# CHAPTER 5

# CONCLUSION

The design, development and field trials of a multibeam forming receiver for the decameter wave telescope at Gauribidanur have been discussed in this thesis. The receiver uses one-bit correlators for obtaining interference patterns between the component antennas. It's simplicity in hardware, dynamic range and the independence of its output on the amplifier gains were the main factors which led to its choice for the receiver system. In the one-bit correlator, the signal is infinitely clipped and hence the amplitude information of the signal is lost. A simple circuit was designed to obtain the amplitude information of the signal too using one-bit correlators. With this it became possible to measure a deflection proportional to the strength of the source. A one-bit correlator without the total power information measures the source strength relative to the background in which it is present.

The receiver uses a DSB system. This made phase calibrations independent of the delay settings used for the delay compensation. A new technique to obtain the quadrature samples of signals required for a DSB system has been developed. This technique requires lesser hardware than the conventional DSB technique and also achieves an improved system performance. R.F.I. was monitored before deciding the centre frequency and the bandwidth for observation. To obtain maximum interference free observation a variable bandwidth receiver for the E-W antenna was built. Deterioration of SNR when two unequal bandwidth signals are correlated in a one bit correlator has been investigated. It was found that for linear phase filters in the E-W array, usage of a simple delay in the N-S signal path results in very little SNR deterioration. To excise interference, filtering after one bit quantisation was also attempted. To indicate and monitor interference, an inteference monitor which functions like a spectrum analyser was built. It had limited utility during the observations described in this thesis as interference was at a minimum presumably due to low solar activity.

A conventional DSB receiver has been built to observe spectral lines using the same set of correlators used for continuum observations. The scheme employed does not need any switching and automatically measures (on-off) spectral line temperatures.

Use of proper grounding and shielding techniques resulted in smooth functioning of the multichannel receiver. On board test equipment like crystal oscillator, noise generator, HP printer interface and a test correlator were found extremely useful during the installation of the receiver system. Field test equipment was designed to replace a vector voltmeter, signal generator and large power inverters and will be extremely useful in antenna testing and it's maintenance. The LO system built for the receiver used TTL buffers as current amplifiers and power splitters. This replacement for RF amplifiers and hybrid power splitters resulted in cost reduction without foregoing system performance.

An astronomical clock described in this thesis gives accurate sidereal time information. The use of Cos/Mos circuits results in a power consumption so low that the entire clock can operate on self contained dry batteries for a period of two days. A lock switch prevents manual interference with the running of the clock. Copies of this clock have also been installed at the Radio Astronomy Centre, Ooty, and the optical observatory at Kavalur. In the absence of a computer at the Gauribidanur observatory,the magnetic tape interface described in this thesis has enabled the receiver system to collect visibility data on an incremental magnetic tape recorder. Its ability to change writing speeds, insert record and file gaps at programmable intervals, and write noise calibration before closing a file were found extremely useful.

Continuum and line observations were successfully carried out using this receiver at Gauribidanur. The maps shown in this thesis clearly indicate the suitability of this system for a large scale survey. Maps presented here are more reliable than those made with movable elements, because at low frequencies ionospheric refraction causes time varying shifts in the position and intensity of the sources. Large scale features are observable in these maps since all the fourier components down to zeroth order were obtained by direct observation. While calibrating these maps, we did not find the discrepancy in the low frequency WKB flux scales suggested by Braude et.al. (1970).

We have been able to detect low frequency recombination lines of carbon in the direction of Cas A. This confirms the earlier observations made by Konovalenko et.al. Observations were also carried out in the direction of Cygnus A, but no spectral lines with $T_L/T_c \geq 2 \times 10^{-3}$ were detectable.

This thesis has suggested a scheme to configure the E-W and N-S arrays for one dimensional synthesis which requires minimum hardware and which can map the entire observable region of sky ($\pm 50^{\circ}$ Z.A. without grating lobes) in one observing session. The hardware required for this reconfiguration is being built. A large scale survey using this receiver system will then be undertaken. Narrow band filters to observe spectral lines with higher resolution are also under construction. Galactic plane and a few other probable regions will be surveyed to detect low frequency recombination lines.

# APPENDIX A

# ONE BIT FILTER

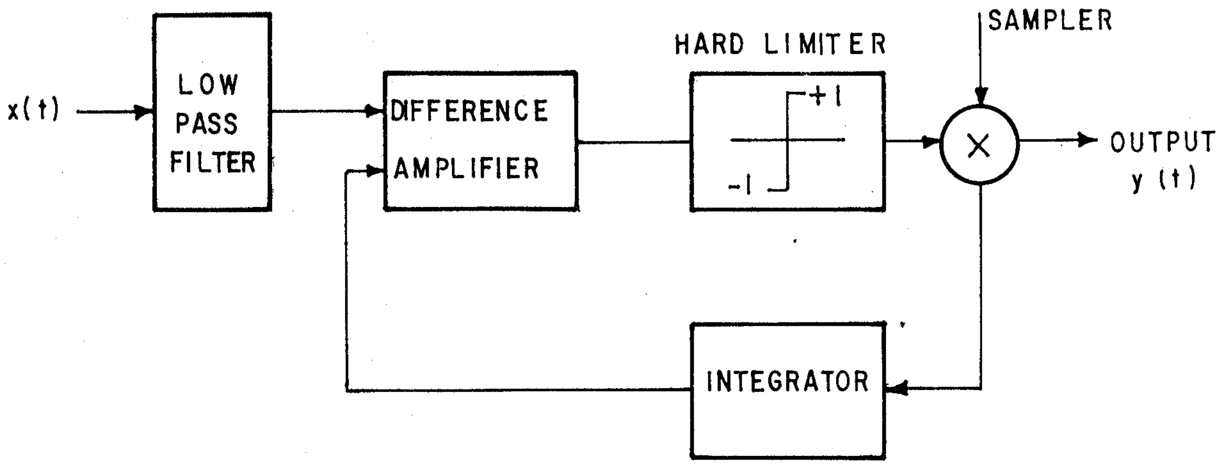
## A.1 INTRODUCTION

Many real-time filtering tasks are performed today using digital logic. Such digital filters have all the traditional advantages associated with digital systems. They are, high accuracy, reliability, repeatability and independence of system performance on the component tolerances, component drifts and to a certain extent on spurious environmental signals. In addition, digital filters can be easily programmed to alter the frequency characteristics by merely changing the filter co-efficients. The main disadvantage of the digital filter is its high cost, which is proportional to the number of bits used, for signal quantization, to represent the filter coefficients, and to perform the required arithmetical operations. If the number of bits for quantization is reduced, it may not be possible to obtain the desired filter characteristics.

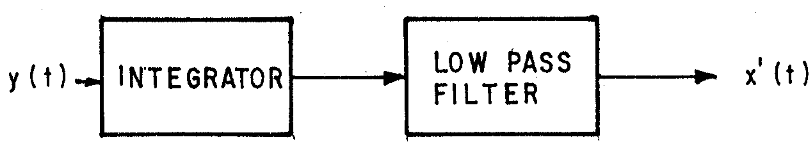
Two well known digital techniques which provide cost effective filters are based on digital filtering using delta modulation, and sequency filtering using Walsh functions.

Delta modulation is an inexpensive means of analog to digital conversion. This produces a binary sequence from which the analog input can be recovered by a simple integration (Ref fig A.1). In delta modulation the sampling function samples a bipolar or a binary quantized difference signal instead of the signal itself (F.de Jager, 1952). In spite of its simplicity, delta modulation did not initially find widespread application because of its dynamic range limitation. Lack of dynamic range was due to the fixed step size used in the modulation process. Adaptive delta modulation which has a variable step size improves the dynamic range (Brolin and Brown, 1968; Abate, 1967). Today adaptive delta modulation is used in various applications like correlation and filtering. ( L.F. Rocha, 1980; D. Lagoyanis, 1981; M. Shafi, 1981).

Walsh functions form an ordered set of rectangular waveforms taking only two amplitude values +1 and -1. They are defined over a limited time interval T, known as the "Time base". Like the Sine and Cosine functions, two arguments only are required for complete definition. They are, a time period t (usually normalised to the time base as t/T), and an ordering number n related to frequency. For most purposes, a set of such functions is ordered in ascending value of the number of zero crossings found within




(a) MODULATOR




(b) DEMODULATOR

FIG. A .1 HARDWARE IMPLEMENTATION OF A DELTA MODULATION SYSTEM the time base(ref fig A.2). One half of the average number of zero crossings per unit time interval is known as sequency. If the function terms are ordered in ascending value of sequency, then it is known as sequency ordering and has similarities to the familiar Sine-Cosine series.

The use of sequency functions for digital filtering allows a simplification in the modelling of the filter transfer characteristics. Sequency filters do not need complex arithmetic and require only a few arithmetic operations. Thus they are very suitable for large scale filtering tasks. Walsh functions are widely used in image processing, coding and in speech processing (K.G. Beauchamp, 1984).

The correlators built for the present work are not based either on delta modulation or sequency analysis. We use a one-bit method of finding the correlation function, (sec 2.4) which we find simpler and more elegant.

In the present correlator the signals are multiplied after infinite clipping to determine the correlation function. The correlation functions of the signal before clipping ( $\rho_A$ ) and after clipping ( $\rho_c$ ) are related by the Van Vleck relation.

$$\rho_A = \sin \frac{\pi}{2} \rho_c \qquad \text{(A.1)}$$

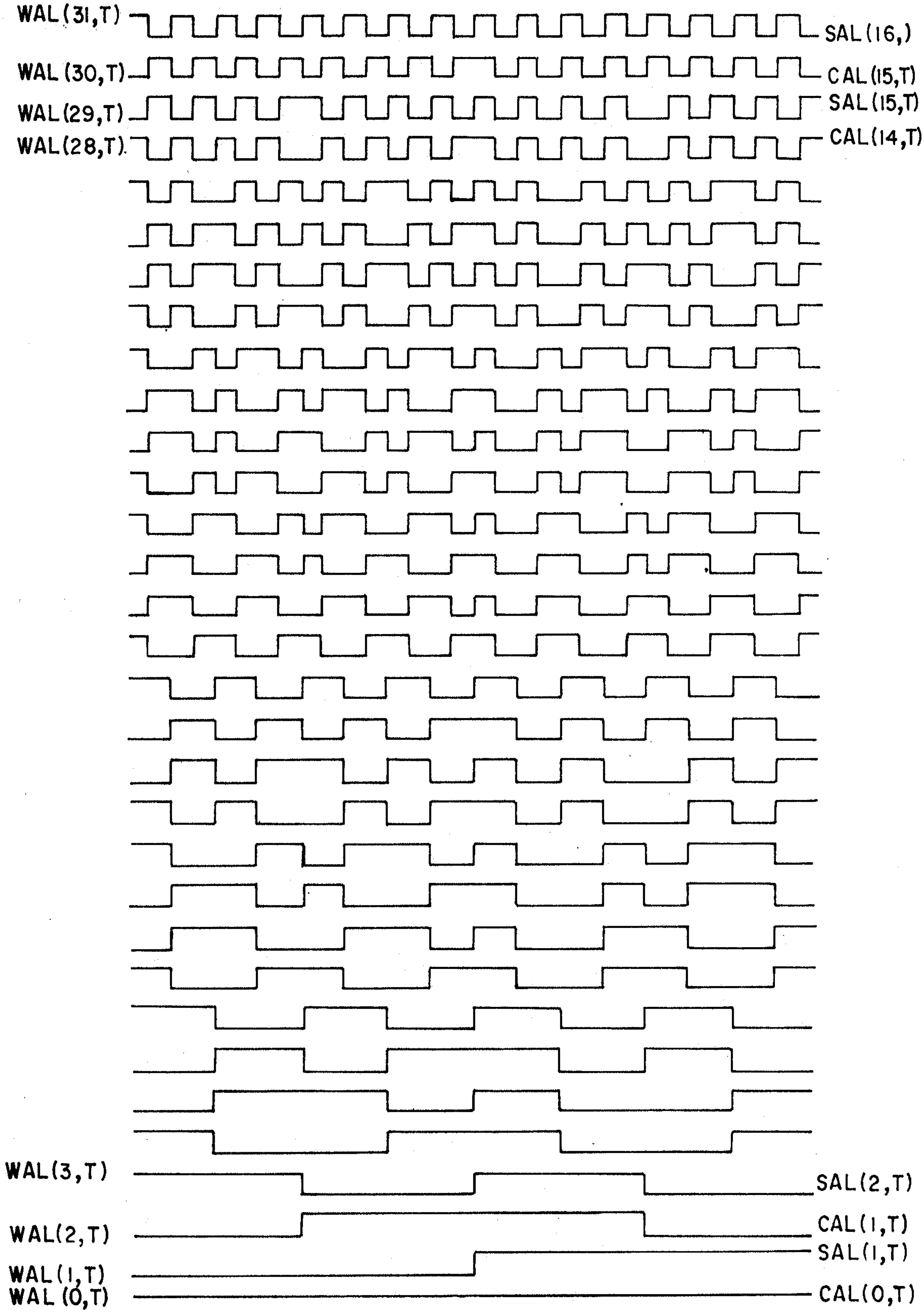



FIG. A.2 A SET OF WALSH FUNCTIONS ARRANGED IN SEQUENCY ORDER. (BEAUCHAMP-1984).

$$f(t) = a_0 \, WAL(0,t) + \Sigma a_n \, WAL(n,t)$$

$$a_n = \frac{1}{T} \int_0^T f(t) \, WAL(n,t) \, dt.$$

The correlation and power spectrum are related by a Fourier transform. Thus the Van Vleck relation clearly indicates that the zero crosses of a noise signal with gaussian statistics has complete information about the spectral contents of the original signal. Thus it should be possible to alter the frequency components by altering the zero crossings of the signal. This requires simple arithmetical operations on the one-bit output of the sampler.

A simple analog or a multibit digital filter is a "linear transversal filter". In such a filter, the present output is a weighted average of the present and the past inputs. The coefficients of the weighting function determine the filter characteristics. The transfer function of a filter which uses a uniform weighting function can be written as

$$C'_n = C_n + C_{n-1} + \cdots\cdots C_{n-(m-1)} \qquad \text{(A.2)}$$

where $C_n, C_{n-1}, \ldots\ldots C_{n+1-m}$ are the present and the past inputs and $C'_n$ is the output of the transversal filter. A block schematic of a simple digital transversal filter is shown in the figure A.3. When each sample is quantized to a single bit, the averaging simply reduces to majority finding operation. Such a filter may be called a "one-bit filter". Due to non-linearity of the one-bit quantization process, a one-bit filter cannot have a response identical to a multibit transversal filter. To study its response, a

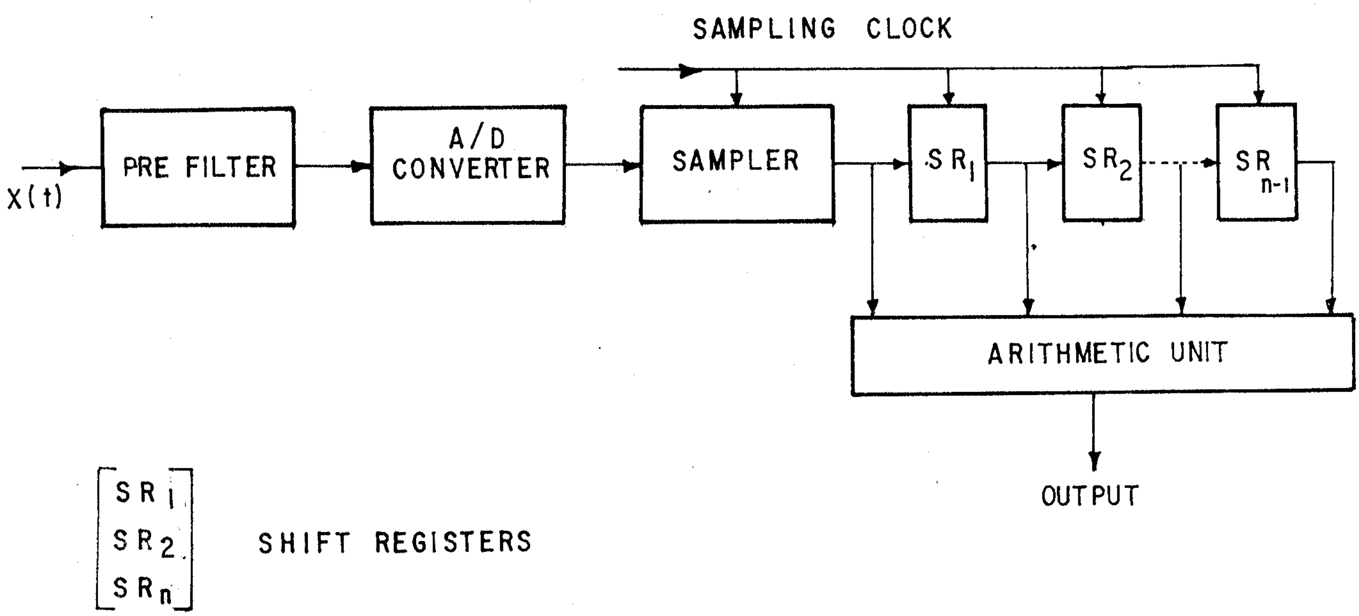
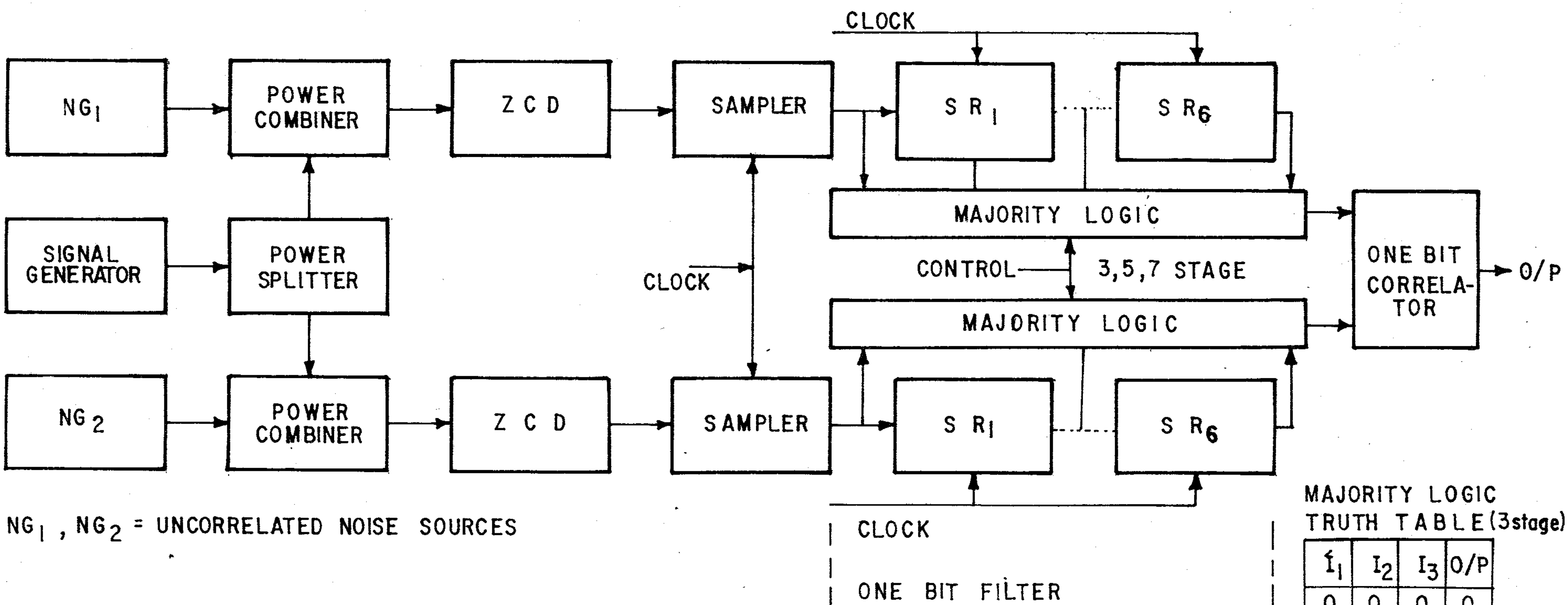



FIG.A.3 HARDWARE IMPLEMENTATION OF AN n STAGE MULTIBIT DIGITAL TRANSVERSAL FILTER simple experiment shown in Fig. A.4 was performed. It was found that the correlation coefficients at the output decreased as the frequency of the continuous wave was increased. This is shown by the dotted lines in the Fig. A.5. This figure clearly indicates that the frequency response of a one-bit filter has the same trend as the frequency response of a multibit transversal filter.

A simple simulation was done on a desk calculator to study the difference between one-bit quantization after averaging and majority finding after quantization. The scheme used is shown in Fig. A.6. The coincidence count between the samples obtained by the two methods is 84%. This result can also be obtained by the following considerations.

Averaging before one-bit quantization and majority finding after quantization give similar results when all the samples under consideration have the same sign. The two methods of averaging can give different results when one of the samples has a different sign from the other two, and has a modulus greater than the sum of the other samples. To find the extent of similarity between the two methods of filtering, one has to find the probability of occurence of such samples. If $X_1, X_2, X_3$ are three gaussian random variables which are uncorrelated, they obey jointly three dimensional gaussian statistics. If their amplitudes are normalized such that

$$X_1^2 + X_2^2 + X_3^2 = 1 \quad \text{(A.3)}$$




MAJORITY LOGIC TRUTH TABLE (3stage)

| $I_1$ | $I_2$ | $I_3$ | O/P |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

FIG. A .4 AN EXPERIMENTAL SET UP TO DEMONSTRATE THE FILTERING ACTION OF ONE BIT FILTERS

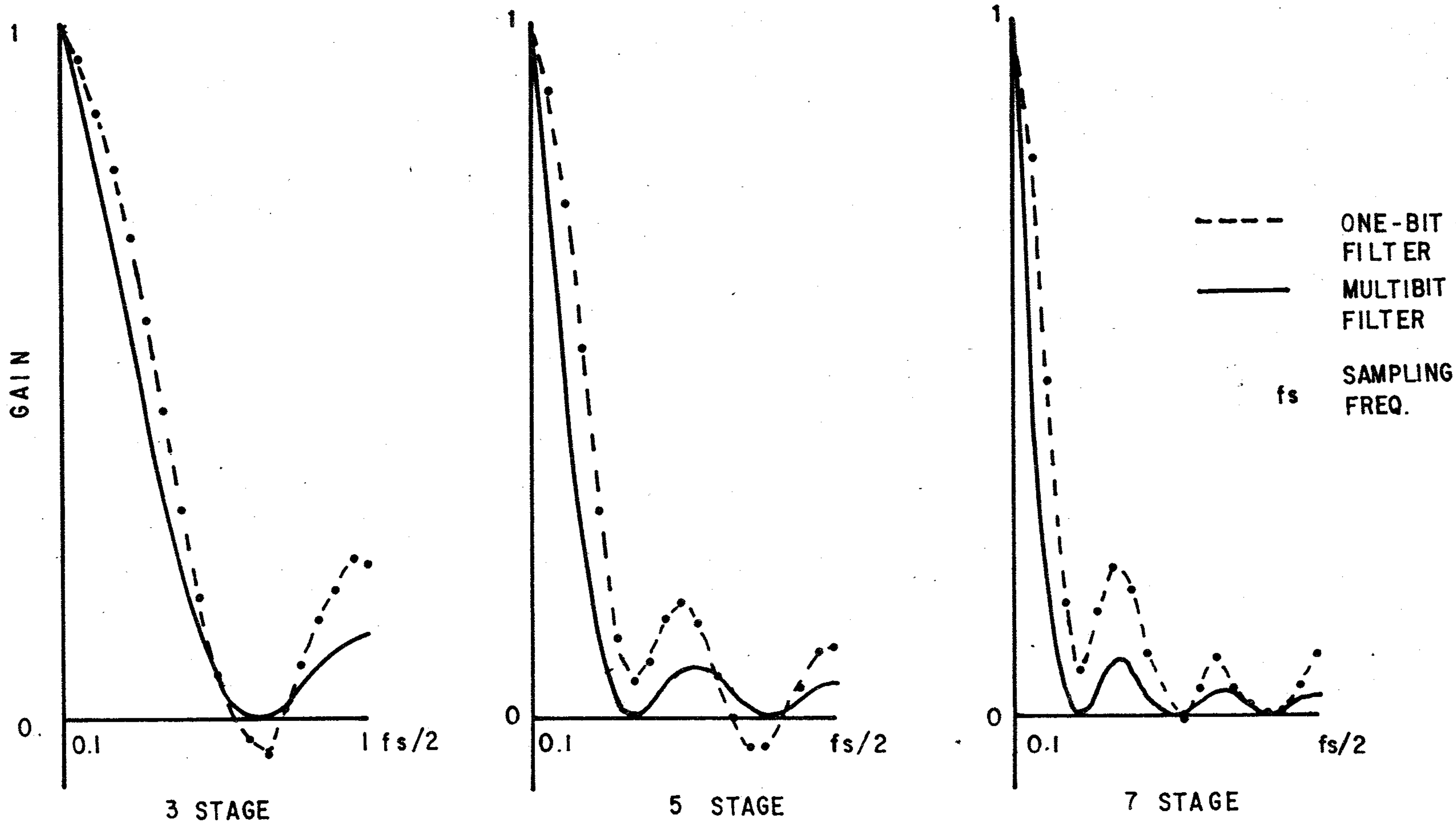



FIG.A.5 FREQUENCY RESPONSE OF MULTIBIT & ONE-BIT DIGITAL FILTERS

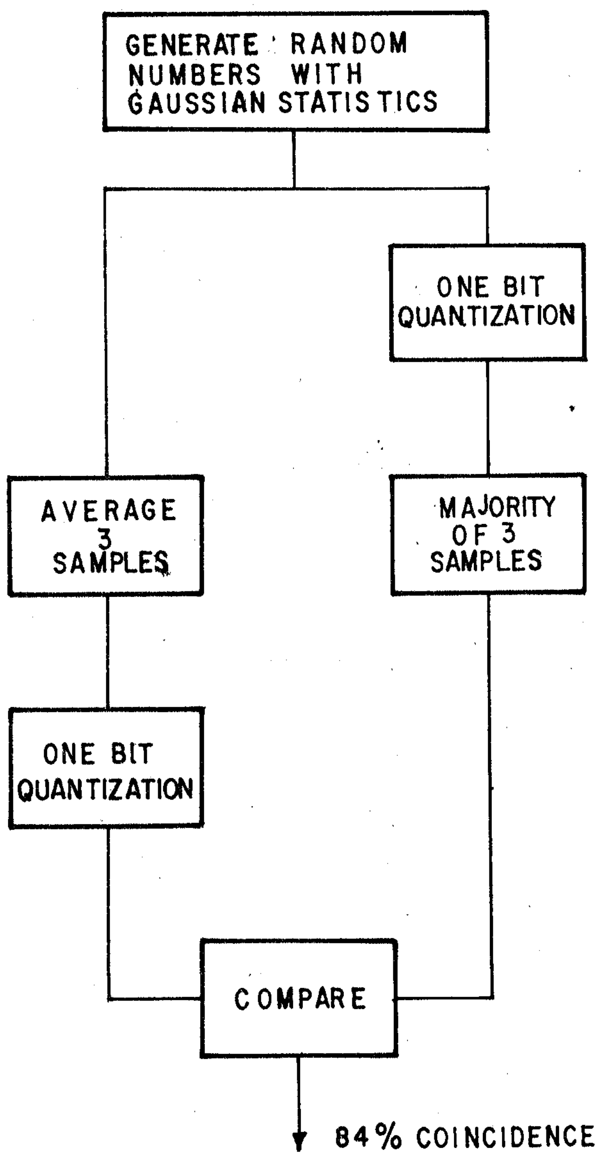
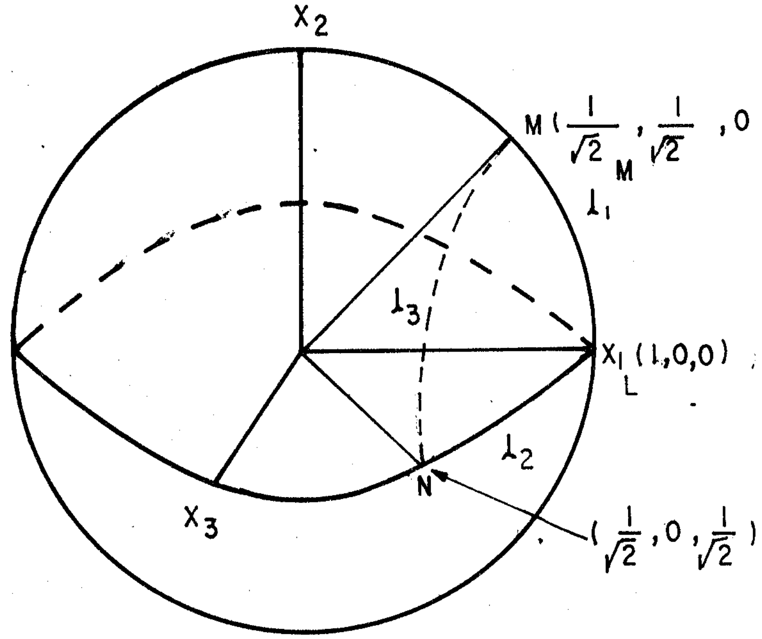



FIG. A.6 SCHEME TO STUDY THE SIMILARITY BETWEEN MULTIBIT AND ONEBIT, 3 STAGE FILTERS

Then the three random variables lie on the surface of a sphere of unit radius. Now it is required to calculate the probability, that $X_1$ has a different sign from $X_2$, $X_3$ and a modulus $> |X_2 + X_3|$. This will be same as $X_1$ having the same sign of $X_2$, $X_3$ and $|X_1| > |X_2 + X_3|$ as $X_1$, $X_2$, $X_3$ are uncorrelated. Let us consider a case when all the three variables are positive. In the $(X_1, X_2)$ plane, $X_3 = 0$ and on the arc $L_1$ shown in the fig. A.7. $X_1 \geqslant X_2$. Similarly, in the $(X_2, X_3)$ plane, $X_1 = 0$ and on the arc $L_2$, $X_1 \geqslant X_3$. On the surface of the sphere, $X_1 \geqslant X_2 + X_3$ is satisfied on the surfce of the spherical triangle bounded by the arcs $L_1 L_2 L_3$. Since the sphere is of unit radius, the ratio of the area of the spherical triangle l,m,n to the area of the sphere gives the probability that $X_1 \geqslant X_2 + X_3$.

$$\frac{\text{Area of the spherical triangle}}{\text{Area of the sphere}} = .027 \qquad \text{(A.4)}$$

Since $X_1$, $X_2$, $X_3$ are uncorrelated all the eight possible combinations of their signs are equiprobable. The two methods of averaging can give different results only when the sign of one of the variables is different from the other two. Six of the eight possible combinations satisfies this requirement.




ANGLE BETWEEN $\lambda_1$ & $\lambda_2$ $= \pi/2$

LENGTHS OF $\lambda_1$ & $\lambda_2$ $= \pi/4$

LENGTH OF $\lambda_3$ $= \pi/3$

AREA OF A SPHERICAL $\Delta^{le}$ = SUM OF ANGLES OF THE $\Delta^{le} - \pi$

$$= \angle \lambda_1 \lambda_2 + \angle \lambda_2 \lambda_3 + \angle \lambda_3 \lambda_1 - \pi$$

$$= \pi/4 + 2 \sin^{-1} \sqrt{2}/3 - \pi$$

$$= 2 \sin^{-1} \sqrt{2}/3 - \pi/2$$

PROBABILITY OF OCCURENCE OF THE EVENT UNDER CONSIDERATION $= \dfrac{\text{AREA OF THE } \Delta^{le}}{\text{AREA OF THE SPHERE}}$

$$= 1/2\pi \ \sin^{-1} \sqrt{2}/3 - 1/8$$

$$= 2.7\,\%$$

FIG.A .7 DERIVATION OF THE PROBILITY THAT $|x_1| \geq |x_2 + x_3|$, WHERE $x_1$ $x_2$ $x_3$ ARE GAUSSIAN RANDOM VARIABLES

Thus the probability of anticoincidences = 6 * .027 (A.5)

or the probability of coincidences = 84%

This is the same number as obtained by computer simulation. This shows that the frequency response of a three stage one-bit filter is not very different from a multibit transversal filter averaging three samples.

## A.2 CORRELATION COEFFICIENT OF SIGNALS AFTER ONE-BIT FILTERING

A one-bit correlator measures the normalized correlation coefficient. This means that identical changes in the bandwidth of signals at the inputs of a one-bit cross correlator do not alter its output. But it can be shown that when one-bit filters are used for filtering the output correlation coefficient changes depending on the number of stages used for filtering. This can be compared to the decrease in the correlation coefficient when one-bit correlators are used for the measurement of the correlation function. By using the Van Vleck relation, one can recover the true correlation coefficient in one-bit correlators. Similarly, it is necessary to determine the correlation coefficient at the output of a one-bit correlator when one-bit filters are used for filtering. For a three stage one-bit filter it can be calculated as follows.

Let $C_1, C_2, \ldots C_n$ and $C'_1, C'_2, \ldots C'_n$ be one-bit Nyquist samples of two bandpass signals ($S_1, S_2$) to be correlated. Let $\rho_c$ be the correlation coefficient after clipping, i.e., the probability that $C'_n = 0$ given that $C_n = 0$ is given by $P = (1+\rho_c)/2$ . Since the samples under consideration are nyquist samples, the successive samples of $S_1$ and $S_2$ are uncorrelated. Thus if one considers a set of three samples $C_n, C_{n-1}, C_{n-2}$, all eight possible combinations of 0's and 1's are equiprobable. Let us consider the output correlation coefficient when these two pulse trains are passed through a 3 stage one-bit filter and then correlated. Let us consider various possibilities.

Case 1. $C_n\, C_{n-1}\, C_{n-2} = 0\ 0\ 0$

Output of the majority finding logic = 0.

The output of the other channel will also be 0, if all the three samples $C'_n, C'_{n-1}, C'_{n-2}$ are zero or only one of them is 1. Given $C_n\, C_{n-1}\, C_{n-2} = 0\ 0\ 0$ .

Probability that $C'_n\, C'_{n-1}\, C'_{n-2} = 0\ 0\ 0$ is equal to $P^3$ (A.6)

and the probability that any one of the samples is 1 is equal to

$$= 3P^2(1-P) \tag{A.7}$$

Thus the probability of coincidence at the output of the filter is given by

$$P^3 + 3P^2(1-P) \tag{A.8}$$

The same argument holds good when all the three successive samples $C_n, C_{n-1}, C_{n-2}$ are 1. Since 0,0,0 and 1,1,1 are equiprobable the probability of coincidence at the output is given by

$$= \frac{2}{8}\left[ P^3 + 3P^2(1-P) \right] \tag{A.9}$$

Case 2: When any one of the bits is 1, say $C_n \ C_{n-1} \ C_{n-2} = 0 \ 0 \ 1$.

Probability that $C'_n \ C'_{n-1} \ C'_{n-2}$ is identical $= P^3$.

Probability that all the three samples $C'_n, C'_{n-1}, C'_{n-2}$ are zero is given by

$$P^2(1-P)$$

Probability that either $C'_n$ or $C'_{n-1}$ is 1 and $C'_{n-2} = 0$ is given by

$$2P(1-P)^2$$

Thus when $C_n \ C_{n-1} \ C_{n-2} = 0 \ 0 \ 1$

Probability of coincidence at the output of the one-bit filter is equal to

$$= P^3 + P^2(1-P) + 2P(1-P)^2 \tag{A.10}$$

The same argument applies when one of the other two bits $C_n$ or $C_{n-1}$ is 1. The same is also true when one of them is zero and the other two bits are 1. Thus the probability of coincidence at the output:

$$= \frac{6}{8}\left[P^3 + P^2(1-P) + 2P(1-P)^2\right] \quad (A.11)$$

Adding all the probabilities, one obtains the probability of coincidence at the output of a 3 stage one-bit filter as:

$$P_3 = P^3 - \frac{3}{2}P^2 + \frac{3}{2}P \quad (A.12)$$

With $P_3$ as the probability of coincidence the correlation coefficient is

$$\rho_3 = 2P_3 - 1 \quad (A.13)$$

Similar equations for a 5 stage and a 7 stage filter can be derived. The probability of coincidence $P_5$ and $P_7$ at the output of a 5 stage and a 7 stage filter are given by

$$P_5 = \frac{1}{8}\left[18P^5 - 45P^4 + 50P^3 - 30P^2 + 15P\right] \quad (A.14)$$

$$P_7 = \frac{1}{16}\left[100P^7 - 350P^6 + 546P^5 - 490P^4 + 280P^3 - 105P^2 + 35P\right] \quad (A.15)$$

The corresponding correlation coefficients are given by:

$$\rho_5 = 2P_5 - 1 \quad (A.16)$$

$$\rho_7 = 2P_7 - 1 \quad (A.17)$$

Fig. A.8 shows the analog correlation coefficient and the corresponding correlation coefficients for a one-bit correlator with no filter and with a 3 stage, 5 stage and a 7 stage one-bit filters.

## A.3 DISCUSSION

1. Using the correction equations described above, one-bit filters can be used to cut down the bandwidth of a signal after infinite clipping.

2. The equations A1.13, 16 and 17 are valid for signals with gaussian statistics. In view of this requirement these filters cannot be used to suppress strong narrow band interference. In the presence of such signals the statistics will no longer be gaussian.

3. The amplitude information of the signal is lost during infinite clipping. The one-bit filter alters the bandwidth. Thus determination of the unnormalized correlation coefficient becomes difficult.

4. Reduction of correlation coefficient deteriorates the SNR. A one-bit correlator suffers a SNR deterioration of 33%. Since one-bit filters reduce the correlation coefficient further it suffers from a higher SNR deterioration. In a one bit correlator, SNR can be improved by over sampling. The same improvement cannot be obtained with one-bit filters, since higher sampling rate requires more number of stages of one-bit filter to obtain the

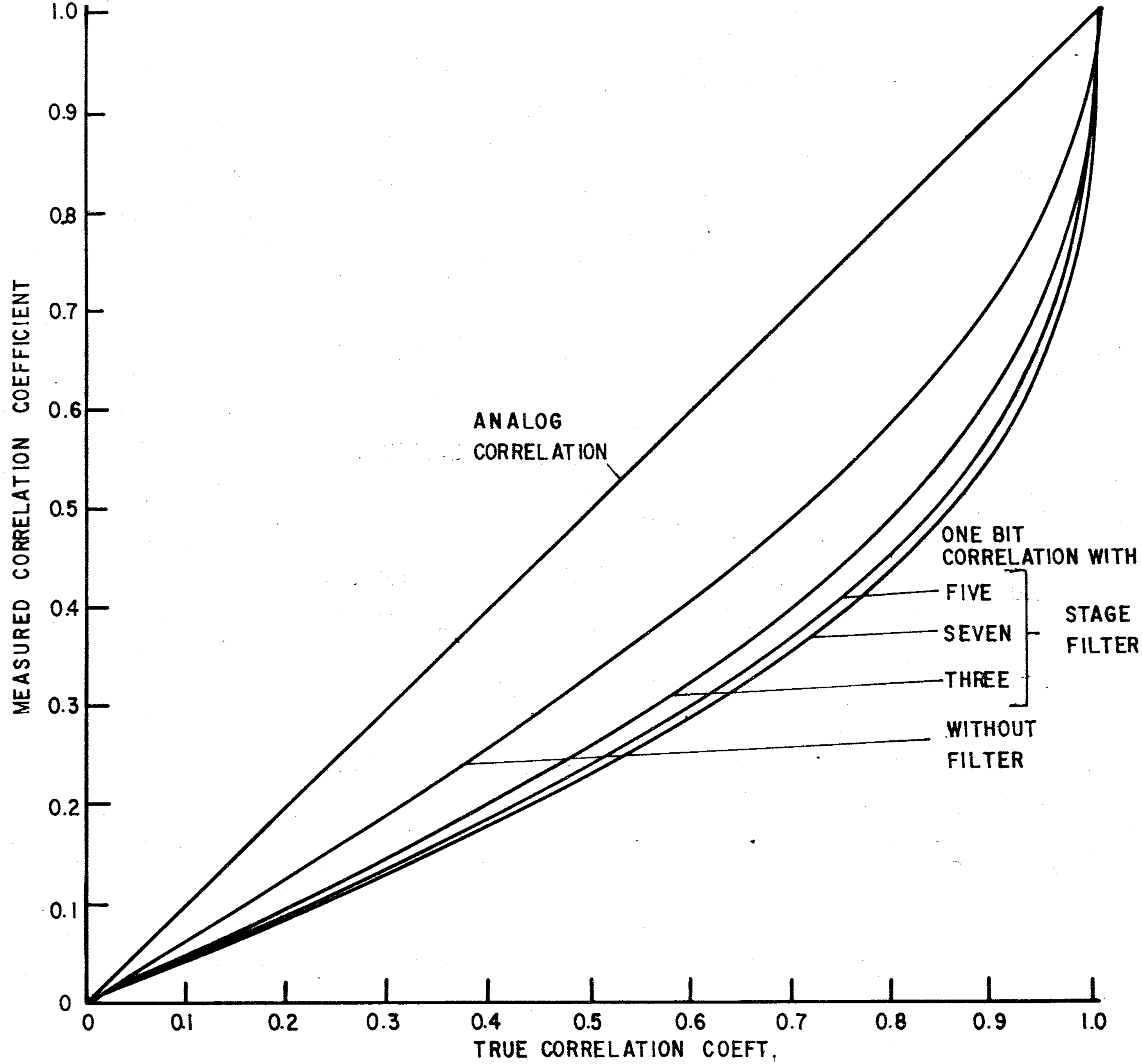



FIG. A.8 TRUE & THE MEASURED CORRELATION COEFFICIENTS desired bandwidth.

5. Thus it is clear that one-bit filters offer simplicity in hardware and can be used in applications, where the signal has gaussian statistics.

## APPENDIX B

## EFFECT OF DISSIMILAR FILTERS ON THE SNR OF ONE BIT CORRELATORS

It was found by R.F.I. monitoring that the increased observation time obtainable by building variable bandwidth filters in each N-S channel is not commensurate with the efforts required to build such filters in each channel (ref. sec 2.6). Thus it was decided to build variable bandwidth filters only for the E-W channel. The effect of dissimilar filters in the path of signals in a one-bit cross correlator on its output is described in section 2.6. The present appendix derives an expression for the statistical error in the measurement of the correlation coefficients for such inputs.

A statistical error, as opposed to an instrumental one, is the uncertainty in the measurements due to the amount of data gathered, the probabilistic nature of the data and the method used in deriving the desired parameter. Suppose $r_e$ is an estimate of a parameter r, If $\overline{r_e} = r$ , $r_e$ is known as an unbiased estimate of $r$. The mean square error of the estimate is defined by

$$\overline{\Delta r_e^2} = \overline{(r_e - r)^2} \qquad \text{(B.1)}$$

$\eta_e$ will be usually a function of the record length T. Then $\eta_e$ is defined to be a a consistent estimate of $\eta$ if:

$$\lim_{T \to \infty} \overline{\Delta \eta_e^2} = 0 \qquad \text{(B.2)}$$

That is, the mean square error decreases as the record length increases

B.1 <u>AN EQUATION FOR THE STATISTICAL UNCERTAINTY IN THE MEASUREMENT OF THE CORRELATION COEFFICIENT</u>

Let us consider two zero mean gaussian processes x(t) and y(t). Let $S_x(f)$ and $S_y(f)$ be their power spectral functions, band limited to a bandwidth B. Let $\sigma_x$ , $\sigma_y$ be the mean square of the two processes. The variance of a random process with zero mean is simply its autocorrelation function R(T) for T = 0. Thus by definition

$$R_x(0) = \int_0^\infty S_x(f)\, df = \sigma_x^2 = \overline{X^2} \qquad \text{(B.3)}$$

$$R_y(0) = \int_0^\infty S_y(f)\, df = \sigma_y^2 = \overline{Y^2} \qquad \text{(B.4)}$$

Let $X_1, X_2, \ldots X_i$ and $Y_1, Y_2 \ldots Y_i$ be the Nyquist samples of the two random processes. Then the cross correlation function for zero delay $R_{xy}(0)$ is defined as

$$R_{xy}(0) = \overline{X_i \cdot Y_i} \qquad \text{(B.5)}$$

The normalised correlation coefficient $\rho_{xy}(0)$ is defined as

$$\rho_{xy}(0) = R_{xy}(0) / \sigma_x \sigma_y \tag{B.6}$$

Let $\rho_{exy}$ be the estimate of $\rho_{xy}$ obtained from observations over a time T. Then the number of sample points observed (N) is given by

$$N = 2BT \tag{B.7}$$

Then $\rho_{exy}(0)$ is given by

$$\rho_{exy}(0) = \sum_{i=1}^{N} X_i Y_i / N \sigma_x \sigma_y \tag{B.8}$$

Now we are interested in the variability $\overline{\Delta \rho_{exy}^2}$ defined by

$$\overline{\Delta \rho_{exy}^2(0)} = \overline{[\rho_{exy}(0) - \rho_{xy}(0)]^2} \tag{B.9}$$

$$= \overline{\rho_{exy}^2(0)} - \rho_{xy}^2(0) \tag{B.10}$$

$$= \frac{\sum_{i=1}^{N} \sum_{j=1}^{N} X_i Y_i X_j Y_j}{N^2 \sigma_x^2 \sigma_y^2} \tag{B.11}$$

Using the relation

$$\overline{X_1 X_2 X_3 X_4} = (\overline{X_1 X_2})(\overline{X_3 X_4}) + \cdots \tag{B.12}$$

Where $X_1, X_2, X_3, X_4$ are four zero mean random variables which follow a four dimensional gaussian distribution (Laning,1956). One can simplify the equation B.11 to

$$\Delta\rho_{exy}^2(0) = (1+\rho_{xy}^2(0))/N \qquad (B.13)$$

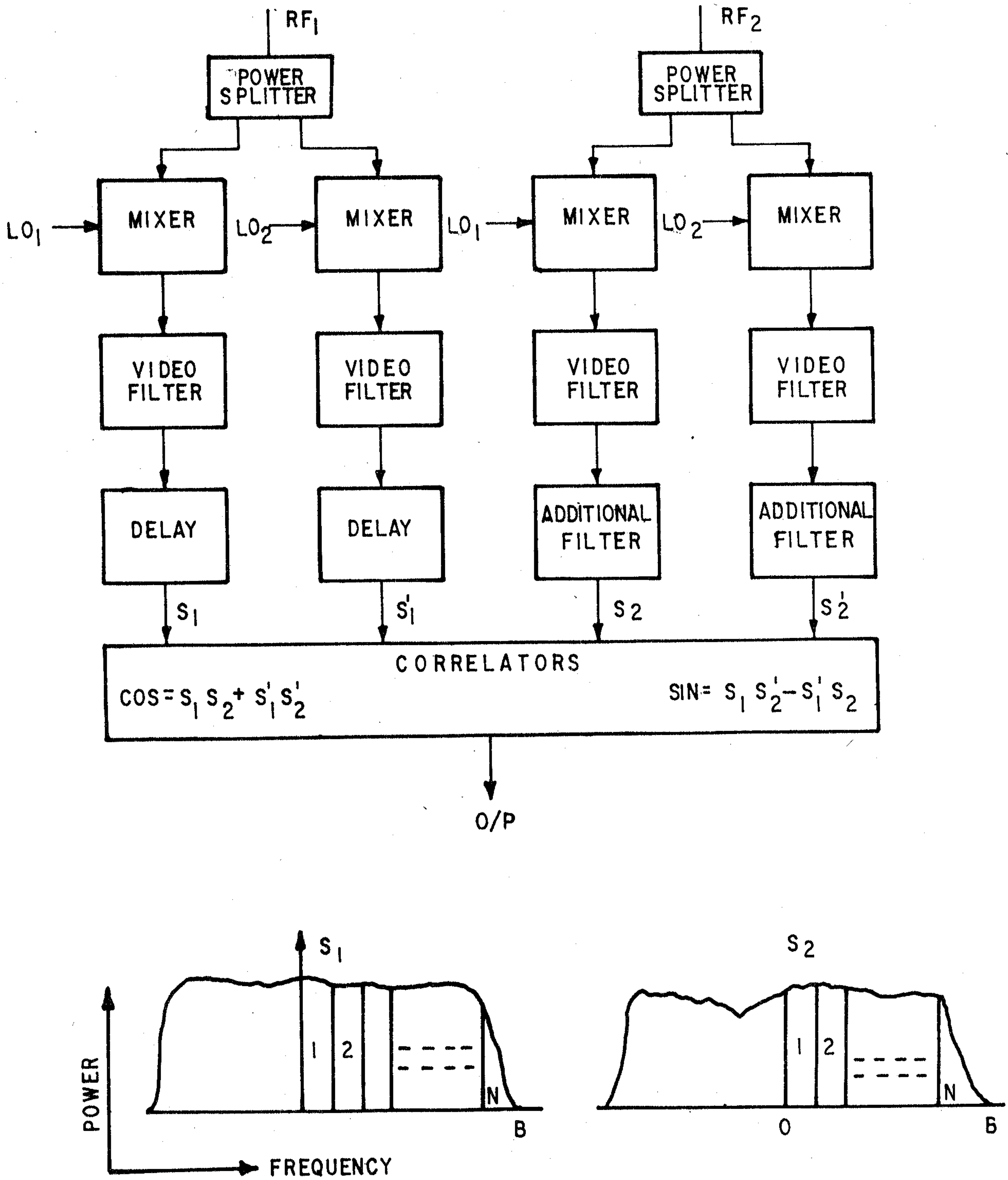
### B.1.1 Statistical Uncertainty When The Signal Bandwidths Are Different

Let $S_x(f)$ and $S_y(f)$ have different band shapes. The normalised correlation coefficient between the two random processes can be considered as the sum of the correlations between infinitesimally small channels into which each band can be split (Ref Fig. B~~A~~2.1). Only the channels with identical frequency components correlate. If each band can be split into N channels, $\rho_{exy}(0)$ can be written as

$$\rho_{exy}(0) = \frac{\text{Sum of the correlations between similar frequency channels on either side}}{\sigma_x \sigma_y} \qquad (B.13)$$

$$= \frac{\sum_{n=1}^{N} \rho_n(f)\,\sigma_{xn}(f)\,\sigma_{yn}(f)}{\sigma_x \sigma_y} \qquad (B.14)$$

$$= \frac{\sum_{n=1}^{N} \rho_n(f)\sqrt{S_{xn}(f)\,S_{yn}(f)}}{\sigma_x \sigma_y} \qquad (B.15)$$




$\sigma_{nf}$ = RMS VALUE OF $n^{th}$ FREQUENCY CHANNEL

$S_{nf}$ = POWER OF $n^{th}$ FREQUENCY CHANNEL

$\rho_{nf}$ = CORRELATION BETWEEN $n^{th}$ FREQUENCY CHANNELS

FIG. B.1 DOUBLE SIDE-BAND SYSTEM WITH DISSIMILAR FILTERS.

Statistical uncertainty can now be written as the sum of statistical uncertainties of each channel since they are independent. Thus

$$\Delta \rho^2_{exy}(0) = \frac{\sum_{n=1}^{N} S_{xn}(f)\, S_{yn}(f)\, df^2\, \overline{\Delta \rho_n^2(f)}}{\sigma_x^2\, \sigma_y^2} \tag{B.16}$$

Substituting

$$\Delta \rho_n^2(f) = \frac{1 + \rho_n^2(f)}{2\, df \cdot T} \tag{B.17}$$

and replacing the summation by an integral assuming the bands to be divided into inifinitely large number of channels

$$\Delta \rho^2_{exy}(0) = \frac{\int S_x(f)\, S_y(f)\, \frac{1 + \rho^2(f)}{2T} \cdot df}{\sigma_x^2 \cdot \sigma_y^2} \tag{B.18}$$

If the correlation coefficient is independent of the frequency channel

$$\Delta \rho^2_{exy}(0) = \frac{1 + \rho^2}{2T \sigma_x^2 \sigma_y^2} \int S_x(f)\, S_y(f)\, df \tag{B.19}$$

$S_x = S_y = 1$ for $f \leq B$ and 0 for $f > B$

$$\Delta \rho^2_{exy}(0) = \frac{1 + \rho^2}{2BT}$$

This is the same as equation B.12.

### B.1.2 Phase Distortion And A D.S.B. System

In radio astronomy one measures both Sine and Cosine correlations (ref.section 1.3). The present reciever system uses a DSB system. The previous section considered a case when the bandshapes are different. In actual cases, the two filters also introduce a phase difference between the two signals which is a function of the frequency. Let us consider two band pass signals centred around $\omega_c$ .

$$RF_1 = U_1 \cos[(\omega_c+\omega)t+\varphi_1] + L_1 \cos[(\omega_c-\omega)t+\varphi_2] \quad (B.20)$$

$$RF_2 = U_2 \cos[(\omega_c+\omega)t+\varphi_3] + L_2 \cos[(\omega_c-\omega)t+\varphi_4] \quad (B.21)$$

To completely represent the band the R.H.S. must be summed for different values of $\omega$ say from -B/2 to B/2 if B is the R.F. bandwidth. If these two signals are multiplied in Cosine and Sine correlators, the outputs will be,

Cos correlator output C =

$$U_1 U_2 \cos(\varphi_1-\varphi_3) + L_1 L_2 \cos(\varphi_2-\varphi_4) \quad (B.22)$$

and the Sine correlator output S =

$$U_1 U_2 \sin(\varphi_1-\varphi_3) + L_1 L_2 \sin(\varphi_2-\varphi_4) \quad (B.23)$$

In the present receiver system the R.F. signal can be considered to have been processed as shown in Fig. B.1 .. Let the two LO's be

$$LO_1 = \cos \omega_c t \quad ; \quad LO_2 = \cos(\omega_c t + \pi/2) \qquad (B.24)$$

The output of the video filters shown in the figure are

$$S_1 = U_1 \cos(\omega t + \varphi_1) + L_1 \cos(-\omega t + \varphi_2) \qquad (B.25)$$

$$S_1' = U_1 \cos(\omega t + \varphi_1 - \pi/2) + L_1 \cos(-\omega t + \varphi_2 - \pi/2) \qquad (B.26)$$

$$S_2 = U_2 \cos(\omega t + \varphi_3) + L_2 \cos(-\omega t + \varphi_4) \qquad (B.27)$$

$$S_2' = U_2 \cos(\omega t + \varphi_3 - \pi/2) + L_2 \cos(-\omega t + \varphi_4 - \pi/2) \qquad (B.28)$$

Now one of the signals passes through a filter which introduces a gain g and a phase $\varphi$ which is a function of the frequency Thus modified, $S_1$ and $S_1'$ may be written as

$$S_1 = g U_1 \cos(\omega t + \varphi_1 + \varphi) + g L_1 \cos(-\omega t + \varphi_2 - \varphi) \qquad (B.29)$$

$$S_1' = g U_1 \cos(\omega t + \varphi_1 + \varphi - \pi/2) + g L_1 \cos(-\omega t + \varphi_2 - \varphi - \pi/2) \qquad (B.30)$$

By assuming that U.S.B. and L.S.B. are identical in the case under consideration after multiplication and integration the Cosine and Sine correlator outputs will be

$$C' = g\, C \cos \varphi \qquad (B.31)$$

$$S' = g\, S \cos \varphi \qquad (B.32)$$

Thus $\rho_c$ and $\rho_s$ , the total Cos and Sin correlations for the entire band can be written as

$$\rho_c = \frac{\int g\, C \cos\varphi \sqrt{S_x S_y}\, df}{\sigma_x\ \sigma_y} \quad \text{(B.33)}$$

$$\rho_s = \frac{\int g\, S \cos\varphi \sqrt{S_x S_y}\, df}{\sigma_x\ \sigma_y} \quad \text{(B.34)}$$

Thus, if the phase distortion can be kept to a minimum, the statistical uncertainty will be less. Let us consider a case, where the bandwidths of x(t) and y(t) are (B,B), (B,B/2) and (B/2,B/2), and a linear phase filter whose phase characteristics can be obtained by a simple delay. If the delay is inserted in the path of the signal which has no filter $\varphi = 0$.

Case 1

For bandwidths B on either side,

$$\rho_c = \frac{\int C \sqrt{S_x S_y}\, df}{\sigma_x\ \sigma_y} \quad \text{(B.35)}$$

$$\overline{\Delta \rho_c^2} = (1 + C^2)/2BT \quad \text{(B.36)}$$

$$SNR = \rho_c^2 / \overline{\Delta \rho_c^2} = (C^2/1+C^2) \cdot 2BT \quad \text{(B.37)}$$

Case 2

For bandwidths B,B/2

$$\rho_c = C/\sqrt{2} \tag{B.38}$$

$$\overline{\Delta\rho_c^2} = (1+C^2)/2BT \tag{B.39}$$

$$SNR = (C^2/1+C^2)\cdot BT \tag{B.40}$$

Case 3

For bandwidths B/2,B/2

$$\rho_c = C \tag{B.41}$$

$$\overline{\Delta\rho_c^2} = (1+C^2)/BT \tag{B.42}$$

$$SNR = (C^2/1+C^2)\cdot BT \tag{B.43}$$

Thus in this oversimplified case, it is seen that even though the signal strength decreases when the two bandwidths are unequal the SNR ratio does not deteriorate when compared to a case where both the bandwidths are decreased.

It was found that for 3 to 4 stage butterworth filters, the phase can be compensated by simple delays. R.M.S. phase errors of $15^\circ$-$25^\circ$ were found very easy to attain .This decreases the SNR by negligeble amounts.

APPENDIX C

WIGHTING SCHEME FOR CAS A ABSORPTION LINE

The spectral line observed in the direction of Cas A (ref. sec. 4.91) is due to an absorbing cloud in front of the strong source. The parameters observable in such an observation are as shown in Fig. C.1 . A spectral line appears as a small perturbation over an underlying intensity due to the background source and the receiver temperatures.

If a wave of intensity $S_1$ passes through an absorbing medium, the intensity of the wave after absorption is given by

$$S = S_1 e^{-\alpha X} \tag{C.1}$$

where $\alpha$ = Attenuation constant m

$X$ = depth of the absorbing medium.

The product $\alpha X$ is called the optical depth and is commonly designated by the symbol $\tau$. At decametric wavelengths the background radiation predominates the system temperature and fills the beam. Thus the observed brightness on and off the spectral line frequencies are given by

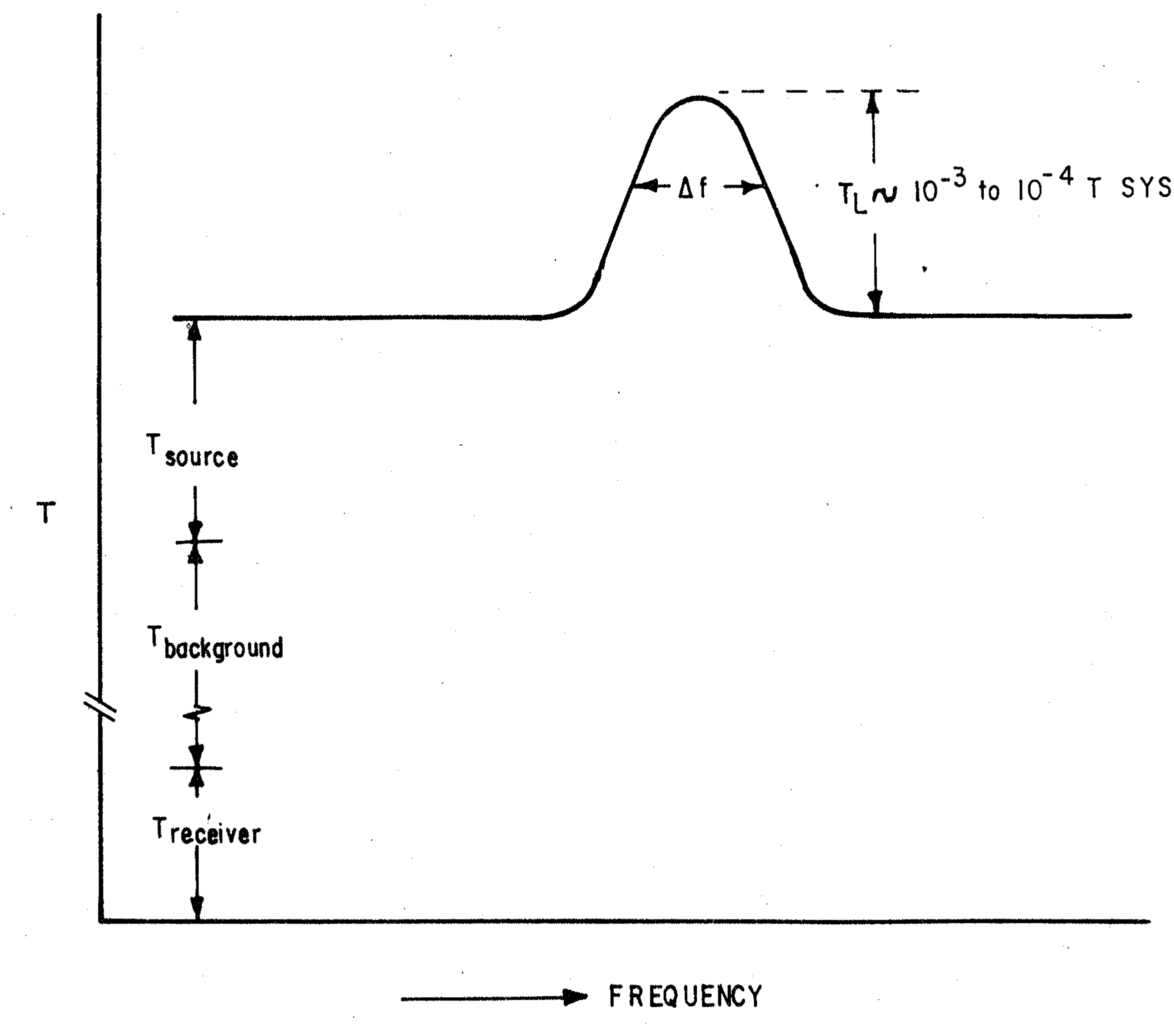



FIG C.1 A TYPICAL SPECTRAL LINE

$$\text{off line} = T_B + T_S \tag{C.2}$$

$$\text{on line} = T_B + T_S \cdot e^{-\tau} \tag{C.3}$$

where $\tau$ = optical depth in the direction of the source.

A DSB line receiver measures (U-L) and (U+L) (ref. section 2.9) (U-L)/(U+L) represents,

$$\tau_{(\text{Measured})} = \frac{\text{ON LINE-OFF LINE}}{\text{OFF LINE}} \tag{C.4}$$

If the spectral line is present only in the U.S.B.

$$\tau_M = \frac{(T_B + T_S e^{-\tau}) - (T_B + T_S)}{T_B + T_S} \tag{C.5}$$

$$= \frac{1 - e^{-\tau}}{1 + \frac{T_B}{T_S}} \tag{C.6}$$

for small values of $\tau$ $1 - e^{-\tau} \approx \tau$

$$\tau = \tau_M \left(1 + \frac{T_B}{T_S}\right) \tag{C.7}$$

For bright sources like Cas A, $T_S$ predominates background and

$$\tau \approx \tau_M \tag{C.8}$$

For the present observation N-S arrray was used. It has a widebeam in the E-W direction and the source contribution to the system temperature varies with time. The noise ($\Delta$T) on the measurement is proportional to (ref.

section 1.2)

$$\Delta T \propto \frac{T_S + T_B}{\sqrt{B \Delta T}} \qquad (C.9)$$

where B = bandwidth used

$\Delta T$ = time of observation

As $T_S$ varies, noise on the measurement also varies. Thus to get the best estimate of $\tau$ absorption spectra at different t, should not be averaged with equal weights. If $T_B(t)$ and $T_S(t)$ are the background and source contribution to the system temperature at any time t; then $\tau(t)$ is given by

$$\tau(t) = \tau_{Meas}\left[1 + \frac{T_B(t)}{T_S(t)}\right] \qquad (C.10)$$

An estimate of $\tau$ with $\tau(t)$ weighted by a weighting function $\omega(t)$ is given by

$$\tau_{estimate} = \frac{\int \omega(t)\,\tau(t)\,dt}{\int \omega(t)\,dt} \qquad (C.11)$$

$\tau$ estimate will be the best estimate if $\omega(t)$ is chosen to minimise the noise on it. The variance on the measurement of $\tau$ estimate is given by

$$\overline{\Delta \tau^2_{estimate}} = \frac{Const. \int \omega^2(t)\left[1 + \frac{T_B(t)}{T_S(t)}\right]^2 dt}{\left[\int \omega(t)\,dt\right]^2} \qquad (C.12)$$

The variance will be minimum if w(t) satisfies the relation

$$\delta\left[\overline{\Delta\tau_e^2}\right] \Big/ \delta\left[\omega(t)\right] = 0 \qquad \text{(C.13)}$$

This condition leads to the equation

$$\omega(t) \propto \left[\frac{T_S(t)}{T_S(t) + T_B(t)}\right]^2 \qquad \text{(C.14)}$$

Thus the weighting function automatically goes to zero when the source is not in the beam.

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