General Relativistic Rotation Effects in Pulsar Radiation

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The purpose of these notes is to estimate the magnitude of two effects which may be observable in pulsar radiation. These effects have to do with rotation in general relativity (GR). They are extremely minute effects and were earlier worked out when the millisecond pulsar was discovered (1982). Even with this rapidly spinning star ($P = 1.6$ millisec.), the magnitudes were considered dismally small and unobservable. In the years since then, a double pulsar has been discovered. This orbit of this system is almost edge on and shows eclipses. Just before and after an eclipse we could expect that radiation from one star passes close to the surface of the other star. Since pulsar radiation is pulsed and polarised an imprint of this passage may remain in the arrival time and polarisation of the received radiation.

These notes are motivated by a discussion with Avinash Deshpande, who has been observing the Double Pulsar from the Ooty Radio Telescope.

Both the effects of interest have are well known in GR. The question is: Can we see them? The effects are:

1. **Rotational time delay**: Rays passing on either side of the pulsar rotation axis suffer different time delays. By studying the arrival time of the pulses from the second pulsar we may be able to detect this effect.

2. **Gravitational Faraday effect**: Radiation traversing a direction along the rotation axis will have its plane of polarisation rotated. This rotation will be independent of frequency (unlike the regular Faraday effect) and could be monitored over the duration of the eclipse.

We first compute the order of magnitude of these effects and then discuss the possibility of observing them. At the present stage, detailed modelling is not appropriate, since the picture will be messy and complicated and obscure the basic idea. A simple order of magnitude estimate is more useful.

Consider two pulsars with masses M_1, M_2, periods P_1, P_2 going around each other. We are interested in the effect of the (Fig. 1) gravitational field of pulsar 1 on the radiation received from 2. ($P_1 = 20$ millisec. $P_2 = 127$ millisec.).

The rotational time delay

Let the spacetime around 1 be described by the following metric ($M = M_1$)

$$\begin{aligned} ds^2 &= -\left(1 - \frac{2M}{r}\right) dt^2 - \frac{4Ma\sin^2\theta}{r}\, dtd\varphi + \left(\frac{1-2M}{r}\right)^{-1} dr^2 \\ &+ r^2(d\theta^2 + \sin^2\theta d\varphi^2) \end{aligned}$$

In the above equation, $a = J/M$ is the angular momentum per unit mass of the star, which has dimensions of length ($G, c = 1$). This is the slow rotation exterior Kerr solution. This slow rotation approximation is justified because the surface velocity ωR of the pulsar is small in units of the speed of light.

$$\omega R << c.$$

Using standard numbers ($R = 10$ km)

$$\frac{\omega R}{c} = \frac{2\pi}{20\times 10^{-3}}\ \frac{10 \text{ km}}{3\times 10^5 \text{ km/sec}} = \frac{2\pi}{6}\times\frac{10}{10^3} = 10^{-2}$$

which is small (1%) and so we are justified in assuming slow rotation.

Let $A_i = \frac{g_{oi}}{g_{oo}}$ $\qquad A_\varphi = \frac{-2aM\sin^2\theta}{(r-2M)}$ $\qquad A_\theta, A_r = 0$

The time delay of interest is given by $\oint A_i\, dx^i$ along the closed curve shown in Fig. 2. Roughly, this measures the gravitational magnetic flux enclosed by the two curves shown. Since the gravimagnetic field falls of as $1/r^3$, most of the contribution comes from near the star and we can replace the integral approximately, by one encircling the equator of the star $\theta = 0$

$$\begin{aligned}
-2aM \oint d\varphi \; \frac{\sin^2\theta}{(r-2M)} \quad &= \quad -2aM \int \frac{d\varphi}{(r - rM)} \\
r \quad &= \quad R = 10 \text{ km} \\
2M \quad &= \quad 2GM/c^2 = 1.4 \times 3 \text{ km} = 4.5 \text{ km}
\end{aligned}$$

time delay

$$\begin{aligned}
T \quad &= \quad \frac{2aM}{5 \text{ km}} \; 2\pi \quad = \quad \frac{4\pi aM}{5 \text{ km}} \quad = \quad \frac{4\pi J}{5 \text{ km}} \\
J \quad &= \quad 2/5 \;\; MR^2\omega \;\; = \;\; 2/5 \;\; MR\,(\omega R) \qquad\qquad \text{putting back c}'\text{s \& G}'\text{s} \\
T \quad &= \quad \frac{4\pi}{5 \text{ km}} \; \frac{2}{5} \; \left(\frac{\omega R}{c}\right) \;\; \times \;\; \frac{1}{2} \; \left(\frac{2GM}{c^2}\right) \; 10 \text{ km} \\
&= \quad 4\pi \; \cdot \; 2 \times \; 10^{-2} \;\; \times \; 4.8 \text{ km} \\
&= \quad \frac{8\pi \times 5 \times 10^{-2} \text{ km}}{3 \times 10^5 \text{ km/sec}} \\
&= \quad 40 \times 10^{-7} \text{ sec} \;\; = \;\; 4 \times 10^{-6} \text{ sec} \;\; = \;\; 4\mu \text{ sec}
\end{aligned}$$

This delay is a tiny fraction of 127 millisec, the period of the second pulsar. The only hope may be if we continuously monitor the arrival time of pulses using some sharp feature and watch the successive delays show a plot like Fig. 3. These will be delays due to gravitational red shift as well as the rotational delays. The former are symmetric with respect to before and after the eclipse but the latter are antisymmetric.

Gravitational Faraday effect

For a ray traversing the geometry

$$\begin{aligned}
ds^2 = \quad - \quad &(1 - \frac{2M}{r}) \, dt^2 - \; \frac{4Ma\sin^2\theta}{r} \; dt d\varphi + (1 - \; \frac{2M}{r})^{-1} \, dr^2 \\
+ \quad &r^2 \, (d\theta^2 \; + \; \sin^2\theta \; d\varphi^2)
\end{aligned}$$

we compute the total change in the plane of polarization as the light ray grazes the stars surface. The most advantageous geometry is shown in Fig. 4. The size of the effect is radians is

$$\Delta\phi = \frac{4 \; \frac{2}{5} \; MR^2\omega}{R^2 - 2MR} \;\; = \;\; \frac{4}{5} \left(\frac{2GM}{C^2}\right) \; \frac{R\omega}{R - 2M} = \frac{4}{5} \times 5 \times \; \frac{10^{-2}}{5} \;\; = \;\; 10^{-2} \;\; \text{Radian}$$

We need a signal to noise ratio of at least 100 to see this tiny change in the plane of polarisation of the radiation.

Remarks

1. Both effects have been computed only in order to magnitude. Detailed modelling is possible, but not appropriate until we have a good idea whether it is worth while.

2. Although the effects are small, they are worth keeping in mind. It may happen that a better observing technique or a better system may become available in the future.

3. The theoretical interest in these observation are due to the fact that they represent gravimagnetic effects – the gravitational effect of matter in motion. Such effects are the gravitational analogue of magnetic effects in electromagnetic theory.

4. The two effects mentioned have, the rotational time delay and the gravitational Faraday effect measure the component of spin angle momentum in the direction perpendicular to the orbital plane, and the other measures the component along the line of sight. Between them, the two effects give us the angular momentum vector of the star.

5. If this measurement can be made it would be the first direct measurement of the moment of inertia of a neutron star. The gravimagnetic effects are related to angular momentum. Since the period is measured, we can determine the moment of inertia.

6. With the binary pulsar, Hulse and Taylor have been able to track if for years and estimate all its orbital parameters. If a similar exercise can be done with the double pulsar, we would first extract all the orbital parameters based on the arrival times of pulses. Then, in the vicinity of the eclipses we would remove the ordinary gravitational time delay, (which is symmetric on either side of the eclipse). The minute residual in the timing noise after these known effects are accounted for may show an asymmetry (with respect to before and after eclipse). This is what we are after. The magnitude of the delays is only a few microseconds.

So unless there are sharp features in the pulse, these delays may be hard to measure.

7. Both the effects described here are independent of frequency. This follows from the equivalence principle. If there are frequency dependent effects these would due to the ordinary Faraday effect acting on the radiation from one pulsar as it passes through the magneto sphere of the other. Such frequency dependent effects have been looked for (Avinash Deshpande, Alak Ray) and not found so far.

8. The nice thing about an eclipse is that the radiation from one star grazes the surface of the other just before and after the eclipse. This radiation is therefore sensing gravitational fields as strong as they can be. Gravimagnetic effects fall off as $1/r^3$ (like any dipole field). The closer we can get the better our chances.

9. On closer examination of the orbit parameters of the double pulsar, it appears that the radiation from the background pulsar is blocked by the *magnetosphere* of the foreground pulsar and not the star itself. This would mean that the received radiation is not grazing the surface of the foreground star, which makes the above estimates too optimistic.

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