Orthogonal Matrices for Multi-Line Addressing

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***Abstract*—Simple orthogonal matrices are proposed to select multiple address lines simultaneously in RMS responding matrix displays. Good brightness uniformity of pixels can be achieved with less number of time intervals in an address cycle.**

***Index Terms*—Liquid crystal displays (LCDs), matrix addressing, multi-line addressing (MLA), multiplexing.**

I. Introduction

MULTI-LINE addressing (MLA) is used to drive liquid-crystal display (LCD) in cell phones, MP3 players, etc. [1], wherein a few (s) address lines are selected simultaneously by applying voltages that are proportional to the elements in a column (select vector) of an orthogonal matrix. Data voltages that are proportional to the dot product of select vector and data of pixels located in selected address lines are applied to columns of the matrix display along with select voltages to the rows. Entire sets of address lines are selected once with all the select vectors to complete a cycle. Polarity of scanning and data voltages is reversed periodically to ensure DC free operation. Walsh functions and Hadamard matrices are used to generate the scanning waveforms in MLA [2]. Number of time intervals to address the display cycle (n_c) is equal to the number of address lines (N) when Walsh functions or Hadamard matrices [2] are used to generate the addressing waveforms if $s = 2^i$ or $(4.i)$, i.e., an integer (i) power of two or multiples of four. The number of time intervals n_c is greater than N for all other values of s.

II. Objective

The main objective of this work is to reduce n_c when $s \neq 2^i$ or $(4.i)$ and several orthogonal matrices are proposed to achieve it. The number of time intervals (n_c) depends on the number of address lines in a matrix display (N), number of address lines that are selected simultaneously (s) and the number of select vectors (n_s), i.e., number of columns in the orthogonal matrix as follows:

$$n_c = \left[n_s.N/(s)\right]. \tag{1}$$

Display has to be refreshed at a rate that is fast enough to suppress flicker. Period of the address cycle should be small as compared to response times (time taken to switch the state of pixels) to ensure rms response of the display. Pixels in LCD are equivalent to capacitors and power is dissipated in output resistances of drivers when pixels are charged or discharged depending on the amplitude of transitions and the number of transitions [3]. Hence, it is preferable to have a low n_c. A low value n_c improves uniformity by increasing T so that pixels that are driven to identical states will have same brightness; because error in rms voltages across pixels due to distortions will be less when the time constant of driver circuit is small as compared to select time $T = 1/(f.n_c)$; wherein f is the frame frequency. DC-free operation is an important but less stringent condition and it is adequate to achieve it in several address cycle or frames. For example polarity of addressing waveforms can be reversed at the end of each scan to obtain DC-free waveforms in $2N$ time intervals. Addressing techniques based on Rademacher functions [4] have a large n_c because $n_s = 2^s$ and, therefore, $n_c = (2^s.N/s)$ increases exponentially with s. However, flicker will not be observed even if the refresh frequency is reduced by a large factor because Rademacher functions have several orthogonal functions in them. For example, four Hadamard matrices are embedded in the matrix corresponding to Rademacher functions for $s = 4$, shown in (2) at the bottom of the next page. It is easy to identify them after rearranging columns of the Rademacher matrix in (2) as shown in (3), at the bottom of the next page. LCD can also be scanned by using just four select vectors, viz., columns (1 to 4) or (5 to 8) or (9 to 12) or (13 to 16) of the matrix in (3) because they are orthogonal matrices. They correspond to Hadamard matrices: H_A, H_B, $-H_A$, and $-H_B$, shown in (4)

$$H_A = \begin{bmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 \end{bmatrix}; \; H_B = \begin{bmatrix} -1 & +1 & +1 & +1 \\ +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \end{bmatrix}. \tag{4}$$

Display refresh frequency can be reduced by a factor of four (without causing flicker) when Rademacher matrix in (3) is used to scan the display. Hence, the reduction in n_c that is

TABLE I
Number of Time Intervals in a Cycle to Address a Matrix Display

S	Order of the Hadamard matrix	n_c - With Hadamard matrices	Percentage increase in the select time (T) if $n_c = N$
3	4	(4N/ 3)	33%
4	4	N	0
5	8	(8N/ 5)	60%
6	8	(4N/ 3)	33%
7	8	(8N/ 7)	14%
8	8	N	---
9	12	(4N/ 3)	33%
10	12	(6N/ 5)	20%
11	12	(12N/ 11)	9%
12	12	N	0

n_c—Number of time intervals for completing an address cycle

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achieved by using Hadamard matrices instead of Rademacher functions is superficial. Objective of this work is to reduce n_c when s is not an integer power of two or an integer multiple of four by using compact orthogonal matrices leading to reduction of n_c and a real increase in select time Tas compared to using Hadamard matrix of higher order that is nearest to s. Number of time intervals to complete a cycle i.e. n_c when Hadamard matrices are used to scan the display is given in Table I. It is higher than $N$when s is not equal to 2^i or $(4.i)$; it could be as high as 60% when $s = 5$. Simple orthogonal matrices are proposed in the next section to reduce n_c when $s \neq 2^i$ or $(4.i)$.

III. Simple Orthogonal Matrices

A. Diagonal Matrices

Let us consider a square matrix of order s with d as its diagonal elements and let r be the rest of its elements as in

$$\begin{bmatrix} d & r & r & r & . & . & r & r \\ r & d & r & . & . & . & r & r \\ r & r & d & . & . & . & . & . \\ r & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & r \\ . & . & . & . & . & d & r & r \\ r & r & . & . & . & r & d & r \\ r & r & . & . & r & r & r & d \end{bmatrix}. \tag{5}$$

The matrix is orthogonal if the following condition is satisfied:

$$(s-2).r^2 + 2.d.r = 0. \tag{6}$$

An infinite number of orthogonal matrices can be constructed to satisfy the condition

$$\frac{d}{r} = -\frac{(s-2)}{2}. \tag{7}$$

Matrices of order 3 are shown in (8) as examples. It is not necessary to normalize these matrices when it is used for scanning matrix displays.

$$\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}; \begin{bmatrix} +1 & -2 & -2 \\ -2 & +1 & -2 \\ -2 & -2 & +1 \end{bmatrix}; \begin{bmatrix} -\sqrt{2} & +\sqrt{8} & +\sqrt{8} \\ +\sqrt{8} & -\sqrt{2} & +\sqrt{8} \\ +\sqrt{8} & +\sqrt{8} & -\sqrt{2} \end{bmatrix} \dots \tag{8}$$

Hadamard matrix of order 4 is obtained as a special case of these matrices when the diagonal elements are -1. Two more matrices of order 5 are also shown as examples in (9)

$$\begin{bmatrix} -3 & +2 & +2 & +2 & +2 \\ +2 & -3 & +2 & +2 & +2 \\ +2 & +2 & -3 & +2 & +2 \\ +2 & +2 & +2 & -3 & +2 \\ +2 & +2 & +2 & +2 & -3 \end{bmatrix}; \begin{bmatrix} +3 & -2 & -2 & -2 & -2 \\ -2 & +3 & -2 & -2 & -2 \\ -2 & -2 & +3 & -2 & -2 \\ -2 & -2 & -2 & +3 & -2 \\ -2 & -2 & -2 & -2 & +3 \end{bmatrix}. \tag{9}$$

B. Orthogonal Matrices Using Kronecker Product

Higher order matrices can be constructed by using the (7) or by using Kronecker product of two lower order orthogonal matrices, as shown in (10)–(12)

$$O_6 = O_3 \otimes H_2 = \begin{bmatrix} -1 & 2 & 2 & -1 & 2 & 2 \\ 2 & -1 & 2 & 2 & -1 & 2 \\ 2 & 2 & -1 & 2 & 2 & -1 \\ -1 & 2 & 2 & 1 & -2 & -2 \\ 2 & -1 & 2 & -2 & 1 & -2 \\ 2 & 2 & -1 & -2 & -2 & 1 \end{bmatrix} \tag{10}$$

$$O_6 = H_2 \otimes O_3 = \begin{bmatrix} -1 & -1 & +2 & +2 & +2 & +2 \\ -1 & +1 & +2 & -2 & +2 & -2 \\ +2 & +2 & -1 & -1 & +2 & +2 \\ +2 & -2 & -1 & +1 & +2 & -2 \\ +2 & +2 & +2 & +2 & -1 & -1 \\ +2 & -2 & +2 & -1 & -1 & +1 \end{bmatrix} \tag{11}$$

$$\begin{aligned} O_9 &= O_3 \otimes O_3 \\ &= \begin{bmatrix} 1 & -2 & -2 & -2 & 4 & 4 & -2 & 4 & 4 \\ -2 & 1 & -2 & 4 & -2 & 4 & 4 & -2 & 4 \\ -2 & -2 & 1 & 4 & 4 & -2 & 4 & 4 & -2 \\ -2 & 4 & 4 & 1 & -2 & -2 & -2 & 4 & 4 \\ 4 & -2 & 4 & -2 & 1 & -2 & 4 & -2 & 4 \\ 4 & 4 & -2 & -2 & -2 & 1 & 4 & 4 & -2 \\ -2 & 4 & 4 & -2 & 4 & 4 & 1 & -2 & -2 \\ 4 & -2 & 4 & 4 & -2 & 4 & -2 & 1 & -2 \\ 4 & 4 & -2 & 4 & 4 & -2 & -2 & -2 & 1 \end{bmatrix}. \end{aligned} \tag{12}$$

The matrices O_6 and O_9 can be used to select 6 and 9 address lines simultaneously and matrices of higher order are not of practical importance.

IV. Analysis

Let the element of the orthogonal matrix of order s be $o_{k,t}$ and let $d_{k,j}$ be the data in the selected set of address lines in column-j. Select voltage applied to k address lines in the

$$\begin{bmatrix} +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} +1 & +1 & +1 & +1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 & -1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & -1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 & -1 \\ +1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 \end{bmatrix} \tag{3}$$

selected set be $o_{k,t}.V_s$ and the data voltage that is applied to column-j is

$$\text{data voltage } d_{t,j} = V_d.\sum_{k=1}^{s} o'_{k,t}.d_{k,j} \tag{13}$$

Energy delivered to a pixel located at the intersection of address line-i and data line-j during the select and nonselect time intervals are as follows:

$$E_{\text{select}} = \sum_{t=1}^{s}\left[o_{k,t}V_s - V_d.\sum_{k=1}^{s} o'_{k,t}.d_{k,j}\right]^2 \tag{14}$$

$$E_{\text{nonselect}} = \left(\frac{N}{s}-1\right).\sum_{t=1}^{s}\left[V_d\sum_{k=1}^{s} o'_{k,t}.d_{k,j}\right]^2 \tag{15}$$

RMS voltage across a pixel is given by the following equation:

$$\begin{aligned} V_{\text{RMS}} &= \sqrt{\frac{E_{\text{select}} + E_{\text{nonselect}}}{s.N}} \\ &= \sqrt{\frac{e\left(V_s^2 - 2d_{k,j}V_s.V_d + NV_d^2\right)}{s.N}}. \end{aligned} \tag{16}$$

Here, e is the sum of squares of the elements in each row. The ratio of RMS voltage across ON pixel to that of OFF pixel is

$$\sqrt{(\sqrt{N}+1)/(\sqrt{N}-1)} \quad \text{when} \quad V_s = \sqrt{N}.V_d \tag{17}$$

It is the maximum that can be achieved by any addressing technique [4]. It can be shown that sum of square of elements in each row (or column) of the diagonal matrix is square of the order of the matrix and the difference between the two elements of the diagonal matrix is equal to the order of the matrix. Supply voltage of the MLA technique that is based on the diagonal matrices is 50% of the conventional line-by-line addressing technique. Supply voltage is $(\sqrt{N}/(\sqrt{N}+1))$ times that of the modified waveforms proposed by Kawakami *et al.* [5] and, hence, it is slightly lower than that of [5].

V. Conclusion

Hardware complexity of row drivers is same as the conventional MLA technique. Drivers that are capable of applying one out of three voltages are adequate; when diagonal matrices and other matrices constructed by using at least one Hadamard matrix are used for scanning the display. Seven voltages are necessary in scanning (row) waveforms when an orthogonal matrix of order 9 (constructed as a Kronecker product of diagonal matrix of order 3 with itself) is used to scan the display. Hardware complexity of the data drivers is highly dependent on the type and order of the orthogonal matrix as shown in Table II. Simultaneous selection of 3 or 4 rows is popular among displays in mobile phones [1] and the diagonal matrix of order 3 is attractive for such applications. Select pulses will be 33% wider when diagonal matrix of order is 3 with three select vectors used, as compared to using Hadamard-type orthogonal matrix with 4 select vectors as in [1]. Supply voltage of the driver circuit will also be less by a factor $(\sqrt{3}/2)$; i.e., about 14% less when N is greater than 22 as compared to using Hadamard matrix [1]. Similarly the orthogonal matrix of order 6 fills the void among the Hadamard matrices of order 4 and 8 to achieve a better brightness uniformity of pixels when the RC time constant of the driver circuit is same. Gray shades can be displayed by adopting successive approximation technique for multi-lines [6] without increasing the hardware complexity of the drivers.

TABLE II
Number of Voltages in the Addressing Waveforms

Type and Order of the matrix	Parameters e / p	Voltages in data waveforms (Normalized to V_c)
Diagonal-3	9 / 3	±5, ± 3, ± 1
Diagonal-4	4 / 2	±4, ± 2, 0
Hadamard -4	4 / 2	±4, ± 2, 0
Diagonal-5	25 / 5	±11, ± 7, ± 5, ± 3, ± 1
Diagonal-6	9 / 3	±7, ± 5, ± 3, ± 1
Kronecker-6	18 / 3	± 10, ± 8, ± 6, ± 4, ± 2, 0
Diagonal-7	49 / 7	±17, ± 13, ± 9, ± 7, ± 5, ± 3, ± 1
Hadamard-8	8 / 2	±8, ± 6, ± 4, ± 2, 0
Diagonal-8	16 / 4	±10, ± 8, ± 6, ±,4 ,±2, 0
Diagonal-9	81 /9	±23,±19,±15,±11,±9,±7 ±5,±3,±1
Kronecker-9	81 / 6	±25, ± 23, ± 21, ± 19,±17,±15, ± 13, ± 11,±9,±7,±5,±3,±1

Parameter e is the sum of squares of the elements in each row and p is the difference of the maximum and the minimum of elements in the rows/columns of the symmetric orthogonal matrix.

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