Information is Not Lost in the Evaporation of 2D Black Holes

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We analyze Hawking evaporation of the Callan-Giddings-Harvey-Strominger black holes from a quantum geometry perspective and show that information is not lost, primarily because the quantum space-time is sufficiently larger than the classical. Using suitable approximations to extract physics from quantum space-times we establish that (i) the future null infinity of the quantum space-time is sufficiently long for the past vacuum to evolve to a pure state in the future, (ii) this state has a finite norm in the future Fock space, and (iii) all the information comes out at future infinity; there are no remnants.

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In his celebrated paper [1], Hawking showed that in quantum field theory on a fixed black hole space-time the vacuum state at past null infinity $\mathcal{I}^-$ evolves to a thermal state on $\mathcal{I}^+$. Thus, in this external field approximation, pure states evolve into mixed; information is lost. Hawking also drew a candidate Penrose diagram including back reaction and suggested that information loss would persist. There has since been a large body of literature on the issue using diverse methods, models and approximations. More recently, the anti–de Sitter/conformal field theory conjecture has been used to argue that information cannot be lost. However, this reasoning requires a negative cosmological constant and even in that case a space-time description of the evaporation process is still lacking.

In this Letter we analyze the issue of information loss using the $1+1$ dimensional Callan-Giddings-Harvey-Strominger (CGHS) model [2]. The model is well suited because it shares most of the conceptual complications of realistic four-dimensional black holes but is technically simpler to analyze. Therefore it drew a great deal of attention in the early nineties (see, e.g., [3] for excellent reviews). Although a firm conclusion could not be reached due to limitations of semiclassical methods that were used, partial results suggested to many authors that information is probably lost.

Our analysis is motivated by the fact that quantum geometry leads to resolution of spacelike singularities in a number of simple models (see, e.g., [4]). This resolution provides an entirely new perspective on the problem [5]. Much of the older discussion assumed, as Hawking did, that the future boundary of the relevant space-time consists not just of $\mathcal{I}^+$ but also a piece of the initial classical singularity (see Fig. 1). Since part of the "in" state falls into the singularity, it is not surprising that the "out" state at $\mathcal{I}^+$ fails to capture the full information contained in the in state at $\mathcal{I}^-$. By contrast, if the singularity is resolved, this potential sink of information is removed. We will argue that in the quantum extension of the classical CGHS space-time, $\mathcal{I}^+$ is long enough to register all the information contained in the in state. Although our considerations are motivated by loop quantum gravity, in this Letter we will use the more familiar Fock quantization since the main argument is rather general.

Classical theory.—Fundamental fields of the CGHS model are the space-time metric g, a dilaton ϕ and a massless scalar field f. The action is given by

$$S(g,\phi,f) = \frac{1}{G}\int d^2V e^{-2\phi}(R + 4g^{ab}\nabla_a\phi\nabla_b\phi + 4\kappa^2) - \frac{1}{2}\int d^2V g^{ab}\nabla_a f\nabla_b f, \tag{1}$$

where R is the scalar curvature of g and κ is a constant (with dimensions of inverse length). Let $M_0 \equiv \mathbb{R}^2$ and fix on it a Minkowski metric η. Denote by $\mathcal{I}^{0\pm}$ its null infinity. We will be interested in physical metrics g which approach η at $\mathcal{I}^{0-}$. Denote by $z^\pm$ the advanced and retarded null coordinates of η so that $\eta_{ab} = -\partial_{(a}z^+\partial_{b)}z^-$ and set $\partial_\pm = \partial/\partial z^\pm$. Finally, set

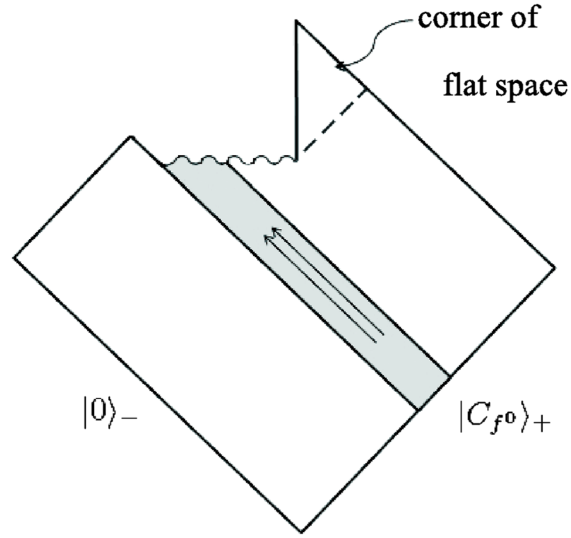

FIG. 1 (color online). A Penrose diagram of an evaporating CGHS black hole, motivated by [1]. Information can be lost in the singularity represented by the wiggly line.

$$\Phi = e^{-2\phi} \quad \text{and} \quad g^{ab} = \Theta^{-1}\Phi\eta^{ab} \equiv \Omega\eta^{ab}. \qquad (2)$$

Our fundamental fields will be Φ, Θ, f. They satisfy:

$$\Box_{(g)}f = 0 \Leftrightarrow \Box_{(\eta)}f = 0 \qquad \partial_+\partial_-\Phi + \kappa^2\Theta = GT_{+-}$$
$$\Phi\partial_+\partial_- \ln\Theta = -GT_{+-} \qquad (3)$$

and

$$\begin{aligned} -\partial_+^2\Phi + \partial_+\Phi\partial_+ \ln\Theta &= GT_{++} \\ -\partial_-^2\Phi + \partial_-\Phi\partial_- \ln\Theta &= GT_{--} \end{aligned} \qquad (4)$$

where T_{+-}, T_{++}, T_{--} are the $z^{\pm}$ components of the stress energy tensor of f. If (4) are imposed at $\mathcal{I}^{0-}$, they are propagated by (3). Therefore we will refer to (3) as dynamical equations and ensure that (4) are satisfied by choosing appropriate boundary conditions at $\mathcal{I}^{0-}$. In the classical theory T_{-+} vanishes identically but in quantum theory it is nonzero because of the trace anomaly.

Because f satisfies the wave equation on (M_0, η), it can be naturally decomposed into left and right moving modes $f_\pm(z^\pm)$. In the sector of the theory of interest to us, $f_- = 0$ and a black hole forms because of the gravitational collapse of f_+ (Fig. 1). To express the solution explicitly, it is simplest to use coordinates $x^\pm$:

$$\kappa x^+ = e^{\kappa z^+}, \quad \text{and} \quad \kappa x^- = -e^{-\kappa z^-}. \qquad (5)$$

Then, for any given f_+, the solution satisfying the appropriate boundary conditions at $\mathcal{I}^{0-}$ is given by [6]:

$$\begin{aligned} \Theta &= -\kappa^2 x^+ x^- \\ \Phi &= \Theta - \frac{G}{2}\int_0^{x^+} d\bar{x}^+ \int_0^{\bar{x}^+} d\bar{\bar{x}}^+ (\partial f_+/\partial\bar{\bar{x}}^+)^2 \\ &\quad - \frac{G}{2}\int_0^{x^-} d\bar{x}^- \int_0^{\bar{x}^-} d\bar{\bar{x}}^- (\partial f_-/\partial\bar{\bar{x}}^-)^2. \end{aligned} \qquad (6)$$

This brings out the fact that the true degree of freedom lies just in the matter field f; the geometry and the dilaton is determined algebraically from f. (The term containing f_- vanishes classically but is important for quantum considerations that follow.)

The solution is regular on all of M_0. How can there be a singularity and a black hole then? To answer this question let us examine the physical metric $g^{ab} = \Omega\eta^{ab} \equiv \Theta^{-1}\Phi\eta^{ab}$. Now, although Ω (and hence g^{ab}) is a well-defined tensor field on all of M_0, Φ vanishes on a spacelike line. Along this line g^{ab} also vanishes and its curvature becomes infinite. Thus $\Phi = 0$ is the singularity of the physical metric g. Is this a black hole singularity? Right null infinity $\mathcal{I}_R^+$ of g is a proper subset of $\mathcal{I}_R^{0+}$ (of η) [3]. However detailed analysis shows that it is complete with respect to g and its past does not contain the singularity. Thus the singularity is hidden behind a horizon with respect to $\mathcal{I}_R^+$. However, left null infinity $\mathcal{I}_L^+$ is incomplete to the future. So, strictly, we cannot conclude that we have a black hole with respect to $\mathcal{I}_L^+$ [7]. Fortunately, $\mathcal{I}_L^+$ does not play a direct role in the analysis of the Hawking effect and information loss.

Quantum theory.—Consider the space of all classical solutions. If $f \neq 0$, the manifold $M_{(g)}$ on which the physical metric g is well defined is a proper subset of M_0, which, however, varies from solution to solution. Therefore, the appropriate arena is the manifold M_0 defined by the fiducial η. This suggests that we represent $\hat{f}_\pm$ as an operator valued distribution on the Fock space $\mathcal{F}_+ \otimes \mathcal{F}_-$ associated with (M_0, η) and define $\hat{\Theta}$ and $\hat{\Phi}$ also on this Hilbert space. Since $f_- = 0$ classically, the quantum sector of interest is spanned by states Ψ of the type $|C_{f^0}\rangle_+ \otimes |0\rangle_-$ on $\mathcal{I}^{0-}$, where f^0 is any suitably regular profile of f_+ and C_{f^0} the coherent state in $\mathcal{F}_+$ peaked at f^0. The span of these states is $\mathcal{F}_+ \otimes |0\rangle_-$.

We will use the Heisenberg picture. The operator $\hat{g}^{ab} = \hat{\Omega}\eta^{ab}$ will define the quantum geometry on M_0. The basic operators $\hat{f} = \hat{f}_+ + \hat{f}_-$, $\hat{\Theta}$, $\hat{\Phi}$ must satisfy the operator version of dynamical Eqs. (3) and appropriate boundary conditions at $\mathcal{I}^{0-}$. More precisely, detailed considerations imply that a mathematical quantum theory of the model would result if we can (i) solve (3) for operators $\hat{f}$, $\hat{\Theta}$, $\hat{\Phi}$, where T_{+-} is replaced by the trace anomaly $T_{+-}(\hat{g})$ defined by the conformal factor $\hat{\Omega}$; and, (ii) ensure that at $\mathcal{I}^{0-}$, $\hat{\Theta}$ and $\hat{\Phi}$ are given by the operator versions of (6), with $(\partial f_\pm/\partial\bar{\bar{x}}^\pm)^2$ replaced by $:(\partial\hat{f}_\pm/\partial\bar{\bar{x}}^\pm)^2:$, where the normal ordering is defined by η. [Operator versions of (4) are then automatically satisfied at $\mathcal{I}^{0-}$.] It is likely that this framework can be made fully rigorous along the lines of the Dütsch and Fredenhagen [8] approach to interacting fields in Minkowski space-time.

The key physical questions are (i) in the solution, are $\hat{\Theta}$ and $\hat{\Phi}$ well defined everywhere on M_0? (ii) Does the operator valued distribution $\hat{\Omega}$ vanish anywhere? If it did, the quantum metric $\hat{g}^{ab} = \hat{\Omega}\eta^{ab}$ could be singular there; and, (iii) what is the physical interpretation of the Heisenberg state in the quantum geometry of $\hat{g}^{ab}$? The third question is crucial for extracting physics from the mathematical framework. While proposals of formulating the quantum theory in terms of operators have appeared in the literature (see, e.g., [9]), to our knowledge our specific formulation is new and the third question, in particular, had not received due attention. In the rest of the Letter we will introduce two approximation schemes to answer these questions. These schemes will also shed light on the exact framework.

Bootstrapping.—Although the quantum versions of the dynamical Eqs. (3) form a closed hyperbolic system for $\hat{\Theta}$ and $\hat{\Phi}$, they are difficult to solve exactly. To develop intuition for the quantum geometry that would result, it is instructive to simplify this task by a bootstrapping procedure. Begin with a seed metric $\hat{g}_0$ and use it to calculate the trace anomaly $\hat{T}_{+-}$, feed the result in the right side of the quantum dynamical equations, solve them, and denote the solution by $\hat{\Theta}_1$, $\hat{\Phi}_1$, and $\hat{g}_1^{ab}$. In the second step, use $\hat{g}_1^{ab}$ as the seed metric and continue the cycle in the hope of obtaining better and better approximations to the closed system of interest.

Let us begin by choosing $\hat{g}_0 = \eta$. Then, the first cycle can be completed. The solution on all of M_0 is $\hat{\Theta}_1 = -\kappa^2 x^+ x^-$ and $\hat{\Phi}_1 = \hat{\Theta}_1 - \frac{G}{2}\int_0^{x^+} d\bar{x}^+ \int_0^{\bar{x}^+} d\bar{\bar{x}}^+ {:}(\partial \hat{f}_+/\partial \bar{\bar{x}}^+)^2{:} - \frac{G}{2}\int_0^{x^-} d\bar{x}^- \int_0^{\bar{x}^-} d\bar{\bar{x}}^- {:}(\partial \hat{f}_-/\partial \bar{\bar{x}}^-)^2{:}$, where normal ordering is defined by η. How does this truncated solution fare with respect to the key physical questions? $\hat{\Theta}_1$ happens to be a c number and $\hat{\Phi}_1$ can be shown to be an operator valued distribution in a well-defined sense. They are regular everywhere on M_0 whence the quantum geometry determined by $\hat{g}_1^{ab}$ is also regular on all of M_0 already at the first approximation. The expectation values $\Phi_1 := \langle \hat{\Phi}_1 \rangle$ and $g_1^{ab} := \langle \hat{g}_1^{ab} \rangle$ turn out to reproduce just the classical solution $\Phi_{\rm class}$ and $g_{\rm class}^{ab}$ given by (6). In particular, g_1^{ab} vanishes along a spacelike line and its Ricci scalar diverges there. However, one can also calculate the fluctuations of $\hat{g}_1^{ab}$ (after suitable smearings since it is an operator valued distribution) and they are very large near that line. Therefore, the expectation value is a poor representation of quantum physics which is perfectly regular there.

The answer to the third physical question is even more interesting. We know that the quantum state of $\hat{f}_-$ is simply the vacuum state $|0\rangle_-$ on (M_0, η). The question is, what is its physical interpretation on the space-time (M_1, g_1) that results at the end of the first cycle? Following [1], one can carry out detailed analysis at late times. There are again two conceptual elements: (i) Since y_1^- defined by the asymptotic time translation on g_1 is nontrivially related to z^-, there is positive and negative frequency mixing between modes of $\hat{f}_-$ defined using z^- and those defined using y_1^-; and, (ii) since $g_1^{ab} = g_{\rm class}^{ab}$, its right future null infinity $\mathcal{I}_R^{1+}$ is a proper subset of $\mathcal{I}_R^{0+}$ of η^{ab}. Hence, one has to trace over modes in $\mathcal{I}_R^{0+} - \mathcal{I}_R^{1+}$. The result is that for the algebra of observables in (M_1, g_1), $|0\rangle_-$ reduces to the density matrix $\hat{\rho}_1 := ({\rm const}) \exp -\beta \hat{H}$, where $\beta = 2\pi/\hbar\kappa$, and $\hat{H}$ is the Hamiltonian of $\hat{f}_-$ at $\mathcal{I}_R^{1+}$. Thus, at this order one recovers the Hawking effect.

To summarize, the regular quantum geometry of $\hat{g}_1$ does not define some exotic sector of the theory, but has the right physical content. Since $\hat{\Theta}_1$, $\hat{\Phi}_1$, $\hat{g}_1$ is an exact solution to the truncated version of full quantum equations, it provides useful intuition for the nature of quantum geometry in the full theory. The next step in the bootstrapping is to start the second cycle using $\hat{g}_1$ as the seed metric. Unfortunately, the resulting quantum equations are now almost as difficult to solve as the exact ones. There is, however, another approximation that is well suited for analyzing the issue of information loss, which we now introduce.

Mean field approximation (MFA).—Rather than using a seed metric, let us return to the closed system of exact quantum dynamical equations, take their expectation values, and solve the resulting equations in the mean field approximation, i.e., by replacing expectation values of the type $\langle F(\hat{\Theta}, \hat{\Phi})\rangle$ by $F(\langle\hat{\Theta}\rangle, \langle\Phi\rangle)$. Viability of this approximation requires a large number N of matter fields so that quantum fluctuations $\hat{\Theta}$ and $\hat{\Phi}$ can be neglected relative to those in the matter fields. This large N approximation has been examined in some detail in the literature [3,10] and initial data near $\mathcal{I}^{0-}$ have been evolved numerically. Examination of marginally trapped surfaces in the resulting solutions shows that the Bondi mass at right null infinity of the mean field metric steadily decreases (essentially) to zero due to quantum radiation. This was often taken to mean that one can attach to the numerically evolved space-time a "corner" of flat space as in Hawking's original guess (see Fig. 1). However, a definitive statement could not be made because, even when N is large, fluctuations of geometry become dominant in the space-time interior making MFA invalid there.

Our new observation is that the key to the information loss issue lies in the geometry near future infinity and MFA should be valid there. Thus, we will assume that (i) the exact quantum equations can be solved and the expectation value $\bar{g}^{ab}$ of $\hat{g}^{ab}$ admits a smooth right null infinity $\mathcal{I}_R^+$ which coincides with $\mathcal{I}_R^{0+}$ in the distant past (i.e., near i_R^0); (ii) MFA holds in a neighborhood of $\mathcal{I}_R^+$; and, (iii) the flux of quantum radiation vanishes at some finite value of the affine parameter y^- of $\mathcal{I}_R^+$ defined by the asymptotic time translation of $\bar{g}$. All three assumptions were standard in previous analyses. Indeed, one cannot even meaningfully ask if information is lost unless the first two hold.

A priori, $\mathcal{I}_R^+$ may be only a proper subset of $\mathcal{I}_R^{0+}$ and no assumption is made about i^+ of $\bar{g}$. However, existence of $\mathcal{I}_R^+$ implies that as we go to large z^+ values along constant z^- lines, $\bar{\Phi} := \langle\hat{\Phi}\rangle$ and $\bar{\Theta} := \langle\hat{\Theta}\rangle$ admit asymptotic expansions of the form

$$\begin{aligned} \bar{\Phi} &= A(z^-)e^{\kappa z^+} + B(z^-) + O(e^{-\kappa z^+}), \\ \bar{\Theta} &= \underline{A}(z^-)e^{\kappa z^+} + \underline{B}(z^-) + O(e^{-\kappa z^+}). \end{aligned} \tag{7}$$

The MFA equations determine $\underline{A}$ and $\underline{B}$ in terms of A and B. Furthermore, y^- adapted to the asymptotic time translation of $\bar{g}$ is given by $\kappa \exp -\kappa y^- = A$. Finally, the MFA equations imply that there is a balance law at $\mathcal{I}_R^+$:

$$\begin{aligned} &\frac{d}{dy^-}\left[\frac{dB}{dy^-} + \kappa B + \frac{N\hbar G}{24}\left(\frac{d^2 y^-}{dz^{-2}}\left(\frac{dy^-}{dz^-}\right)^{-2}\right)\right] \\ &\quad = -\frac{N\hbar G}{48}\left[\frac{d^2 y^-}{dz^{-2}}\left(\frac{dy^-}{dz^-}\right)^{-2}\right]^2. \end{aligned} \tag{8}$$

It is natural to identify the quantity in square brackets on the left side as Gm_B, where m_B the Bondi mass, and the right side as the energy flux at $\mathcal{I}_R^+$. These definitions have the desired properties that the energy flux is positive definite and m_B vanishes in flat space (which is an MFA solution). The first two terms in the expression of m_B yield Hayward's formula [11] of Bondi mass in the classical theory; the third term is a quantum correction.

A key question now is how large is $\mathcal{I}_R^+$ compared to $\mathcal{I}_R^{0+}$? By assumption they coincide in the distant past near i_R^0. One can show that $y^- = Cz^- + D$ (with C, D constants) on the entire future region of $\mathcal{I}_R^+$ where the quantum flux vanishes. Hence $\mathcal{I}_R^+ = \mathcal{I}_R^{0+}$ (see Fig. 2). This implies that to interpret $|0\rangle_-$ at $\mathcal{I}_R^+$ we no longer have to trace over

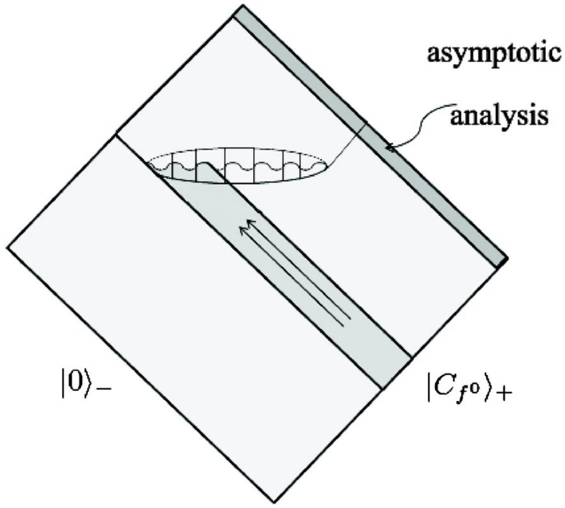

FIG. 2 (color online). Proposed Penrose diagram. The mean field approximation is used in the shaded region near $\mathcal{I}_R^+$. Quantum fluctuations of geometry are large in the interior region around the wiggly line representing the putative classical singularity.

any modes; in contrast to the situation encountered in our bootstrapping discussion, all modes of $\hat{f}_-$ are now accessible to the asymptotically stationary observers of $\bar{g}$. The vacuum state $|0\rangle_-$ of η is pure also with respect to $\bar{g}$.

But is it in the asymptotic Fock space of $\bar{g}$? Calculation of Bogoluibov coefficients shows that the answer is in the affirmative because $y^- = Cz^- + D$ in the future and boundary conditions imply that y^- approaches z^- exponentially quickly in the distant past. Thus, the interpretation of $|0\rangle_-$ with respect to $\bar{g}$ is that it is a pure state populated by pairs of particles at $\mathcal{I}_R^+$. There is neither information loss nor remnants whose quantum state is correlated with the state at $\mathcal{I}_R^+$.

Summary.—A key simplification in the CGHS model is that the matter field satisfies just the wave equation on (M_0, η^{ab}). Therefore, given initial data on $\mathcal{I}^{0-}$, we already know the state everywhere both in the classical and the quantum theory. However, the state derives its physical interpretation from geometry which is a complicated functional of the matter field. We do not yet know the quantum geometry everywhere. But already at the end of the first cycle of bootstrapping we found that $\hat{g}_1^{ab}$ is well defined (and nowhere vanishing) everywhere on M_0. So it seems reasonable to assume that the full $\hat{g}^{ab}$ would also be singularity-free. To pose questions about information loss, one has to assume that its expectation value $\bar{g}$ admits future right null infinity $\mathcal{I}_R^+$ which, *a priori*, may be only a portion of $\mathcal{I}_R^{0+}$ of η. But then the MFA equations imply that $\mathcal{I}_R^+$ in fact coincides with $\mathcal{I}_R^{0+}$ and the exact quantum state $|0\rangle_-$ is a pure state in the asymptotic Fock space of $\bar{g}^{ab}$. The S matrix is unitary and there is no information loss. The Penrose diagram (Fig. 2) we are led to is significantly different from that based on Hawking's original proposal (Fig. 1). In particular, the quantum space-time does not end at a future singularity and is larger than that in Fig. 1. The singularity is replaced by a genuinely quantum region and, in contrast to an assumption that was often made, space-time need not be flat to its "future." Finally, although $\hat{g}^{ab} = \hat{\Omega}\eta^{ab}$, $\hat{\Omega}$ is an operator and is not required to be positive definite. Around the wiggly line of Fig. 2, quantum fluctuations of $\hat{\Omega}$ are large and of either sign (where the negative sign corresponds to interchanging timelike and spacelike directions). Thus, the global causal structure is not that of Minkowski space-time.

We emphasize, however, that a full solution to the quantum equations is still lacking. This is needed to prove the validity of our assumptions and to calculate, everywhere on $\mathcal{I}_R^+$ the function $y^-(z^-)$ that determines the detailed physical content of $|0\rangle_-$ at $\mathcal{I}_R^+$. Nonetheless, using what we already know, we can answer the oft raised question—when does the "information" come out? Following the standard strategy, let us use a basis at $\mathcal{I}_R^+$ analogous to that of [1], trace over modes to the future of the point where the Bondi mass vanishes, and ask if the resulting state is approximately pure. In our framework the answer is in the affirmative. Thus, most of the information comes out with the quantum radiation. This issue, as well as several others that have been raised in the literature, will be discussed in a detailed paper.

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