}$$

Let one of the N address lines be selected with the following waveform:

$$\{+V_r, +V_r, -V_r, -V_r\}. \tag{19}$$

It corresponds to the first wavelet $\langle +1, +1, -1, -1 \rangle$ with it energy equal to 4 for the MSB. Data waveform is same as that of the row select waveform except for its amplitude when the MSB of the pixel in the selected row and a column is logic-0 as shown in the following expression:

$$\{+V_c, +V_c, -V_c, -V_c\}. \tag{20}$$

The data waveform that is applied to the column will be out of phase with the select waveform when MSB is logic-1 as shown in

$$\{-V_c, -V_c, +V_c, +V_c\}. \tag{21}$$

The select and data waveforms for all the three wavelets are summarized in Table I. It is not necessary that the wavelets have to be orthogonal if the rows are selected sequentially with one wavelet at a time. Period of the addressing waveforms will be 12N time intervals if the three waveforms (shown in Table I) are used to select each address line sequentially one after the other. Period of the addressing waveforms can be reduced with compact wavelets with less number discrete elements. It can also be reduced to 8N time intervals by suppressing the intervals when the amplitude is zero. However, it is preferable to reduce the

TABLE I
SELECT AND DATA WAVEFORMS FOR 8-GRAY SHADES

Select waveform	Data waveform for logic-0	Data waveform for logic-1
$\{+V_r,+V_r,-V_r,-V_r\}$	$\{+V_c,+V_c,-V_c,-V_c\}$	$\{-V_c,-V_c,+V_c,+V_c\}$
$\{+V_r,-V_r,0,0\}$	$\{+V_c,-V_c,0,0\}$	$\{-V_c,+V_c,0,0\}$
$\left\{0,0,\frac{+V_r}{\sqrt{2}},\frac{-V_r}{\sqrt{2}}\right\}$	$\left\{0,0,\frac{+V_c}{\sqrt{2}},\frac{-V_c}{\sqrt{2}}\right\}$	$\left\{0,0,\frac{-V_c}{\sqrt{2}},\frac{+V_c}{\sqrt{2}}\right\}$

TABLE II
DATA WAVEFORMS FOR 8-GRAY SHADES

Gray shade MSB to LSB (3-bits)	Data waveform
000	$\left\{2V_c,0,\frac{-(\sqrt{2}-1).V_c}{\sqrt{2}},\frac{-(\sqrt{2}+1).V_c}{\sqrt{2}}\right\}$
001	$\left\{2V_c,0,\frac{-(\sqrt{2}+1).V_c}{\sqrt{2}},\frac{-(\sqrt{2}-1).V_c}{\sqrt{2}}\right\}$
010	$\left\{0,\ 2V_c,\frac{-(\sqrt{2}-1).V_c}{\sqrt{2}},\frac{-(\sqrt{2}+1).V_c}{\sqrt{2}}\right\}$
011	$\left\{0,\ 2V_c,\frac{-(\sqrt{2}+1).V_c}{\sqrt{2}},\frac{-(\sqrt{2}-1).V_c}{\sqrt{2}}\right\}$
100	$\left\{0,\ -2V_c,\frac{+(\sqrt{2}+1).V_c}{\sqrt{2}},\frac{+(\sqrt{2}-1).V_c}{\sqrt{2}}\right\}$
101	$\left\{0,\ -2V_c,\frac{+(\sqrt{2}-1).V_c}{\sqrt{2}},\frac{+(\sqrt{2}+1).V_c}{\sqrt{2}}\right\}$
110	$\left\{-2V_c,0,\frac{+(\sqrt{2}+1).V_c}{\sqrt{2}},\frac{+(\sqrt{2}-1).V_c}{\sqrt{2}}\right\}$
111	$\left\{-2V_c,0,\frac{+(\sqrt{2}-1).V_c}{\sqrt{2}},\frac{+(\sqrt{2}+1).V_c}{\sqrt{2}}\right\}$

number of time intervals further so that a large number of gray shades can be displayed without flicker even when the number of address lines is large. Number of time intervals in a cycle can be reduced with orthogonal wavelets because energy corresponding to several or all the bits of gray shade can be delivered simultaneously by multiplexing as against delivering the energy corresponding to each bit sequentially one at a time as in the previous scheme. Such a reduction in number of time intervals can be achieved with orthogonal wavelets both in line-by-line addressing and multi-line addressing as described next.

A. Line-by-Line Addressing With Wavelets

Let the gray shade of a pixel be represented with g-bits. Then g-orthogonal wavelets are employed to multiplex information corresponding to bits of the gray shade. Data waveforms for displaying the eight gray shades are proportional to the multiplexed data waveforms shown in Table II. They are obtained by adding data waveform corresponding to each bit. Column waveforms can be obtained by multiplying each wavelet with data assigned to the bit and adding them. Hardware implementation is simple because the data assigned to the bits is just $+1$ or -1 and considering that the number of wavelets is finite even for a large number of gray shades; the column drivers select one of a few voltages that are generated with a voltage level generator (resistor network) using the gray shade bits and it is not necessary to multiply or add wavelets as described in [7].

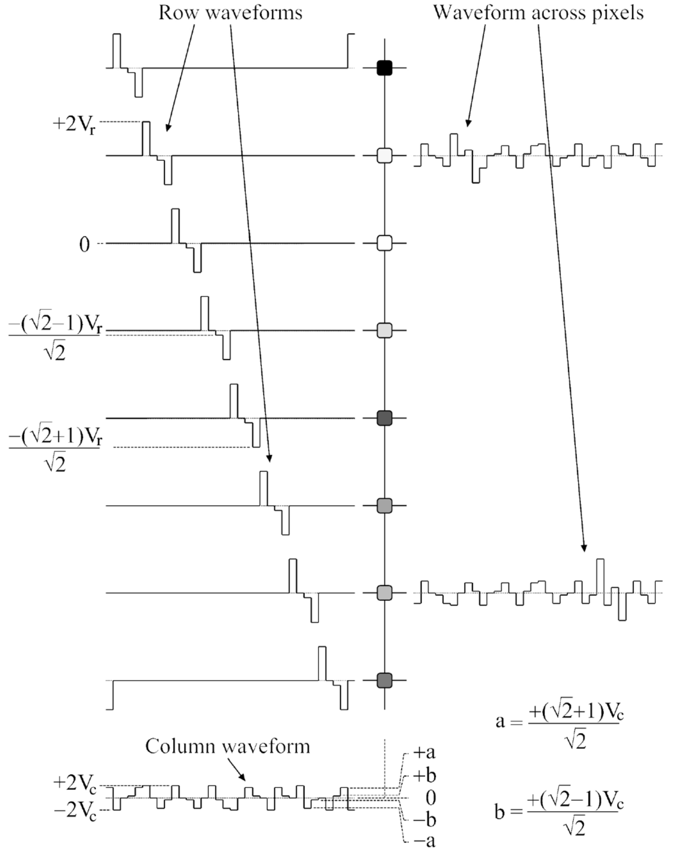

Fig. 5. Typical waveforms of wavelet based line-by-line addressing technique. The four pulses in the scanning waveforms are clustered in this figure; but it could be distributed, as shown in Fig. 6.

The multiplexed data is de-multiplexed by selecting the row with a waveform that is proportional to the sum of all the wavelets. The rest of the $(N-1)$ non-selected address lines (rows) are connected to the ground. Select waveform to de-multiplex and display eight gray shades is shown in

$$\left\{+2V_r, 0, -(\sqrt{2}-1)V_r/\sqrt{2}, -(\sqrt{2}+1)V_r/\sqrt{2}\right\}. \quad (22)$$

A cycle is complete when all the rows are selected once. Number of time intervals in a cycle is reduced to 4NT by multiplexing all the gray scale bits simultaneously and the number of time intervals depends on the orthogonal wavelets and the number of gray shade bits. Typical waveforms of the line-by-line addressing are shown in Fig. 5. Select waveform can either be clustered as shown here or it could be distributed as select pulses, as shown in Fig. 6. It is useful to suppress frame response in displays with fast response times. RMS voltages across the pixels will be the same but the frame frequency to avoid flicker in the display and power dissipation in the driver circuit will be different for the distributed and clustered waveforms. Supply voltage of the driver will be high for line by

TABLE III
COMPARISON OF ADDRESSING TECHNIQUES

Parameter for comparison	Frame and Pulse width modulation and (line-by-line)	Amplitude modulation (line-by-line)	Successive approximation (line-by-line)	Successive approximation (multi-line)	Wavelets based technique (line-by-line)	Wavelets based techniques (multi-line)
Analog multiplexers (Column Drivers)	M numbers of 2:1 multiplexers	M numbers of $2(2^g-1)$:1 multiplexers	M numbers of 2:1 multiplexers	M numbers of (s+1):1 depending on s,	M numbers of 2^s:1 multiplexers	M numbers of 2^s:1 multiplexers
Shift register and latches (Column Drivers)	(1-bit).M	(2g-bits) .M	(1-bit).M	(2 to 4-bits) .M For 4 to 256 gray shades	(3-bits or less).M *	(3-bits or less).M*
Analog multiplexers per stage of row driver:	N numbers of 2:1 multiplexers	N numbers of 2:1 multiplexers	N numbers of 2:1 multiplexers	N numbers of 3:1 multiplexers	N numbers of 2:1 multiplexers	N numbers of (2w+1):1 multiplexers
Shift register and latches (row drivers)	(1-bit).N	(1-bit).N	(1-bit).N	(2-bits).N	(1-bit).N	(2 to 3-bits).N

Note: s is the number of rows selected simultaneously, $*$: depends on no. of nonzero element in each select vector and w-number of active wavelets at a time..

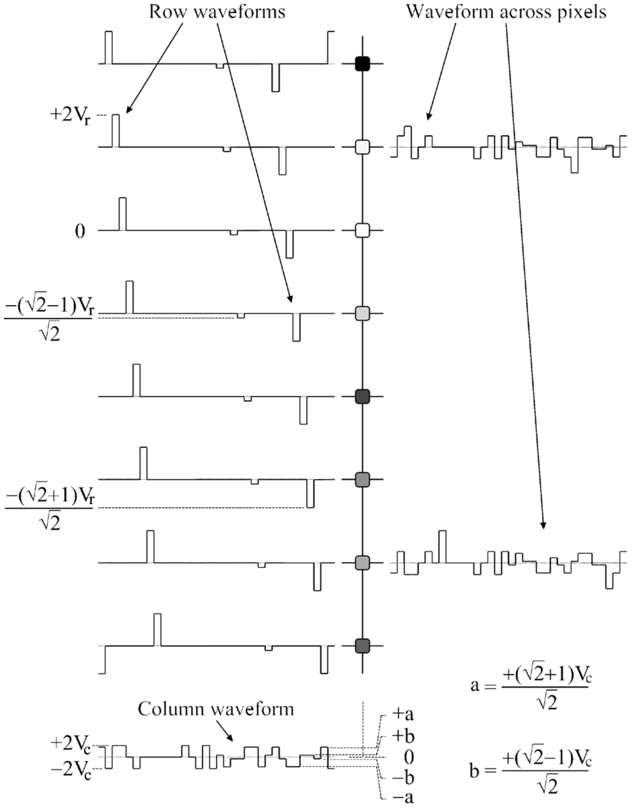

Fig. 6. Typical waveforms of wavelet based line-by-line addressing with select pulses distributed. Frame response phenomenon in the fast responding liquid crystal displays can be suppressed with distributed waveforms.

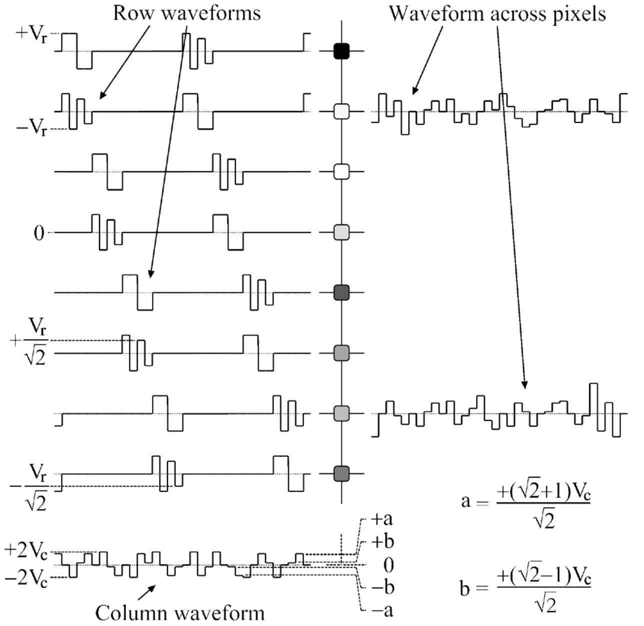

Fig. 7. Waveforms of wavelets base addressing technique when a few address lines are selected simultaneously with orthogonal wavelets. Four select pulses are clustered in this example.

line addressing because the peak-to-peak swing in the select waveform is large when several wavelets are added to obtain the select waveform. Techniques to reduce the supply voltage of wavelets based line-by-line addressing can be found in [7]. Multi-line addressing will have all the advantages of wavelet addressing with a lower supply voltage of the driver circuit as compared to that of line-by-line addressing with wavelets.

B. Multi-Line Addressing

Several address lines are selected simultaneously in multi-line addressing techniques. For example, one of the address lines can be selected with the waveform corresponding to the first wavelet i.e. $\{+V_r, +V_r, -V_r, -V_r\}$ and another address line with $\langle +V_r, -V_r, +V_r/\sqrt{2}, -V_r/\sqrt{2}, \rangle$. It is obtained by concatenating two wavelets of least significant bits

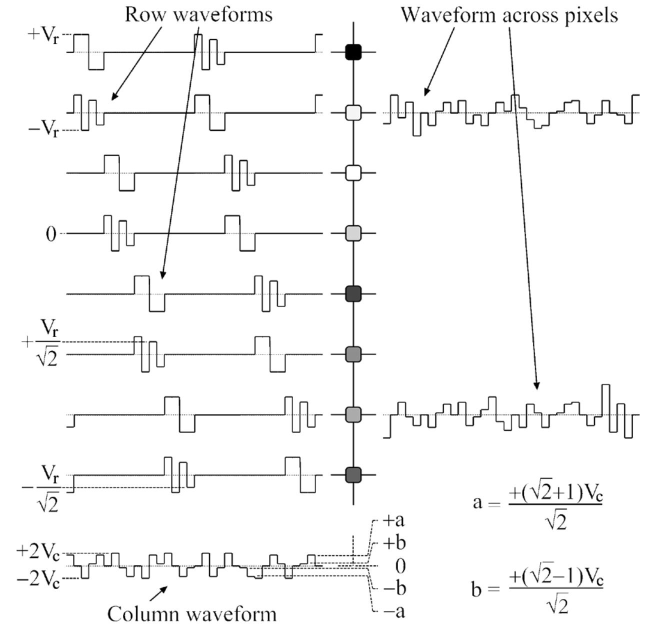

Fig. 8. Typical waveforms of wavelet addressing based on selecting multiple address lines. Orthogonal wavelets are used in when multiple address lines are selected to reduce supply voltage of the driver circuit. Select pulses are clustered in this illustration but it could be distributed in the address cycle.

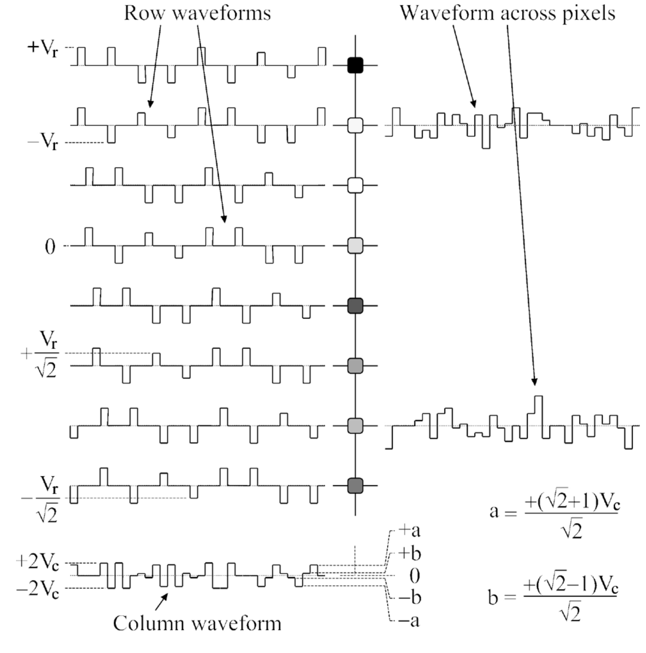

Fig. 9. Waveforms of wavelets based addressing technique when the select pulses are distributed in the address cycle. Frame response phenomenon can be suppressed in displays with fast response and the refresh frequency to avoid flicker will be the lowest for the distributed pulses.

by suppressing zeros in both wavelets. The select waveforms are orthogonal to each other and therefore images will be crosstalk free even if two rows are selected simultaneously. MSB is the data bit of first row. The middle bit is the data bit for the second row during the first two time intervals and it is the LSB for the 3rd and 4th time intervals. Typical waveforms of multi-line are shown in Figs. 7– 9. Data waveforms are the same as that in Table II. However the data bits used to generate the column waveforms are not the same line-by-line addressing and multi-line addressing. Select pulses can be distributed either as one, two, or multiple pulses in an address cycle; RMS voltages will be same but power consumption and the minimum refresh frequency to avoid flicker will depend on the addressing sequence that decides the distribution of select pulses. A cycle is completed when all rows are selected with both the select waveforms. Number of time intervals will be the same as that of line-by-line addressing. A few scanning lines (s-address lines) are grouped and they are selected simultaneous; the matrix display with N address lines will have (N/s) nonintersecting sets each consisting of s rows. In multi-line addressing, one set is selected with waveforms that are derived from orthogonal wavelets; while the rest of the address lines $(N - s)$ in the non-selected sets are grounded.

IV. ANALYSIS

Let the data to be displayed in a pixel is represented by g-bits and let $\sum_{i=0}^{g-1} d_{inm} 2^i$ be one of the 2^g shades gray shades. Here d_{inm} is the data bit 'i' of a pixel located in the row 'n' of the selected subgroup and column 'm' of the matrix display. Each bit is assigned a value of '$+1$' or '-1' corresponding to 'logic-0' or 'logic-1' of the data bit. The column signal is the sum of 'g' products obtained by multiplying the bit 'i' of the data in the selected row with the corresponding wavelet $\Psi_i(t)$.

$$\mathrm{S}_m = \sum_{i=0}^{g-1} \Psi_i(t) d_{inm}. \tag{23}$$

Column signal can also be obtained as the dot product of select pattern with the data vector of the column that is formed by using just one bit of each pixel of that column [2]–[7].

The bit used for each pixel corresponds to energy of the wavelet to be used for selecting that row. The multiplication of data d_{inm} with the wavelet is similar to multiplying the signal with the carrier in communication. Here it reduces to changing the sign of the wavelet since data corresponding to each bit of gray shade is either $+1$ or -1. Column voltage that is applied to the column 'm' is equal to

$$S_m V_c = \left(\sum_{i=0}^{g-1} \Psi_i(t) d_{inm} \right) V_c. \tag{24}$$

The energy across a pixel in a row that is selected with a waveform $\Psi_i(t) V_r$ is proportional to

$$E_i = \int_0^T \left(\Psi_i(t) V_r - V_c \sum_{i=0}^{g-1} \Psi_i(t) d_{inm} \right)^2 dt. \tag{25}$$

Most of the terms in the above equation vanish because the wavelets are orthogonal and hence integral of the product of two different wavelets is zero.

$$E_i = \int_0^T \Psi_i^2(t) V_r^2 dt - 2 d_{inm} \sum_{i=0}^{g-1} \int_0^T \Psi_i^2(t) V_r V_c dt + \int_0^T \left(V_c \sum_{i=0}^{g-1} \Psi_i(t) \cdot d_{i,n,m} \right)^2. \tag{26}$$

On further simplification we get

$$E_i = 2^i\left(V_r^2 - 2d_{inm}V_rV_c\right) + V_c^2\sum_{i=0}^{g-1} e_i. \tag{27}$$

The middle term on right-hand side of the equation in (26) depends on the data bit d_i and, hence, an term that is proportional to the weight of the binary bit is added or subtracted to the energy delivered to the pixel depending on the sign of the corresponding data bit of the pixel. Energy corresponding to the gray shade can be delivered to the pixel by selecting the row with all the wavelets $\Psi_0(t)$ to $\Psi_{g-1}(t)$ in a cyclic manner and it is important to note that few more rows are simultaneously selected with other wavelets and the images will be free of cross talk because the wavelets are orthogonal.

$$E = \sum_{i=0}^{g-1} E_i = \sum_{i=0}^{g-1}\left[2^i\left(V_r^2 - d_{inm}V_rV_c\right) + V_c^2(2^g - 1)\right] \tag{28}$$

$$E = (2^g - 1)\left[V_r^2 + gV_c^2\right] - \sum_{i=0}^{g-1} 2^i d_{inm} V_r V_c. \tag{29}$$

The first term in the above equation is constant and the second term corresponds to the gray shade data. Energy that is proportional to 2^i, the weight of the bit i is added or subtracted depending on whether d_{inm} is "-1' or '$+1$," respectively. The desired gray shade can be displayed by using g wavelets. In case of multi-line addressing one set of rows are selected while the other non-selected rows are grounded.

Energy across the pixel when rows in other subgroups are selected is obtained by substituting $V_r = 0$ in (25). This term corresponds to the cross talk in matrix displays that contributes to lower contrast in passive matrix displays when the number of rows in the matrix display (N) is increased.

$$\int_0^T \left(\sum_0^{g-1} \Psi_i(t) d_{inm} V_c\right)^2 dt = (2^g - 1)V_c^2. \tag{30}$$

A cycle is complete when all the rows the matrix display are selected with all the wavelets once. This ensures that energy delivered to a pixel is g, which corresponds to all the gray shade data bits. The energy delivered to a pixel during a cycle is obtained by summing the energies delivered by all the data bits. The RMS voltage across the pixel is given in

$$V_{\text{pixel}}(rms) = \sqrt{(E_{\text{select}} + E_{\text{non-select}})/(gN)} \tag{31}$$

The RMS voltage across the ON and OFF pixels is given in the following equations and RMS voltages of pixels that driven to gray shades are within the RMS voltages of OFF and ON pixels:

$$V_{\text{ON}}(rms) = \sqrt{\left(2(2^g - 1)(N + \sqrt{N})\right)/(gN)} \tag{32}$$

$$V_{\text{OFF}}(rms) = \sqrt{\left(2(2^g - 1)(N - \sqrt{N})\right)/(gN)}. \tag{33}$$

The ratio of RMS voltages across ON pixels to that across OFF pixels is a measure of discrimination achieved between the two extreme states of the pixel. This ratio is referred to as the selection ratio (SR) and is given by

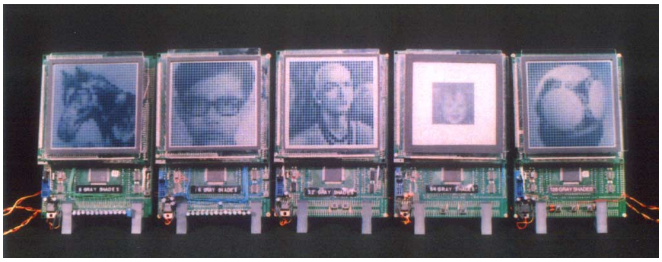

Fig. 10. Photographs of displays that are scanned with waveforms derived from wavelets. From left to right: prototypes that are capable of 8, 16, 32, 64, and 128 gray shades. All of them are based on multi-line addressing with wavelets except for the fifth one that uses wavelets for line-by-line addressing.

$$\text{SR} = \sqrt{(\sqrt{N} + 1)/(\sqrt{N} - 1)}. \tag{34}$$

Hence, this technique has the selection ratio that is the maximum achievable by any passive matrix addressing technique [8], [9].

V. CONCLUSION

Several variants of the wavelets based addressing technique were demonstrated in references [2]–[7]. Photograph of the prototypes that are capable of displaying 8–128 gray shades is shown in Fig. 10. The first [2] and second [5] prototype use Haar wavelets to display 8 and 16 gray shades by selecting three rows at a time [2]. The third display is driven by modified slant wavelets to demonstrate that complexity of the controller does not depend on the complexity of the wavelets and just the data bits are adequate (not necessary to take the orthogonal transform of data) and the supply voltage is lower than that of multi-line successive approximation technique [6]. The fourth is based on multi-line addressing [3] while the fifth display is driven by line-by-line addressing [7] to demonstrate the capability and advantages of wavelets for a large number of gray shades. Hardware complexity of the driver circuit is a design parameter and it depends on the number of nonzero elements in the orthogonal matrix. For example; drivers that are capable of applying 1 of 8 voltages can be used as row drivers and column drivers along with some analog multiplexers for both line-by-line [7] and multi-line [3] implementation of the wavelets based addressing technique. Comparison of these techniques with other techniques for displaying a large number of gray shades viz. amplitude modulation [10], pulse height modulation [11] and successive approximation (multi-line) [12] as well as successive approximation line-by-line [13] can be found in references [3]–[7]. Detailed analysis of supply voltage and its comparison with other techniques, response time measurements and hardware implementation can be found in these references. Successive approximation technique [13] can be viewed as a specific case of wavelets based technique wherein just the amplitude (but not the period) of a pulse is varied to control the energy delivered to the pixel as compared to varying both the amplitude and period in wavelets based techniques. Wavelets have the unique advantage of allowing modulation of both amplitude and period depending on the data bit. Selecting wavelets with larger period for most significant bits and smaller period for least significant bits will lead to lower supply voltage. Orthogonal wavelets allow multi-line addressing, reduce supply voltage, reduce to number of time intervals to complete a cycle to avoid flicker and reduce hardware complexity of the driver circuit.

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