Chapter 4

NEUTRON STAR ROTATIONAL DYNAMICS

4.1 Introduction : Some consequences of the pinning between fluxoids and vortices

The rotational dynamics of a neutron star, as for any other system, is decided by its structure: its constituents and how they interact and couple to each other. The standard picture might be sketched as follows.

The neutron superfluid in the interior (ie. the core) of a neutron star coexists with superconducting protons and "normal" degenerate electrons. The interior charged plasma including the lattice of proton vortices (fluxoid) is expected to be strongly coupled to the lattice of nuclei and the electrons in the crust due to the strong magnetic field present, with a coupling time scale ≤ 10 s as described in § 3.2.1. All the charged components of the star are hence assumed to be always co-rotating and will be referred to as the "crust". An isolated neutron star is subject to an "external" torque, the dipole torque (cf. § 1.4), which acts on its magnetic field and hence on the "crust". The core superfluid would be influenced by the "external" torque and driven to follow the spin behavior of the crust through their mutual coupling mechanism that would operate between the "crust" and the superfluid vortices (cf. § **4.3**). The "external" force exerted by the "crust" on the neutron vortices (responsible for either spin-up or spin-down of the core superfluid) is usually considered to be due to scattering of the electrons off the vortex cores. The dominant scattering effect is that of the induced magnetization of the vortices caused by a "drag" between the proton and neutron condensates. The associated vortex velocity relaxation time scale $\tau_{v-e} \sim 10P_s - 20P_s$, where P_s is the spin period of the star.

Moreover, in the quantum liquid interior of a neutron star a neutron vortex is expected to "pin" to a fluxoid should the two structures overlap; an effect which we have explored in previous chapters in our study of the magnetic field evolution in neutron stars (cf. Fig. 1.4, § 1.2, § 2.2, § 2.2.1, § 3.1, and § 3.2). This is in fact the ***point*** of *departure* from the existing theories for our study on the rotational dynamics of neutron stars in the present chapter.

To recall, the strength of the pinning energy barrier which would impede any relative crossing motion between the fluxoids and the vortices is estimated to be $E_P \sim 0.1$ – 1.0 MeV. The mechanism of the pinning is associated with either the proton density perturbations or the magnetized nature of both the vortices as well as the fluxoids. The effective length scale of the pinning interaction d_P is considered to be of the order of the coherence length ξ_p or the London penetration depth λ_p of the proton superconductor for the above two pinning mechanisms, respectively. The effective pinning force (per intersection) $f_P \sim \frac{E_P}{d_P}$ is therefore expected to be roughly equal for the two interaction mechanisms since the value of E_P due to the magnetic interaction is larger than that of the density perturbation by the same ratio as their corresponding d_P values is smaller, ie. $\frac{\lambda_P}{\xi_P} \sim 10$.

The consequences of pinning between the vortices and the fluxoids with respect to the *radial* motion of the vortices have been already discussed in the literature. The pinning would however also impede any relative *azimuthal* motion of the two lattices of the vortex lines. As the fluxoids are expected to be rigidly co-rotating with the rest of the "crust", the pinning energy barriers would therefore act as a source of an azimuthal frictional force which will try to restore co-rotation between the vortices and the "crust". *This is a new effect of the pinning between the vortices and the fluxoids in the core of a neutron star which is addressed here for the first time.* In addition to the viscous drag force on the magnetized neutron vortices due to scattering of the electrons we are invoking a different frictional force on the vortices caused by their "collision" with the fluxoids (the fluxoid "scattering" mechanism). Both the mechanisms will try to restore the state of co-rotation between the vortices and the fluxoids, viz. between the neutron vortices and the "crust". This is how the bulk superfluid communicates with the "crust" and feels the applied torque on itself (cf. § **4.3).** It is recalled that the "crust" consists of the actual crust of the star plus the charged components, ie. protons and electrons, in the core as well as the fluxoids all corotating together.

In this chapter we discuss the rotational dynamics of neutron stars taking into account also the above effects (both radial, as well as azimuthal) of the pinning of the vortices in the core of the star. The observed glitches in radio pulsars provide unique "laboratory tests" for the ideas about the interior of neutron stars. Any study of the rotational dynamics of these stars would naturally concern at least partly, if not completely, to the phenomenon of glitch. **A** brief review of the important aspects of the observational data on the glitches and the related theoretical models is therefore given in Section 4.2.1. Section 4.2.2 then addresses *a problem,* a dilemma with the data on the observed post-glitch relaxations of radio pulsars. The data indicates a fraction of the moment of inertia of the star much larger than that of the crust of the star to be involved in the relaxation, hence calling for a *pinned* superfluid in the core. This is particularly contradicting glitch theories which try to explain the glitch recovery in terms of only a superfluid component in the *crust* of neutron stars. However a pinned core superfluid also might seem to be ruled out by the data, since it would generally be expected to remain *decoupled* over time scales much larger than that observed !.

Our results for the behavior of the core superfluid are presented in Section **4.3,** after giving a brief account of how a *superfluid* become aware of the presence of a torque acting on it **!.** In 4.3.1 the expected steady state relative rotation of the different components of a neutron star are described, distinguishing between the cases of *pinned* and *unpinned* vortices in the core superfluid. The steady state value of the rotational lag between the superfluid and the vortices in a spinning down neutron star is then discussed. The *pinned* core superfluid is argued to take part in the steady state slowing down of the star, hence ruling out doubts expressed regarding the inferred strengths of the magnetic fields of pulsars on the account of a pinned core superfluid for the star. The vortex velocity relaxation time scale caused by the fluxoid "scattering" which is found to be much shorter than that of the electron scattering in all cases of interest, is derived in Section **4.3.2.** The dynamical time scale for the coupling of rotation of the core superfluid (consisting the dominant fraction of the star's moment of inertia) to the "crust" is discussed in Section 4.3.3. The response of the core superfluid to a jump in the rotation frequency of the "crust" is argued to be different for the pinned and the unpinned cases, and also would depend on the magnitude of the jump for the pinned superfluid. A necessary modification in the existing derivation for the dynamical time scale due to Alpar & Sauls (1988) is pointed out which results in shorter time scales even considering the vortex relaxation by the electron scattering alone.

In Section **4.4** some consequences of the predicted behaviour of the core superfluid as applied to the glitches are presented. The post-glitch recovery behavior expected according to the views advanced here is presented in Section **4.4.1.** Although the predictions are not quantitative in all details they nevertheless serve to resolve the dilemma with the glitch data discussed in Section 4.2.2. The key issue here would be to appreciate the particular nature of the pinning sites in the core of neutron stars. The pinned superfluid in the core of a neutron star is finally shown in Section 4.4.2, to lead to a new possibility for the *triggering cause* of the glitches being driven by the long-term evolution of the superfluid rotational lag. The proposed glitch-inducing "jumping lag" model is further argued to be also consistent with other observed features of the glitches in radio pulsars, and might have solutions for some of the yet unsolved problems in the data.

4.2 Glitches

4.2.1 A Review

Glitches are observed in radio pulsars as sudden changes $\Delta\Omega_c$ in Ω_c followed by a recovery or relaxation back towards the pre-glitch behavior of Ω_c over time scales of days to years (Radhakrishnan & Manchester 1969, Lyne 1995). The observed values of the jump $\frac{\Delta\Omega_c}{\Omega_c}$ range between 10^{-9} and 10^{-6}. In addition to the amplitude of the jump and the recovery time scale τ_R, a third characteristic quantity is the recovery amplitude expressed in terms of a percentage recovery factor Q. **A** glitch function representing the time evolution of the observed rotation of the star subsequent to a jump $\Delta\Omega_0$ at $t = 0$ is thus defined (see Fig. 4.1) as

$$\Omega_c(t) = \Omega^{no}(t) + \Delta\Omega_0 \left[Q e^{-\frac{t}{\tau_R}} + 1 - Q \right] \tag{4.1}$$

where $R^{nO}(t)$ is the extrapolated pre-glitch value of the rotation frequency. Such a recovery behavior has been explained on the basis of a 2-component model of the neutron star (Baym et al. 1969). The observed behavior of various pulsars after a glitch have been however different (see Fig. 4.2), requiring very different values of the parameters to fit the form of Eq. 4.1. In cases where detailed post-glitch observations are available, as for the Crab and Vela pulsars, more than one exponential component have been necessary to fit the data (Flanagan 1995; Lyne, Graham Smith & Pritchard 1992). Different values for the recovery factor Q have been observed which seem to be also correlated with the age of the pulsar (see Fig. 4.3). In younger pulsars the jump in Ω_c is accompanied by an increase in the spin-down rate $\dot{\Omega}_c$ both of which approach their pre-glitch values over the following months ($Q \sim 1$). For the older pulsars on the other hand there is very little recovery in $\Delta\Omega_c$ over years ($Q << 1$), and the small amount of the recovery depends inversely upon the characteristic age of the pulsar (see Fig. 4.3). The Crab pulsar has however been unique in that part of the jump in the $\dot{\Omega}_c$ remains persistently, hence the recovery in Ω_c overshoots $R^{nO}(t)$, $Q > 1$! (see Fig. 4.2). The spin-down rate of the Crab pulsar has consequently increased by a total fraction of $\sim 10^{-3}$ over a period of 23 years of the observations (Lyne et. al. 1992). Moreover, the

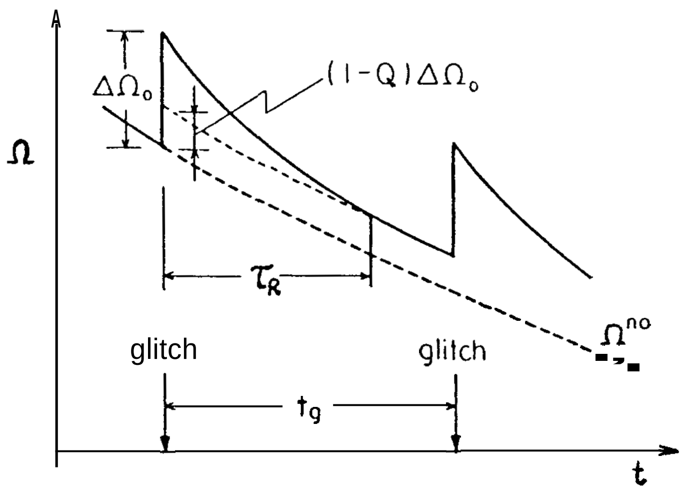

Figure 4.1- The typical behavior of a pulsar rotational frequency at a glitch. The jump at the glitch and its subsequent relaxation is shown by the solid line, while the dashed line represents the linear extrapolation of the pre-glitch behavior which is used to define the recovery factor Q. The recovery time scale τ_R and the interval between successive glitches t_g are indicated. [from Takatsuka & Tamagaki 1989]

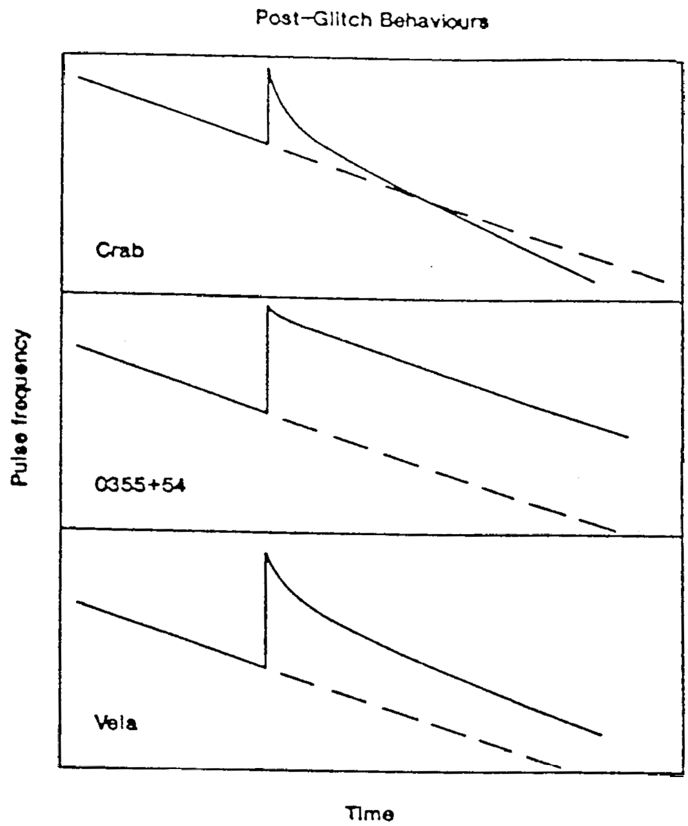

Figure 4.2- Schematic diagram illustrating the different observed post-glitch behaviors, not to scale, of the rotation frequency for the Crab pulsar, PSR 0355+54, and the Vela pulsar. [from Manchester 1992]

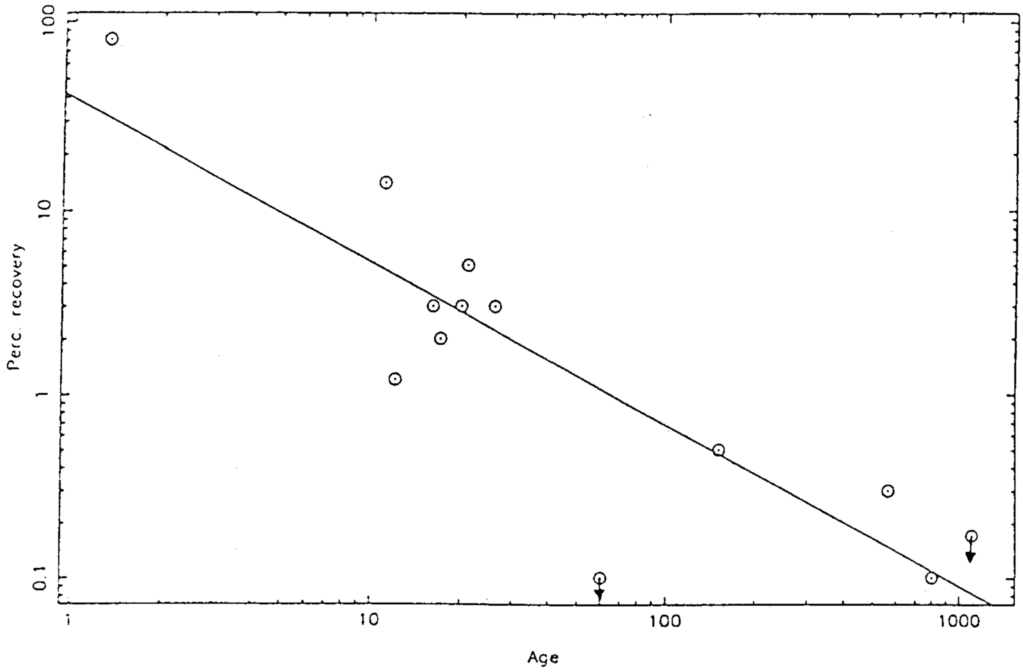

Figure 4.3- The percentage recovery of the initial frequency jump for the different observed glitches in pulsars, as a function of the pulsar characteristic age in units of thousand years. [from Lyne 1995] observations of 1989 glitch in Crab has also revealed another unusual recovery behavior which could be fitted with two exponentially *increasing* components of Ω_c with time scales of 0.8 day and 265 day, in addition to a normal decaying component with a time scale of 18 day. The 0.8 day component, in particular, might be an extraordinary case showing the onset of the rising up of Ω_c at a glitch which has been otherwise unresolved in all other observed events, although the interpretation of the data is not unique (Lyne et. al. 1992).

Since the radio emission of a pulsar is thought to be linked with the magnetic field of the star which is in turn assumed to be frozen in the crust the observed jump at a glitch is hence interpreted as that of the crust rotation rate. Glitches, on the other hand, have not shown any correlation with the electromagnetic signature (intensity, polarization, pulse profile, etc.) of a pulsar. The *cause,* as well as the *recovery behavior* observed after a glitch are therefore *attributed to mechanisms related to the internal structure of the star.* In addition to the starquake mechanism (Baym et. al. 1969) models based on the properties of the superfluid part of the crust have offered, independently or combined with a starquake component, successful explanation for many aspects of the observational data of glitches in many pulsars. The starquake, invoked for both the crust or a possible solid core in a neutron star, may of course offer by itself an explanation only for the triggering cause of a glitch; the observed behavior at the jump and the post-glitch recovery depend further on the assumed dynamical coupling mechanism between different components of the star.

Different models of the glitch recovery, including the vortex creep theory, have indeed focused attention on the implications of a superfluid component in the *crust* of neutron stars (Pines & Alpar 1985; Jones 1991; Epstein, Link & Baym 1992). The rest of the star including the core superfluid is assumed in these models to be co-rotating on time scales larger than few minutes. This has been motivated largely by the observed values of $\frac{\Delta\dot{\Omega}_c}{\Omega_c}$ at the glitches which in most, *but not all* !, cases amount to values in the range of 10^{-3}– 10^{-2}; curiously similar to the fractional moment of inertia of the superfluid component in the crust with respect to the total moment of inertia of the star (Alpar 1992). The properties of a "pinned" superfluid component in the crust have also been considered in connection with the driving cause of the glitches. A sudden release and rapid outward motion of a large number of the pinned superfluid vortices is suggested as the source of the angular momentum given to the crust at a glitch which is observed as the jump in Ω_c. Suggested mechanisms for the sudden release of a large number of otherwise pinned vortices include "catastrophic" unpinning due to an intrinsic instability, breaking down of the crustal lattice by magnetic stresses, and thermal instability resulting in increase in the mutual friction between the vortices and the superfluid (Anderson & Itoh 1975; Greenstein 1979; Ruderman 1976; Link & Epsein 1996).

In contrast, the core superfluid has not played any interesting role in the glitch theories, particularly in the more recent years. **A** main objective of this chapter is to point out the interesting consequences of the core superfluid with respect to the glitches.

4.2.2 The Problem

The *pinning* of the superfluid vortices in the interior of a neutron star might seem to be in contradiction with the observational data on glitches in various pulsars. The observed post-glitch values of $\frac{\Delta\dot{\Omega}_c}{\dot{\Omega}_c} \lesssim 1$ after a few minutes indicate that the interior superfluid has already coupled to the crust. In contrast, the coupling of a pinned superfluid in the core for it to follow the spinning down of the crust is usually expected to start only after the recovery of the superfluid rotational lag w to its critical value w, : $w \rightarrow w$,,. This should take more than tens of days for the typical spin-down rates of pulsars and the expected values of w, (cf. § 4.3.1). On the other hand, a fast coupling of the core superfluid in the *absence* of *any pinning* poses difficulties in understanding some of the post-glitch observations which have revealed values of $\frac{\Delta\dot{\Omega}_c}{\dot{\Omega}_c} \sim 10\%–60\%$ on time scales of few hours to tens of days. Recent glitches in Vela have shown a component of $\frac{\Delta\dot{\Omega}_c}{\dot{\Omega}_c} > 10\%$ which recovers, over time scales of few hours (Flanagan 1995). Also, values of $\frac{\Delta\dot{\Omega}_c}{\dot{\Omega}_c} \gtrsim 10\%$ have been observed in the case of PSR 0355+54, with a recovery time scale of $\sim$ 44 days (Lyne 1987). The data hence indicate that a part of the star with a fractional moment of inertia $\gtrsim 10\%$ is involved in the observed post-glitch response of the star. This is in contradiction with the expectations based on the vortex creep model, and any other model, in which only the superfluid in the crust of a neutron star plays a role during the post-glitch recovery; the core superfluid being assumed to be coupled to the crust over time scales of minutes. It is noted that the moment of inertia $\mathbf{I}_s$ of the crust superfluid is expected to be $\sim 10^{-3} - -10^{-2} \times \mathbf{I}$ (Alpar 1995), where $\mathbf{I}$ is the total moment of inertia of the star; much smaller than being responsible even for values of $\frac{\Delta\dot{\Omega}_c}{\Omega_c} \sim$ few %. While for the moment of inertia I_s of the core superfluid $\frac{I_s}{I} \sim 90$ %; stressing the obvious point that the latter side of the problem (ie. large values of post-glitch spin-down rates) does not concern a model in which the core superfluid takes part in the post-glitch recovery behaviour of the star.

Thus, in some cases (which are increasing in number as more glitches are being observed soon after they take place) the post-glitch data *is inconsistent with a recovery due solely to the crust superfluid*; a pinned core superfluid is implied (part of) which is decoupled from the "crust" to explain the observed large values of $\frac{\Delta\dot{\Omega}_c}{\Omega_c}$. But the expected *recovery time* for the associated rotational lag of a *pinned core superfluid* is also apparently in sharp contrast with even a larger body of observational data. The core superfluid would be expected to remain decoupled over times much larger than that permitted by the observations. This is the dilemma.

In the following we mention four different suggestions (I – IV, below) in the literature as a resolution of this issue. The first one is based on the effects due to the crust superfluid alone and tries to explain the observed large values of $\frac{\Delta\dot{\Omega}_c}{\Omega_c}$ without bringing in any contribution from the core component. The other three take the opposite stand by allowing for a contribution from (a fraction of) the large moment of inertia of the core. They consider pinning of the superfluid vortices in the core to the fluxoids, and make an attempt to show how the small values of $\frac{\Delta\dot{\Omega}_c}{\Omega_c}$ could come about. Nevertheless, we find all of these solutions to be unsatisfactory and in some cases self-contradicting with their adopted models as is discussed below.

I- Crust Superfluid

It has been argued that the observed value of $\frac{\Delta\dot{\Omega}_c}{\dot{\Omega}_c} \sim 16\%$ for the 1988 glitch in Vela is because part of the crust superfluid is "temporarily" being *spun up* by the "crust" (Alpar, Pines & Cheng 1990). A fraction of the crust superfluid has been assumed to support a small steady state value of the lag $\omega_\infty \sim 3.5 \times 10^{-6}$ rad s^{-1} and hence a glitch of a size $\frac{\Delta\Omega_c}{\Omega_c} \sim 10^{-4}$ rad s^{-1} would result in $\Omega_c >> \Omega_{sp}$ in the corresponding region, where Ω_{sp} is the rotation rate of the assumed superfluid region. The subsequent *spin-up* of the assumed part of the superfluid by the "crust" is thus argued to act as an extra spinning-down torque on the "crust" until w_∞ is restored. However, *the spin-up of the superfluid when the instantaneous $|\omega| >> w_\infty$ is expected to take place over microscopic time scales,* also according to the vortex creep model. The negative lag induced by the jump in Ω_c will therefore first decay rapidly leading to a state with $w \equiv \Omega_{sp} - \Omega_c = -\omega_\infty$. Thereafter the superfluid will remain decoupled till $w \sim \omega_\infty > 0$ is reached due only to the spin-down of the "crust". The spin-up of the assumed superfluid component in the crust from a state of $w << -\omega_\infty$ to $w = -\omega_\infty$ is indeed the reverse of the same process as the original jump in Ω_c (viz. the glitch), which is assumed in the vortex creep model to be induced by another part of the *crust superfluid* and over *microscopic time scales.* The spin-up time scale of the crust (say, at a glitch) by the freely moving vortices scattering off the nuclei has been estimated to be $\lesssim 5$ s, for the Vela pulsar (Epstein, Link & Baym 1992). The spin-up of the assumed superfluid component for which initially at a glitch $w << -\omega_\infty$ should also occur over similar time scales, and the observed large values of $\frac{\Delta\dot{\Omega}_c}{\dot{\Omega}_c}$ over time scales of hours to days cannot be attributed to this process.

It is important, and a very interesting point of the hydrodynamics of supeduid rotation, to note that the above spin-up of the crust to rotation rates larger than that of the (part of the crust) superfluid and the spin-up of the superfluid by the crust *are not two successive processes; both are achieved simulataneously.* Since the superfluid vortices (being pinned) to the nuclei *are also spun up* along with the crust any spinning up of the superfluid has to take place during this process itself. The bulk superfluid *cannot be spun up* by the crust while the latter is spinning down (or even while it is rotating at a constant rate, for that matter). This is simply because the crust (while being spinning down or rotating at a constant rate) could not exert a *spinning-up torque* on the vortices (and hence on the bulk superfluid) which are already co-rotating with it. This discussion should also clear any doubt about the possibility of *an inward* creeping motion of the vortices and hence the above suggested spinning-up of the superfluid during the period when the crust and the vortices are rotating faster than the superfluid but with a lag whose magnitude is smaller than the steady state value of the lag, ie when $0 > w > -\omega_\infty$.

The large magnitude of the observed change of the spin-down rate of the crust at glitches (particularly in Vela) therefore indicates the contribution of the core in the recovery; *the observations hence indicate the pinning of the vortices in the core of neutron stars,* since *an* unpinned superfluid component would follow the crust on the dynamical time scale (cf. § **4.3).**

II- Creeping Vortices

Thermal creep of the *core-superfluid* vortices over the pinning sites might seem to provide a way for a partial coupling of the superfluid and thus explain, in this case the small, observed changes in the rotation rate of the "crust" soon after a glitch. However, due to the exponential dependence of the rate of the creep (hence v_n and $\dot{\Omega}_s$) on the value of the lag ($v_n \propto e^{\omega}$) the *creep process is essentially stopped until* $w \sim \omega_{cr}$ *is reached.* The inconsistency with the fast coupling of the core as indicated by observations therefore persists, except allowing for arbitrary small values of ω_{cr}. Thus, values as low as $\omega_{cr} \sim 10^{-6}$ rad s^{-1} corresponding to the core magnetic fields $B_c \sim 10^8$ G have been inferred for Vela, assuming an upper limit of 35 min for the coupling of the core superfluid as implied by the data on its 1988 glitch (Chau et al. 1992). Still smaller values of $\omega_{cr} \sim 2 \times 10^{-7}$ rad s^{-1} and $B_c \sim 10^7$ G would be however required for the much shorter time limit of $\lesssim 7$ min imposed by observations of the 1991 Vela glitch (Flanagan 1995). But, values of $\omega_{cr} \lesssim 10^{-6}$ rad s^{-1} are too small to be useful !. The assumed creep process would be of no consequence for the long term post-glitch recovery of Ω_c, and also the core superfluid could not be responsible for the triggering of a large glitch as observed in Vela and other pulsars. Furthermore, the implied values of B_c (G) $\sim 10^7$–10^8 for the Vela pulsar which are unusually small are not consistent even with the magnetic evolution of neutron stars as implied by the same pinning mechanism for the interior superfluid. The spin-down induced flux expulsion, as the fluxoids are pulled out of the core along with the vortices, requires a much larger time scale than the age of Vela for even an order of magnitude reduction in the core field (Srinivasan et al 1990; see also Fig. 3.1). Surprisingly, a value of $B_c \sim 10^8$ G for Vela is stated (Chau et al. 1992) to be consistent with the predictions of the model of spin-down induced flux expulsion as studied by Ding, Cheng & Chau (1993). It has to be noted that the rate of flux expulsion as prescribed in the original model (Srinivasan et al 1990) is in fact larger than in the work of Ding et al., for times $\lesssim 10^6$ yr. Nevertheless, final values as low as $B_c \sim 10^8$ G have been obtained in their model, for an assumed value of field decay time scale in the crust $\tau_{\rm Ohm} \lesssim 10^7$ yr, only for a hypothetical case in which initial core and surface field strengths are set as $B_c = 10^9$ G and $B_s = 5 \times 10^{12}$ G, respectively. Although the unusual combination of the assumed initial values of B_c and B_s is hard to believe, however in this case too a decrease in the core field by about an order of magnitude is seen (Fig. **3** in Ding et al. 1993) to take a time $\gtrsim 10^5$ yr.

III- Sliding Vortices

A further possibility for a fast coupling of the pinned core superfluid has been suggested by considering the azimuthal anisotropy of the pinning barriers against the radial motion of the vortices in different directions (Sauls 1989). In regions where radial motion of the vortices does not make a large angle with the fluxoids (see Fig. 1.4) the vortices are suggested to slide out along the fluxoids (Jones 1991) resulting in only a small fraction of the superfluid being effectively pinned and decoupled. The effect however does not seem to reduce the pinned fraction of the vortices to $\lesssim 10\%$ of the total vortices, as would be required for an assumed ratio of $\frac{I_c}{I_s} \sim 0.1$ and the post-glitch values of $\frac{\Delta\dot{\Omega}_c}{\dot{\Omega}_c} < 1$. Moreover, such an anisotropy in the radial motion of the vortices would be also questionable on the account that it results in an *azimuthally nonuniform distribution of the vortices.*

If a rotating superfluid could sustain an equilibrium state with an azimuthally nonuniform vortex number density the superfluid rotation velocity would be likewise nonuniform, which requires a *radial* circulation to be also present in the superfluid velocity field. If so, the Magnus force on the vortices due to this superfluid radial motion in the regions where their radial motion is prohibited by the fluxoids (the region of over-dense vortices) would be then such that to drive them azimuthally (along the fluxoids) into the free regions where they can move radially out, as is assumed. Consequently, an assumed excess in the vortex number density might be expected to be washed out, and *no substantial rotational lag between the vortices and the superfluid could build up anywhere in the core of a neutron star.* Namely, the radial barriers presented by the fluxoids would be side passed by the vortices taking successive azimuthal-radial steps !, should a departure from the azimuthally uniform vortex distribution be permitted. On the other hand, the effect of any closed circulation path on the vortices in the underdense (and also in the outer) regions would be such that to drive them *into* the pinned (and closer to the axis) regions. It is hard to see whether the two effects would cancel out and allow for a lag to be present, or the lag is washed out by a larger vortex current out of the pinned region. It is left to a detailed study of the induced superfluid circulating current for plausible given azimuthal anisotropy of the vortex density (if permitted at all) to decide about these possibilities.

IV- Huge Lag

Finally, very large values of $\omega_{cr} >> 10^{-2}$ rad s^{-1} such that the superfluid would remain decoupled throughout the inter-glitch intervals have been also considered (Muslimov & Tsygan 1985). Even though in this case the post-glitch values of $\dot{\Omega}_c$ are not expected to differ much from the steady state values, however such values of ω_{cr} are ruled out by other observational constraints. A lag of this size, which is not consistent with the current estimates for E_P (Sauls 1989, Jones 1991) cannot however be considered as a possible cause of the observed glitches, even though it has been assumed (and is inevitable !) in this scenario. The induced jump in Ω_c would be much larger than the largest observed events, even if only a small fraction, say one percent, of the superfluid moment of inertia is assumed to be involved. Also, large and rapid variations of Ω_c should have been observed *just before* the glitches which are to be triggered as $w \rightarrow \omega_{cr}$ since the superfluid would start to couple then.

The existing problem addressed in this subsection therefore remains that the observed post-glitch values of $\frac{\Delta\dot{\Omega}_c}{\dot{\Omega}_c}$ seem to be *too large* to be explained by the decoupling of the crust superfluid alone and at the same time *too small* for a pinned core superfluid to play a role. In Section 4.4.1 we will however argue that the *pinning of the neutron vortices to the fluxoids in the core of neutron stars* seems to provide a resolution for the dilemma.

4.3 The Core Superfluid

Superfluid vortices are expected to move with the local superfluid velocity except when there is an external force acting on a vortex. For a given external force F_{ex} per unit length of the vortex its equation of motion is given (Hall 1960, Sonin 1987) as (note the difference with the expression for the Magnus force as in Eq. 3.3)

$$\vec{F}_{ex} = \rho_s \vec{\kappa} \times (\vec{v}_s - \vec{v}_L) \tag{4.2}$$

where ρ_s is the superfluid density, $\vec{\kappa}$ is the vorticity of the vortex line directed along the rotation axis (its magnitude $\kappa = \frac{h}{2m_n}$ for the neutron superfluid, where m_n is the mass of a neutron), and $\vec{v_s}$ and $\vec{v_L}$ are the local superfluid and the vortex line velocities. In particular, the spinning down (or up) torque on a rotating superfluid is associated with a radial outward (or inward) motion of the vortices. The azimuthal Magnus force exerted by the superfluid on the vortices due to this motion which is balanced by F_{ex}, has an equal and opposite reaction on the superfluid which explains how a "mutual" friction between the superfluid and its environment is realized (Hall & Vinen 1956).

4.3.1 The Steady-state; relative rotation of the different components

During the steady state *spinning down* of a neutron star, the vortices are expected to be co-rotating with the "crust" including the proton condensate and the lattice of fluxoids in the core of the star. Strict co-rotation of the vortices with the crust is not however possible for an assumed vortex relaxation process due to the electron scattering. **A** lag, though negligibly small, is required between rotation of the crust and the vortices. The drag force responsible for the spin-down torque on the superfluid implies, of course, a mean relative velocity between the vortices and the electron gas to persist. The pinning force exerted by the fluxoids could, on the other hand, maintain a state of rigid rotation of the vortices along with the fluxoids since the pinning force could impart torque on the vortices even when they are practically co-rotating with the fluxoids. In fact, since the mean spacing between the fluxoids d_f is larger than the size of pinning interaction region d_P, a co-rotation of the two lattices is the stable steady state configuration which could impart a larger torque on the superfluid than otherwise. This is because in a co-moving phase the vortices can adjust their positions (within a length scale of a d_f) such that each of them lies within a pinning interaction region.

An excess in the number density of the vortices compared to the equilibrium value corresponding to the rotation frequency of the vortex lattice is, however, maintained due to the presence of the pinning energy barriers which impede on the radial outward motion of the vortices as the pulsar spins down. The excess number density is such that the associated lag between rotation frequencies of the superfluid and the vortices would account for the required radial Magnus force to overcome the barriers.

The expected relative rotation of the "crust", the core superfluid, and its vortices is sketched in Fig. 4.4. Each plot indicates the conditions during the steady state (S.S.), at a jump in the rotation frequency of the "crust" (JUMP), and after the core

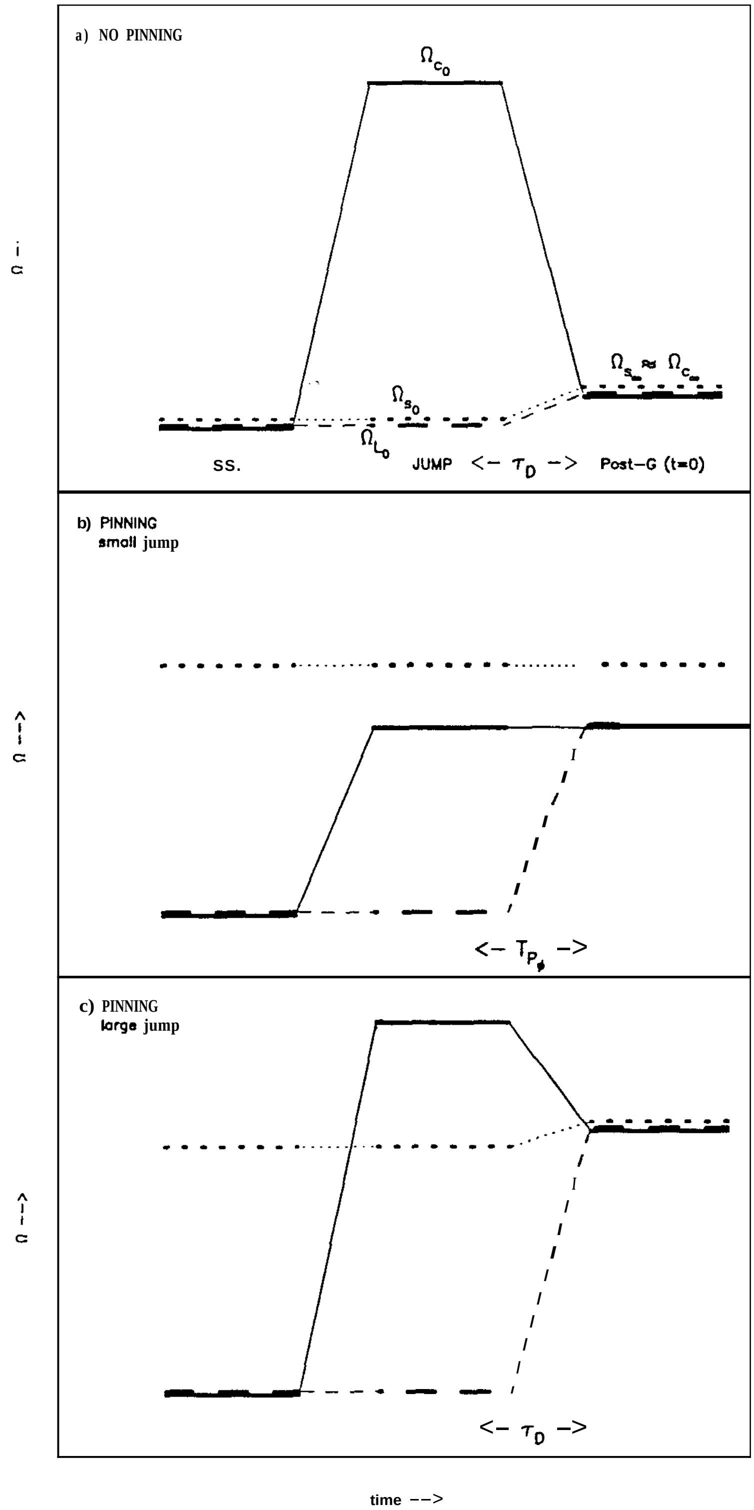

Figure 4.4- Schematic representation of the relative values of angular velocities of the superfluid Ω_s (dotted line), the vortices Ω_L (dashed line), and the "crust" Ω_c (full line) before (S.S.), at (JUMP), and after (pos-G) a sudden jump in Ω_c. **(a)** Vortex lines are free (no pinning) and in the steady-state they co-rotate with the superfluid bulk matter. (b) & (c) Due to the pinning of the vortices a steady-state lag is required in order for the superfluid to be spinning down at the same rate as the "crust". The assumed magnitude of the jump is different for' the case shown in (b) and (c). The superfluid response time (between the jump and the initial, t = 0, post-glitch conditions) is shown in each case.

has responded to that jump (post-G). The cases of pinned and free vortices are shown separately, as there is also a further distinction depending on the size of an assumed jump in the rotation frequency of the "crust" which will be discussed later on (cf. § 4.3.3).

Magnitude of the lag ω_∞

In the steady state of a spinning down superfluid as for the neutron condensate in a pulsar, the azimuthal Magnus force (Eq. 3.3) $F_{\mathrm{M}\phi} = \rho_s \kappa v_n$ associated with the outward radial velocity of the vortices v_n is balanced by an azimuthal drag $F_{\mathrm{D}\phi}$. The latter is exerted by the "normal" fluid (the "crust" in the case of a neutron star) on the vortices, resulting in the torque on the superfluid. As noted earlier, while for the frictional force on the vortices due to the pinning barriers $\Omega_c = \Omega_L$ is strictly true, in general a velocity dependent drag force requires $\Omega_c \neq \Omega_L$. The difference is however negligibly small and might be neglected. Similarly, the radial Magnus force per unit length of a vortex $F_{\mathrm{Mr}} = \rho_s \kappa r \omega$ is balanced by the radial frictional force F_{Dr}, where r is the distance from the rotation axis, and the Magnus force is directed outward for $w > 0$, and vice versa. *The steady state lag w, is thus determined from $F_{\mathrm{Mr}} = F_{\mathrm{Dr}}$, for a given value of v_n ($\propto \dot{\Omega}_s$).* For the typical observed values of $\dot{\Omega}_s = \dot{\Omega}_{SS}$ the drag force due to the electron scattering would again require a negligibly small value of w, However, in the case of pinning of the vortices the dominant frictional force is that of pinning ($F_{\mathrm{Dr}} \equiv \frac{f_P}{d_P}$) and $\omega_\infty = \omega_{cr}$ is determined from the condition $F_{\mathrm{Mr}} = \frac{f_P}{d_P}$. Thermal creeping of the vortices over the pinning barriers could in principle serve to establish a value of $\omega_\infty < w_{,,}$ however for the core superfluid $\omega_\infty \sim w$, (Chau et al. 1992). In any case we will be neglecting the thermal creeping of the vortices in the core (which was argued in § 4.2.2 to have no significant effect) and hence assuming $w, = w_{,,}$ throughout our discussion. Thus as discussed in § 3.2.2 (Eq. **3.8**) $\omega_{cr} = \frac{f_P}{\rho_s \kappa r d_P} \sim 1.6 \times 10^{-4} (\text{ rad s}^{-1}) \frac{B_{12}^{1/2}}{R_6}$, where R_6 is the value of r in units of 10^6 cm. For the typical magnetic fields of young pulsars then one finds

$\omega_{cr} \gtrsim 10^{-4}$ rad s^{-1} at the outer regions of the stellar core (for an assumed value of $R_6 \sim 1$), and

$\omega_{cr} \gtrsim 10^{-3}$ rad s^{-1} at a distance of $R_6 \sim 0.1$ which includes $\sim 1\%$ of the moment of inertia of the core.

The steady state spinning down will be hence established while $\Omega_c = \Omega_L < \Omega_s$, $\omega_\infty = \omega_{cr} > 0$ (with the above estimates for ω_{cr} which also indicate a state of differential rotation for the core superfluid), and $\dot{\Omega}_s = \dot{\Omega}_c = \dot{\Omega}_{SS}$.

The pinning in the core of a neutron star and the existance of a lag $\omega_\infty (\equiv \omega)$ during the spin-down of the superfluid has been argued to imply an overestimation in the inferred surface fields of radio pulsars by a factor $\sqrt{\frac{I}{I_c}}$ (Muslimov & Tsygan 1985, Chau et al. 1992), where I_c is the moment of inertia of the "crust". This is because the instantaneous value of ω in a pulsar is assumed in this argument to be smaller than $\omega_\infty = \omega_{cr}$ which is required for a spin-down of the core superfluid along with the crust. The superfluid is hence expected, in this assumption, to be *always decoupled* from the spinning down "crust". And when $\omega = \omega_{cr}$ is achieved a glitch is induced, resulting in a decrease in ω and decoupling of the core; the cycle repeated. However, values of $\omega_\infty = \omega_{cr} \lesssim 10^{-3}$ rad s^{-1} would be recovered within $\lesssim 100$ day, given the typical values of $|\dot{\Omega}_{SS}| \sim 10^{-10}$ rad s^{-2} in young pulsars. Most of the moment of inertia of the star could not therefore remain decoupled over typical time scales of years between successive glitches in a pulsar, for the assumed values of ω_{cr}. Such time scales ($\sim$ tens of days) for the coupling of a large fraction of the moment of inertia are, of course, ruled out by the observed post-glitch behavior of $\dot{\Omega}_c$. The much larger values of $\omega_{cr} >> 10^{-2}$ rad s^{-1} which would not be recovered within typical inter-glitch intervals were, on the other hand, already argued in § 4.2.2 not to be acceptable.

Hence, it is concluded that *most of the superfluid in the interior of a neutron star takes part in the steady state spinning down of the star,* and no major correction in the inferred magnetic fields of pulsars is required on this ground.

4.3.2 Coupling of the Core Superfluid to the Crust : A new mechanism

An assumed departure from the steady state co-rotation of the vortices with the fluxoids would thus result in theircontinually crossing one another, with an energy cost of E_P per intersection. The total number of crossings per relative rotation cycle of the two lattices having a spherical boundary surface is $\sim N_f N_v \sin\chi$, where N_f and N_v are the total number of the fluxoids and the vortices, and χ is the angle between the two families of lines, namely the angle of inclination between the rotation and the magnetic axes of the star. Excluding the cases of exactly parallel lattices corresponding to $\chi \sim 0$ or 180 deg which is not expected in most of the observed pulsars, the correction due to the $\sin\chi$ factor will be neglected, assuming a perpendicular geometry between the two lattices, for simplicity. The average frictional force (per unit length) on a vortex F_D while cutting through the lattice of uniformly distributed fluxoids is estimated to be

$$\begin{aligned} F_D &\sim \frac{E_P}{d_f^2} \\ &\sim 6 \times 10^{12}(\mathrm{dyn\ cm^{-1}}) E_{\mathrm{MeV}} B_{12} \end{aligned} \tag{4.3}$$

where E_{MeV} is the value of E_P in units of MeV, and B_{12} is the strength of the average magnetic field in the stellar core B_c in units of 10^{12} G. The force in **Eq. 4.3** which is independent of the relative velocity of the two families of lines represents a time averaging of the effective force on a vortex as it travels between successive pinning centers being $\sim d_f$ apart. And the number of potential crossings points per unit length of a vortex is also assumed to be $\sim \frac{1}{d_f}$.

It is noted that the force per unit length corresponding to the pinning force f_P (per intersection) which is larger than F_D in Eq. **4.3** by a factor of $\frac{d_f}{d_P}$ ($d_P \sim \xi_p$ or λ_p) is the relevant frictional force on the vortices only during an equilibrium phase whence the vortices are free to adjust their velocities instantaneously as they move through pinning and free regions successively. The vortices would then spend almost all of the time inside pinning regions and fly across the free spacings between the fluxoids rapidly, since the drag force due to the electron scattering in the free inter-fluxoid spacing is much smaller than that corresponding to $f_{\rm P}$. However, for a transient relaxation the given initial velocity of the vortices would be the same within the free as well as the pinning regions, and hence $F_{\rm D}$ in Eq. **4.3** would apply.

The time $T_{\rm P}$ for a given relative velocity between the fluxoids and vortices to be dissipated might be determined, using the EOM of a unit volume of the charged component gas co-rotating with the fluxoids (the "crust"), through

$$n_{\rm v} F_{\rm D} \;=\; \rho_{\rm c} \frac{1 \;\; - v_{\rm c}}{T_{\rm P}} \tag{4.4}$$

where $n_{\rm v}$ is the number density of the vortices per unit area, and $\rho_{\rm c}$ an $v_{\rm c}$ are the effective density and the velocity of the "crust". Note that since $F_{\rm D}$ is a velocity-independent force, $T_{\rm P}$ is the total time for the decay of the velocity, and not an exponential time constant as in the case of $\tau_{\rm v-e}$. For a given initial jump $\Delta\Omega_{\rm c}$ in the rotation frequency of the crust $\Omega_{\rm c}$ out of its steady state value in co-rotation with the vortices, the time for the decay of the relative azimuthal velocity of the vortices is given by

$$T_{\rm P\phi} \;\sim\; 830({\rm s}) \frac{\Delta\Omega_{\rm c}}{\Omega_{\rm c}} \frac{\rho_{13}}{E_{\rm MeV} B_{12}} \tag{4.5}$$

where ρ_{13} is the value of $\rho_{\rm c}$ in units of $10^{13}{\rm gcm}^{-3}$, and the dependence on the distance from the rotation axis r (v $= r\Omega$) has been averaged out, neglecting any possible differential rotation of the vortices during the relaxation.

For typical glitches observed in the Vela pulsar $\frac{\Delta\Omega_{\rm c}}{\Omega_{\rm c}} \sim 10^{-6}$, and assuming $E_{\rm MeV} \sim \rho_{13} \sim 1$, one obtains $T_{\rm P\phi} \sim 10^{-4} - 10^{-3}$ s, which is much shorter than the corresponding time scale due to the electron scattering $\tau_{\rm v-e} \sim 1 - 2$ s. However, since $T_{\rm P}$ depends on the initial value of the induced relative velocity, in the limit of very large disturbances (much larger than observed even in the giant gitches of Vela) the relevant time scale would be determined by $\tau_{\rm v-e}$. The effective vortex velocity relaxation time scale τ_v might be thus estimated by considering the combined effects of the electron scattering and the fluxoid scattering as

$$\frac{1}{\tau_v} \;=\; \frac{1}{T_{\rm P}} + \frac{1}{\tau_{\rm v-e}} \tag{4.6}$$

4.3.3 The Response Time of the Core Superfluid

The rotational velocity of the vortices does not by itself determine that of the superfluid mass current which is decided rather by the number density of the vortices. The relaxation time discussed above for vortex velocity is likewise different from the dynamical time scale for the coupling of the bulk superfluid. The latter is the time needed for the simultaneous re-adjustment of the vortices in both radial and azimuthal directions in response to the torque on the superfluid.

The equilibrium positions of the vortices might be determined from a solution of their equation of motion (Eq. 4.2), subject to the boundary conditions that in the steady state before and after a jump in Ω_c the superfluid rotational frequency $\Omega_s = \Omega_c$, in the absence of any pinning. Following Alpar & Sauls (1988), Eq. 4.2 is solved by substituting F_D evaluated from Eq. 4.4 for F_{ex} which results in the following solutions for the radial $r_v(t)$ and the azimuthal $\phi_v(t)$ components of the vortex position in polar coordinates, as a function of time t. The time t $= 0$ corresponds to the jump epoch and the final equilibrium positions might be derived from the t $\rightarrow \infty$ behavior of the solutions (see also Fig. 4.4a, note the difference in time $t = 0$ on the figure with that used here). The solutions are :

$$r_v(t) = r_0 \left[\frac{\Omega_{s0}}{\Omega_{c0}} + \left(1 - \frac{\Omega_{s0}}{\Omega_{c0}} \right) e^{-t/\tau_d} \right]^{1/2} \tag{4.7}$$

$$\phi_v(t) = \phi_0 + \Omega_{c0} t + \mathrm{K} \ln \left(\frac{r_v(t)}{r_0} \right) \tag{4.8}$$

where 0-subscripts indicate initial values at $t = 0$ of the corresponding quantities, and $\mathrm{K} = \frac{\rho_s \kappa n_v}{\rho_c} \tau_v$. The solutions are given here to correct for the errors in the expressions given by Alpar & Sauls (1988; their Eqs 8 & 9). The equilibrium vortex positions, hence the coupling of the superfluid to the "crust" is seen from Eqs 4.7 & 4.8 to be approached with a dynamical time constant τ_d which is, same as in Alpar & Sauls (1988),

$$\tau_d = \frac{K + \frac{1}{K}}{2\Omega_{c0}} = \frac{\tau_v}{x} + \frac{x}{16\pi^2} \frac{P_s^2}{\tau_v} \tag{4.9}$$

where $x = \frac{\rho_c}{\rho_s}$ is the ration of the density of the "crust" to that of the core superfluid, and $\kappa n_v = 20. \approx 20, = \frac{4\pi}{P_s}$ has been used. Using a value of x $\sim$ 0.1 and $\tau_v \sim T_P \sim T_{P\phi} \sim 10^{-3}$ s for a Vela type glitch, as is expected due to the fluxoid-vortex pinning, a value of $\tau_d \sim$ 0.01 s is obtained which is much shorter than that due to the electron scattering. To repeat this should be compared to $\tau_d \sim$ 2 min derived by Alpar & Sauls.

Pinned Superfluid

On the other hand, if a lag w $(= \Omega_s - \Omega_L$, where Ω_L is the rotation frequency of the vortex lines) between rotation of the superfiuid and the vortices exist in the steady state, due to a pinning of the vortices, a jump in Ω_c and hence in Ω_L will change the value of w. The subsequent relaxation of the vortices to equilibrium positions in the r-direction *might be presumed* to be possible, as in the steady state, only when $|\omega|$ is larger than a certain critical value ω_{cr} (note again that we neglect any possible thermal creeping of the vortices for simplicity of the present discussion and take the steady state value of the lag ω_∞ = w,). Application of the above dynamical time scale to the case of pinned superfiuid might be questionable on this ground.

However, since the crossing through barriers is inevitable until the state of co-rotation of the fluxoids and the vortices is reached (for rigid vortex lines) the required relaxation in the radial direction becomes possible for **any** value of w. The steady state lag is therefore washed out during the relaxation of the vortices, bringing the superfluid to a co-rotating state with the flwoids and the crust. For a jump in $\Omega_c \rightarrow \Omega_{c0}$ (a glitch) such that $\Delta\Omega_{c0} = \Omega_{c0} - \Omega_c > \omega_{cr}$ (see Fig. 4.4c) the superfiuid will be hence spun up by the crust on a dynamical time scale until the equilibrium state $\Omega_s = \Omega_L = \Omega_c$ is reached, as is also expected for the case with no pinning of the vortices (Fig. 4.4a).

In contrast, should the initial jump $\Delta\Omega_{c0} < \omega_{cr}$ (Fig. 4.4b) the vortices cannot move radially *outward* while being spun up to come into co-rotation with the flwoids, as is required for an assumed final co-rotation of the superfluid and the crust to be established. The co-rotation of the vortices with the crust is hence achieved with no angular momentum being transferred between the superfiuid and the crust, neglecting the inertia of the vortices as is usually assumed. The freedom of the vortices for a radial motion, due to their azimuthal relative velocity with the fluxoids, does not, of course, satisfy by itself the requirements for their outward motion. Vortices may move out, and Ω_s decreases, provided they rotate or tend to be rotating slower than the fluxoids (the "normal" fluid) so that the mutual friction force on the superfluid has the right sign for a slowing down torque.

The latter case of the relatively smaller disturbances as shown in Fig. 4.4b will be therefore followed by only a subsequent rise in Ω_L which takes place over a time scale $T_{P\phi}$ (Eq. 4.5); *no dynamical coupling time scale would be involved.* This is expected to be the case for all the observed glitches in pulsars including the largest events detected in Vela assuming values of $\omega_{cr} \gtrsim 10^{-4}$ rad s^{-1} for the pinned core superfluid. In addition, in contrast to the case of $\Delta\Omega_{c0} > \omega_{cr}$ which has to be induced by some "external" cause, possibly the pinned superfluid component in the crust, a decrease in 52, (ie. the spin rate of the *core* superfluid) itself could as well be the cause of the events with $\Delta\Omega_{c0} < \omega_{cr}$. This would further support the proposed model (cf. § 4.4.2) for the cause of the glitches being due to the core superfluid in neutron stars. The various cases discussed for the expected behavior of 52, following a sudden jump in Ω_c are sketched in Fig. 4.4, where the relevant time scale is also indicated in each case. The three cases may be further associated with the conditions expected for a glitch :
i) caused by *crust* superfluid and with an *unpinned* core superfluid, (Fig. 4.4a)
ii) caused by *pinned core* superfluid, (Fig. 4.4b) and
iii) caused by *crust* superfluid and with a *pinned* core superfluid, (Fig. 4.4c),
respectively. Note that the line representing Ω_s in Fig. 4.4b should be slightly brought down for this latter interpretation.

We further emphasize that the *response time* of the core superfluid to a jump in the rotation frequency of the "crust" *smaller* than ω_∞ is only that of the *vortex azimuthal velocity relaxation* and *not the dynamical coupling time.* This might be said differently as follows : the core superfluid *does not respond* to such small jumps, namely no angular momentum is transferred between the "crust" and the core. The expected behavior of the crust *over time scales smaller* than the assumed dynamical time scale of the core would be in this case (Fig. 4.4b) drastically different than for the unpinned case (Fig. 4.4a) or that of the large jump (Fig. 4.4c). A large decrease in the frequency of the "crust" following its initial jump is envisaged in the latter cases due to its angular momentum trade off with the heavy core of the star which is absent in the case of a small jump and pinned vortices. Observational data with a time resolutions smaller than the assumed dynamical time scale for coupling of the core superfluid (ie. resolution of few seconds) covering the exact epoch of a glitch event could distinguish between the two possibilities. Note that, since for a large jump as assumed here the core superfluid could not be the cause of the event, the above suggested observational test could in fact decide the presence (but not the absence !) of the *pinning* in the core superfluid. The case of small jump if indicated by such data would vote for the pinning in the core, however in the opposite case both possibilities remain. Such a distinction might be in fact provided by the observation of the 1989 glitch of the Crab pulsar (cf. § 4.2.1) if the interpretation of a delayed rise in the rotation frequency of the crust is confirmed and the the resolution in time and the amplitude is high enough.

The dynamical time scale

In the above derivation of τ_d (cf. Eq. 4.9) it has been assumed, following Alpar & Sauls (1988), that while during the relaxation of the vortices to new equilibrium radial positions Ω_s changes from $\Omega_{s0} \rightarrow \Omega_{s\infty} = \Omega$ however for the "crust" $\Omega_{c0} = \Omega_{c\infty} = \Omega$ (see Fig. 4.4a). Indeed, since the relative value of the initial jump is small ($\frac{\Omega_{c0} - \Omega_{s0}}{\Omega_{c0}} << 1$) the relative difference between Ω_{c0} and $\Omega_{c\infty}$ is also negligible ($\Omega_{c\infty} = \Omega_{c0} \frac{1 + \frac{I_s}{I_c} \frac{\Omega_{s0}}{\Omega_{c0}}}{1 + \frac{I_s}{I_c}} \approx \Omega_{c0}$, where I_c and I_s are the moments of inertia of the "crust" and the superfluid, respectively). It might therefore seem justified to approximate Ω_c being constant during the relaxation process. However, if Ω_c and hence v_c in Eq. 4.4 are treated as variables, as they should, the coupling time constant is instead found to be

$$\tau_D = \frac{K + \frac{1}{K}}{2\Omega_{c0}\left(1 + \frac{I_s}{I_c}\frac{\Omega_{s0}}{\Omega_{c0}}\right)} = \frac{1}{1 + \frac{I_s}{I_c}\frac{\Omega_{s0}}{\Omega_{c0}}}\tau_d$$
$$\approx \frac{I_c}{I}\tau_d \qquad (4.10)$$

where $\mathbf{I} = I_c + I_s$. The point is that even though the change in Ω_c during the spin-up of the superfluid is negligibly small compared to its absolute value, nevertheless it is still much larger than that of the superfluid which have been taken into account; $|\Omega_{c\infty} - \Omega_{c0}| \sim \frac{I_s}{I_c}|\Omega_{s\infty} - \Omega_{s0}|$. That τ_D (Eq. 4.10), and not τ_d (Eq. 4.9), has the correct relation with τ_v can be seen from a comparison with the relation expected between the corresponding quantities in the general "2-component" model of a neutron star (Baym et al 1969). In the limit of $\tau_v > P_s$ (which is the limit of neglecting the effect of the radial motion of the vortices and hence appropriate for comparing with the results of the 2-component model) $\tau_d \sim \frac{\tau_v}{x} \sim \frac{I_s}{I_c}\tau_v$ while $\tau_D \sim \frac{I_s}{I}\tau_v$, the latter being in agreement with that of the 2-component model (Baym et al 1969, Shapiro & Teukolsky 1983).

The time scale for coupling of the superfluid core to the crust is thus reduced by more than an order of magnitude (ie. by a factor of $\frac{I_c}{I}$) compared to the earlier estimates (Alpar & Sauls 1988, Pines & Alpar 1992) even for an assumed vortex relaxation mechanism due only to the electron scattering. The predicted values of $\tau_d \sim 400P_s - 10^4 P_s$ (Alpar &Sauls 1988) have already been questioned on the account of the observed early responses of the Vela pulsar following two of its recent glitches. These were interpreted to indicate a core coupling time scale of $\sim$ 2 min (Pines & Alpar 1992) as the observed relative change in the spin-down rate of the 'crust" $\frac{\Delta\dot{\Omega}_c}{\dot{\Omega}_c}$ is much smaller than that expected during relaxation of the core. The 1988 and 1991 glitches of the Vela pulsar have measured values of $\frac{\Delta\dot{\Omega}_c}{\dot{\Omega}_c} \sim 0.16$ and 0.60 at the beginning of the post-glitch observations which were started at $\sim$ 35 min and $\sim$ 7 min after the corresponding glitches, respectively (Flanagan 1995). The existing difficulty with the implied upper limit for the core coupling time scale by these observations (2 (min)$\lesssim \tau_d$ for Vela) does not however persists for τ_D. The requirement of 2 (min) >> τ_D for Vela is satisfied, whether one allows for the vortex relaxation due also to the pinning or only that of the electron scattering.

Furthermore, the suggested observational constraint of $\sim$ 2 min for the core coupling time constant might be an overestimate since the initial pre-relaxation $\frac{\Delta\dot{\Omega}_{c0}}{\Omega_c} \sim 10^4$ which implies a total relaxation time $\sim$ **10** x τ_D, and hence requires $\tau_D \lesssim \frac{7}{10}$ (min) $\sim$ **40 s**. The above estimate may be verified by substituting for the initial value of $\dot{\Omega}_{s0} = \frac{A}{\tau}$ (cf. Fig. 4.4a) in the general "2-component" relation $I_c\dot{\Omega}_c = I\dot{\Omega}_{SS} - I_s\dot{\Omega}_s$ which reduces to

$$\dot{\Omega}_{c0} \sim \dot{\Omega}_{SS}\left(\frac{I}{I_c} + \frac{I}{I_s}\frac{\Delta\Omega_{c\infty}}{\Omega_c}\frac{t_{sd}}{\tau_D}\right) \sim 10^4\,\dot{\Omega}_{SS} \tag{4.11}$$

where $\dot{\Omega}_{SS}$ is the steady state spin-down rate, $\Delta\dot{\Omega}_{c0} = \dot{\Omega}_{c0} - \dot{\Omega}_c \sim \dot{\Omega}_{c0} - \dot{\Omega}_{SS}$, and values of $t_{sd} = \frac{\Omega_c}{\dot{\Omega}_{SS}} \sim 10^4$ yr, $\frac{\Delta\Omega_{c\infty}}{\Omega_c} \sim 10^{-6}$, and $\tau_D \sim 2$ min have been used for the Vela pulsar.

4.4 The Influence of Magnetic Field Evolution on the Rotational Dynamics

4.4.1 Post-glitch Recovery

As we argued in § **4.2.2** a pinned core superfluid for a neutron star is apparently inconsistent with the observed small values of the change in the spin-down rate of the crust soon after a glitch. The pinned superfluid, if present, should however remain decoupled over comparatively much larger time scale until its rotational lag is increased to its pre-glitch value, through the spin-down of the 'crust" alone (accompanied by the vortices) . Nevertheless here we argue that the pinned superfluid in the interior of neutron stars could become coupled to the crust soon after a glitch and even while $\omega < \omega_{cr}$ because of the moving nature of the pinning sites and also the τ-dependence of ω_{cr}. The dependence of the value of the lag on the radial distance implies that *all* of the core superfluid *need not be involved* in the expected *decoupling* after a disturbance of the steady state lag in some part of the superfluid. The motion of fluxoids driven by the buoyancy (and/or possibly other outward forces) guarantees a *minimum* value for the outward radial velocity of the vortices, and therefore maintains a partial coupling of the core superfluid even when w < w,,.

The vortices, after a glitch, could not move slower than fluxoids since that would require a negative lag w $= -\omega_{cr}$ which is not the case in a young pulsar (cf. Fig. 3.1 and the related discussion) and at a glitch. Since *the vortices cannot move* slower *or faster than the fluxoids for the ezpected post-glitch values of the lag* $|\omega_G| <$ w, *they move together* and the superfluid spin-down rate ($\dot{\Omega}_s \propto v_n$) is therefore determined by the radial velocity of the fluxoids v_p. The change in $\dot{\Omega}_s$, and hence $\dot{\Omega}_c$, due to a glitch in a pulsar at a given age $t_{sd} = t_G$ might be therefore determined from a comparison between the *steady state values* of v_n at $t = t_G$ and v_p corresponding to w, $= \omega_G$.

The steady state motions of the fluxoids at a given *r* might be determined by solving the equation of motion of the fluxoids, given the spin-down torque on the star which determines $\dot{\Omega}_s(t) \equiv \dot{\Omega}_c(t)$ and thus the radial velocity of the vortices v_n, as we did in Chapter **3.** The flwoid equation of motion was constructed by balancing the pinning force exerted by the vortices on them at the crossing points, as well as an outward buoyancy force on the fluxoids and an inward drag due to the scattering of electrons. For a single pulsar subject to the standard dipole torque the predicted steady state time behavior of the vortex radial velocity v_n, fluxoid radial velocity v_p, the lag w, and the critical lagw, at distances of $r = R_c$ and $r = \frac{R_c}{10}$ are shown in Fig. 4.5a & b, respectively, where R_c is the radius of the core. These are the results from computations same as that described in Chapter 3, where we have repeated the same procedure (cf. § 3.2.4) for the above two radial distances. Three successive phases of the relative motion between the lines are realized during the lifetime of the star, in which the vortices move faster, together, and slower than the fluxoids. During the co-moving state (v, $= v_n$) the lag changes sign from its positive value throughout the previous phase w, $= \omega_{cr}$ to a negative value *w,* $= -\omega_{cr}$ which will hold true during the subsequent phase.

Following a glitch in a pulsar younger than $\sim 10^6$ yr the force on the fluxoids due

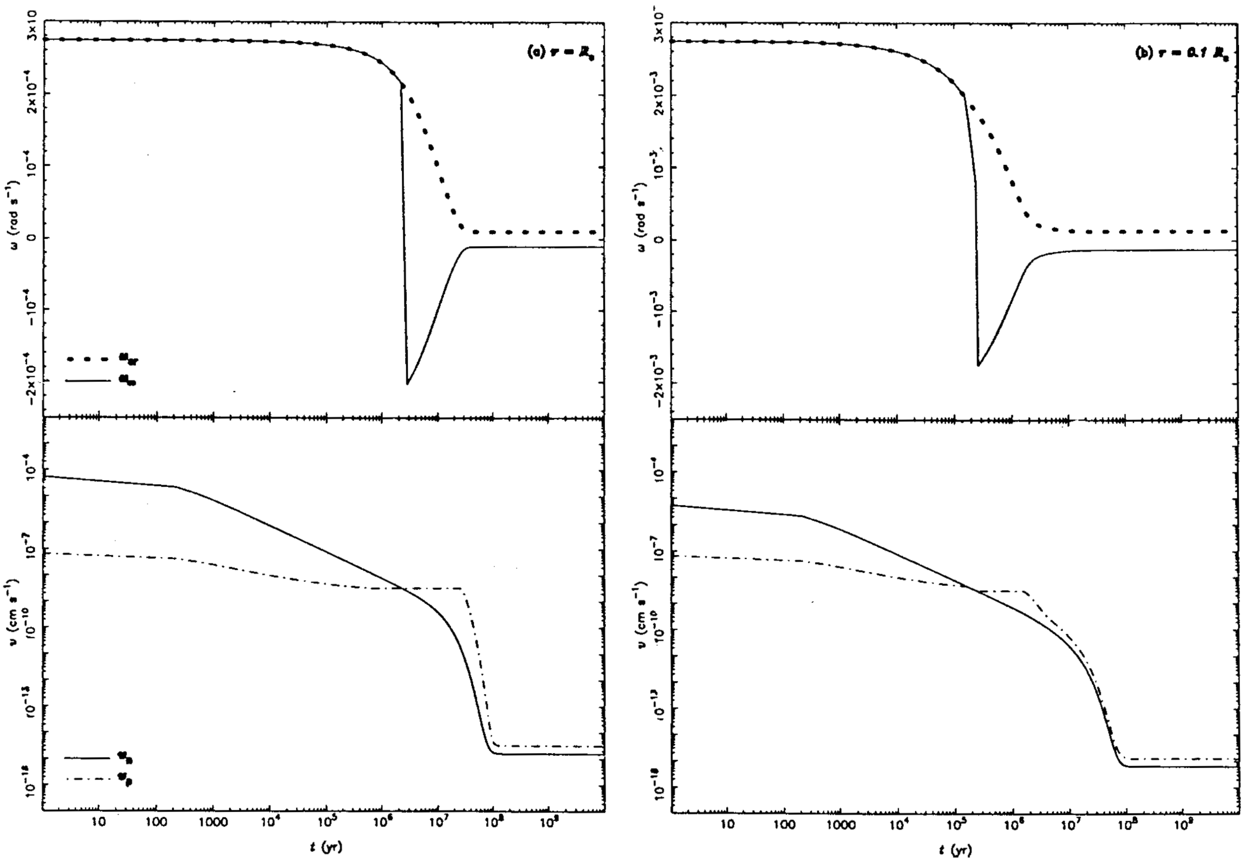

Figure 4.5- Time evolution of the superfluid rotational lag w (*upper* graphs) and the radial velocity v of the fluxoids and the vortices *(lower* graphs) at a distance r from the rotation axis: *(a)* is for the conditions at $r = R_c$; **(b)** is for $r = 0.1R_c$, where R_c is the radius of the stellar core. Note the difference in scales for w between *(a)* and (b).

to the vortices which are now co-moving with them is smaller than in the steady state, therefore the opposite drag force and hence v_p are also reduced correspondingly. However, because of the inverse proportionality of ω_{cr} with r the glitch induced departure of w from its steady state value $(= w_\infty)$ could be very small except for an outer region of the core superfluid with an assumed limiting value of $\omega_{cr} \sim 3 \times 10^4$ rad s^{-1}. In addition to the $\frac{1}{r}$ behavior due to the form of the Magnus force, ω_{cr} might also depend on the density as well as the magnitude of the superfluid energy gap. Assuming that the contributions of the latter two effects cancel out, $\omega_{cr} \propto \frac{1}{r}$ will be considered as a first approximation.

Hence for the inner regions where $\omega_G \sim \omega_{cr}$ after a glitch, the steady state value of v_p (Fig. 4.5) at the given time $t = t_G$ might be used as an estimate for the post glitch values of v_n. As can be seen from Fig. 4.5 the change in v_n due to a glitch might be therefore expected to be very small, in particular for the inner regions (Fig. 4.5b). The post-glitch value of v_n in the outer regions with a substantial difference between ω_G and ω_∞ would be however equal to v_p corresponding to $\omega_\infty = \omega_G$. An estimate of the proper values of v_p applicable to the latter case might be obtained from the steady state values during the co-moving phase shown in Fig. 4.5. The decrease in v_p during this period (at times $10^6 < t$ (yr) $< 10^7$) compared to its earlier values is, nevertheless, seen to be not very large and hence the superfluid in the outer regions might also be expected to be slowing down with a rate not much smaller than its steady state value.

A quantitative evaluation of the post-glitch response of the superfluid and the crust would require a more realistic simulation of the time evolution of v_n and v_p, taking into account also the r-dependence of the various quantities. The extent to which the collective motion of either of the vortex lines is maintained and influence the dynamics of the rest of the corresponding lattice need also to be specified and taken into account in a 2D geometry, self-consistently. In addition, it is noted that the results in Fig. 4.5 which were used to infer the variation in v_n due to a change in w have been derived for apriori given values of v_n. The superfluid response following a jump in Ω_c and hence for a given value of $w = \omega_G \neq \omega_\infty$ should be, of course, determined from a simultaneous solution to the EOM of both the vortices and the fluxoids. In particular, the unrealistic jump in v_n as implied by the above discussion, at a value of $w = w$, is due to the neglect of the drag force acting on the vortices due to the electron scattering.

4.4.2 Does the "lag" decrease continuously or discontinuously?

As may be seen from fig. 4.5, as the neutron star ages there is a secular decrease in the rotational lag w between the "crust" and core superfluid. This is a result we had obtained earlier in Chapter **3,** and we have reproduced it here for ready reference. Further, the lag w between the crust and the core superfluid varies as a function of the perpendicular distance from the rotational axis. In fig. 4.5 we have explicitly plotted the variation of the rotational lag at two radii viz., $r = R_c$ and $r = 0.1R_c$ where R_c is the radius of the fluid core. The important point to notice is that at all radii the rotational lag decreases with time and finally becomes vanishingly small around an age $\sim 10^7$ yr. The basic reason for this is the decrease in the number density of fluxoids as the flux continues to be expelled from the core. More explicitly, the *pinning force per unit length* of *a* vortex is inversely proportional to the spacing between the fluxoids, and consequently decreases as the fluxoids are expelled from the core. As may be recalled from the discussion in Chapter **3,** since the pinning force on the vortex exerted by the fluxoids is balanced by the ***Magnus*** *force,* a reduction in the former implies a reduction in the latter force which in turn is proportional to the rotational lag *w.* We wish to mention in passing a curious feature of the rotational lag as plotted in fig. 4.5 viz., that it suddenly becomes "negative". As may be seen by comparing the two upper panels with corresponding lower panels, this is merely a reflection of the fluxoids moving ahead of the vortices (v, > *v,).* As was explained in the previous chapter, this is a consequence of the dominance of the buoyance force at late times. *In models in which the buoyancy force is neglected altogether the rotational lag will lever become negative.* Nevertheless, in all models the lag will decrease till eventually the superfluid in the core comes into near corotation with the rest of the star.

A comparison of fig. 4.5a and 4.5b will show that the magnitude of the rotational lag is a function of the radial distances from the rotation axis. In other words, the core superfluid is in fact "rotating" differentially. Further, the secular decrease in the rotational lag w occurs first at inner radii and later at the outer radii.

The predicted relative values of the angular velocity of the core superfluid (Ω_s) and that of the crust (R,) as a function of the radial distances from the rotation axis is plotted in fig. 4.6. Two cases are considered: (a) a relatively young pulsar with a characteristic age comparable to the Crab pulsar, and (b) a relatively old pulsar with a characteristic age $\geq 10^7$ yr. As may be seen by comparing the two figures, by an age $\sim 10^7$ yr the superfluid comes into near corotation with the rest of the star. The excess angular momentum it had must obviously have been shared with a crust, resulting in a secular spinning up of the crust superimposed on the secular spinning down due to the radiation torque acting on it. *The key question* **is** *whether the change in the rotational lag indicated in* fig. *4.6 occurs in a gradual and continuous manner* ***or*** *in discrete steps.* If the latter situation is the relevant one then it provides an interesting origin for discrete spin-up events of the crust. It is tantalizing to **ask** whether some of the observed glitches are of this origin?! As was pointed out long time ago by Packard (1972) and Anderson & Itoh (1975) it is quite likely that pinned vortices in a neutron star exhibit metastable equilibrium states, as observed in laboratory experiments of superfluid (Tsakadze & Tsakadze 1975). It may be recalled that in the conventional model glitches are due to catastrophic unpinning of the vortices in the ***crustal*** *superfluid.* What we have argued above provides an alternative or additional(?) mechanism for discrete spin-up events.

If on the other hand the rotational lag w decreases continuously then it would manifest itself as a change in the breaking index of pulsars ($n = \frac{\Omega_c \ddot{\Omega}}{\dot{\Omega}_c^2}$). In any case, we believe that this is the first time anyone has pointed out the possible interplay between the secular field evolution of a neutron star and its rotational dynamics.

Having pointed out this intriguing effect, we shall trace in smaller steps in time the change in the rotational lag. This is done in fig. 4.7. Based on this one can make some

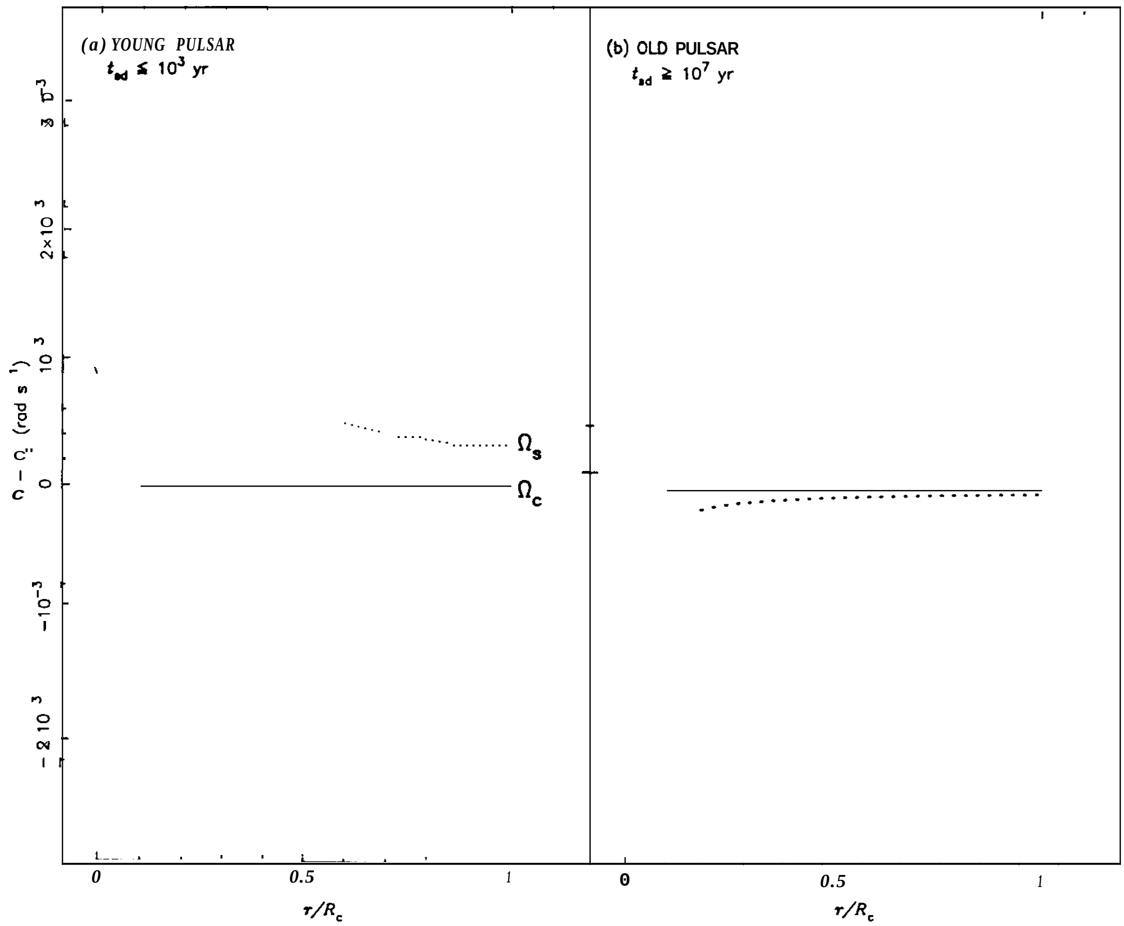

Figure 4.6- Relative profile of the the angular velocity of the neutron superfluid (dotted line) with respect to that of the "crust" ***(full*** line) in the core of a neutron star when it is **(a)** very young, and (b) very old.

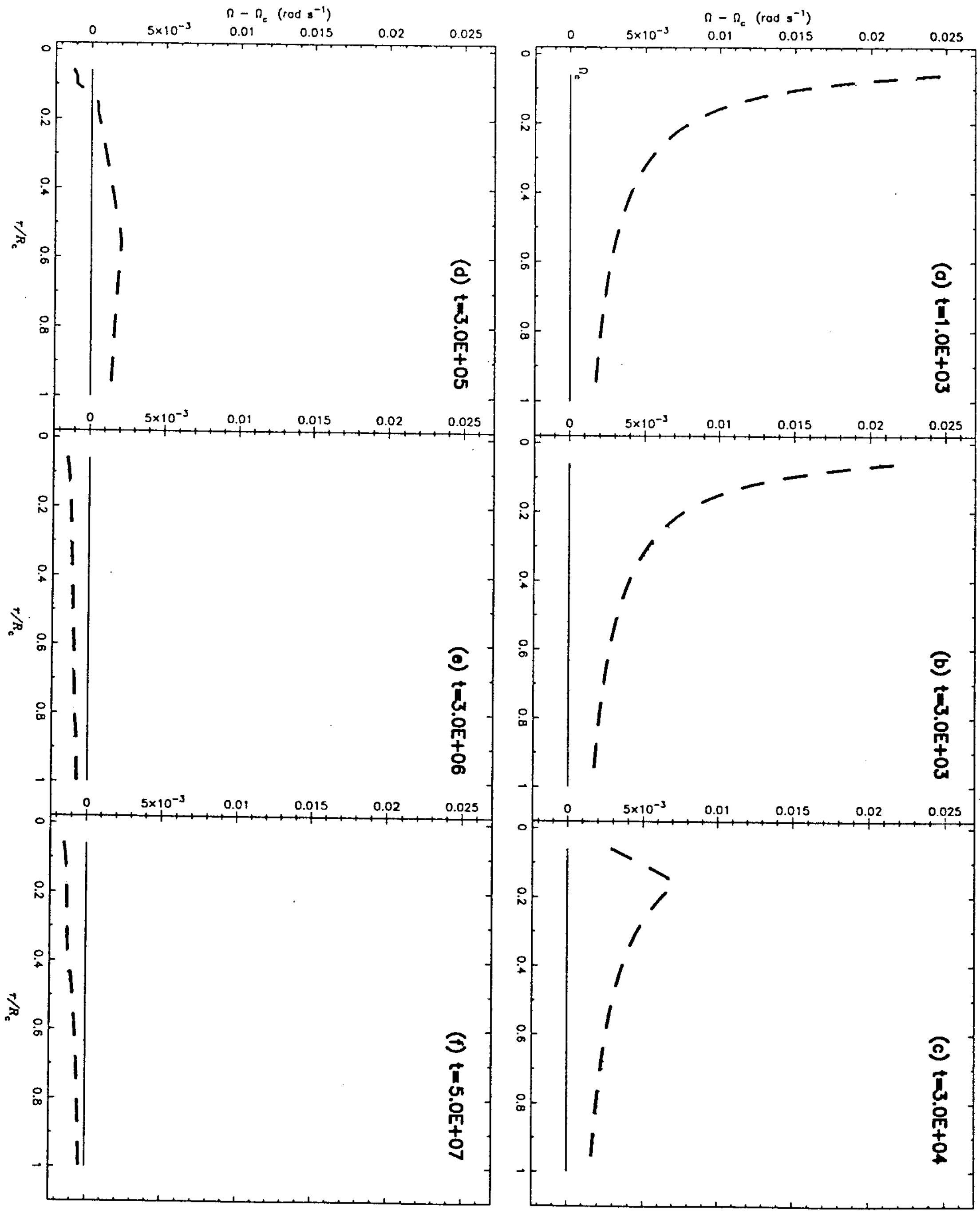

Figure 4.7- Relative profile of the the angular velocity of the neutron superfluid (thick dashed line) with respect to that of the "crust" (fullline) in the core of a neutron star at the different times marked on each graph in units of years. A value of the pinning energy $E_P = 1.8$ M eV has been used.

broad statements.

- The induced jumps in the rotation rate of the crust due to discontinuous decrease in rotational lag is expected to be smaller in younger pulsars. This is because of the smaller moment of inertia of the pinned core superfluid in the central regions of the star.

- In older pulsars the core superfluid in the innermost regions would have already reached corotation with the crust, and it is only the outer regions that still have a differential rotation with respect to the crust. Therefore in older pulsars any induced change in angular velocity of the crust must be due to the outer regions of the core superfluid.

- The sudden change in the angular velocity of the core superfluid *in a certain region of the core* may or may not manifest as a sudden rise in the rotation rate of the crust, or in other words a "glitch". If the excess vortices released originate from the outer region of the core superfluid then they might move outwards fast enough to induce a standard "glitch" viz., a rapid rise in the rotation rate of the crust. Thus, if glitches are due to a sudden decrease in the angular velocity of the core superfluid then *such events are more likely in an older pulsar.*

  As already remarked, in young pulsars it is the inner regions of the core superfluid that first come into corotation with the crust (see fig. 4.6 and 4.7). In order for such an event to manifest itself as a sudden spin-up of the crust the vortices in the entire core of the star must be able to re-adjust to a number density appropriate to a reduced value of the angular velocity of the core superfluid. If pinning with fluxoids is rather strong this may not happen "instantaneously". Rather, it might take a relatively long time to transfer the excess angular momentum from the inner regions of the core superfluid to the crust. It is conceivable that the slow rise in the angular velocity of the Crab pulsar (over a period of many days) following a sudden increase (such as observed in the 1989 glitch) may be due to the effect that we have described here.

Post-glitch Recovery : the unrelaxed component

A glitch induced by a sudden decrease in ω_∞ moreover provides a natural explanation for the observed large unrelaxed component of Ail, in various pulsars (Lyne 1995). The unrelaxed part of a glitch is parametrized as $(1-Q)\Delta\Omega_c$, with $0 \leq Q \leq 1$, while values of $Q \lesssim 10^{-2}$ have been observed in Vela and older pulsars. At a glitch, $\Omega_s(r)$ in some part of the superfluid jumps to a lower value corresponding to its current expected value of $\omega_\infty(r)$. This, however results in an increase in Ω_c which will further reduce the lag in that region as well as in the rest of the core to values smaller than their steady state values, $\omega_G(r) < \omega_\infty(r)$ everywhere. The observed post-glitch recovery of a Q fraction of the initial jump will correspond to the long term build up of $\omega_G(r)$ due to a lower spin-down rate of the superfluid until $\omega_G(r) = \omega_\infty(r)$ is established throughout the core, as discussed earlier. The $(1-Q)$ part of the glitch, on the other hand, corresponds to the sudden decrease in the value of $\omega_\infty(r)$ which will show up as an offset above the pre-glitch extrapolated values of Ω_c. Furthermore, Fig. 4.5a and Fig. 4.5b show that the predicted decrease in $\omega_\infty(r)$ occurs at later times for larger T-values. Since the moment of inertia of the outer regions which undergo a jump in ω_∞ at later times are also larger, smaller Q values would be hence expected in older pulsars as is observed (Lyne 1995). The accumulated effect of the unrelaxed parts of glitches $\Delta\Omega_{\rm total}$ between times t_1 to t_2 might be estimated as

$$\Delta\Omega_{\rm total} = \int_{t_1}^{t_2} (1 - Q(t))\ \Omega(t)\ A(t)\ {\rm d}t \tag{4.12}$$

where the glitch activity parameter $A(t)$ is the fractional increase in the rotation frequency (per year) due to glitches, and t is in units of yr. Using the fitted lines to the observational data given by Lyne 1995, the functions $Q(t) \sim 176t^{-0.88}$ and $A(t) \sim 4.87 \times 10^{-3} t^{-1.04}$ are derived, where the latter applies to times $t \gtrsim 10^4$ yr. Also the observed pulsar periods might be fitted as $\Omega_c \sim 7000({\rm rad\ s^{-1}})t^{-0.5}$ for an assumed standard dipole torque. Neglecting the contribution from times $t < 10^4$ yr which is small because of the large values of Q, Eq. 4.13 then results in a value of $\Delta\Omega_{\rm total} \sim 0.3$ rad s^{-1}, due to all glitches during a pulsar lifetime. In order for the unrelaxed parts of the glitches to be associated with the decline of *w,* during a pulsar lifetime, an average initial value of $\omega_\infty = \omega_{cr} \sim 0.03$ rad s^{-1} is thus required for the core superfluid. Two possible corrections have to be however noted while comparing this with the results in Fig. 4.5, in addition to the poor statistics of the data used to derive the expressions for A and Q. Firstly, the values of ω_{cr} in Fig. 4.5 correspond to an assumed lower limit of $E_P \sim 0.3$ MeV which could as well be larger by a factor of $\gtrsim 6$; since $\frac{\lambda}{\xi} = \sqrt{2}$ has been used instead of $\frac{\lambda}{\xi} \sim 10$ for the proton superconductor in the core, hence $\frac{\ln 10}{\ln\sqrt{2}} \sim 6$ due to $\ln\frac{\lambda}{\xi}$ dependence of E_P (see Fig. 4.8). Secondly, the accumulated angular momentum in a superfluid component having a steady state slow-down rate less than the rest of the star is expected to contribute to the unrelaxed parts of the glitches. This is indeed the solution of the vortex creep model to the observed small Q-values, assumed to be caused by a permanently pinned fraction of the crust superfluid which is referred to as the "capacitor region" (Alpar 1995). In the case of the jumping lag model even though the superfluid is not decoupled completely, however the regions wherein ω_∞ and ω_{cr} decrease earlier (Fig. 4.5) might be expected to support a smaller *steady state* value of v_n and hence a smaller $|\dot{\Omega}_s|$ than the outer regions. The excess lag which is therefore built up in the inner regions between successive glitches would have a similar effect, though smaller, as suggested in the vortex creep model for the completely pinned crust superfluid.

4.5 Summary and conclusions of this Chapter

In Chapters 2 and **3** we explored in detail the consequences of the interpinning between vortices and fluxoids for the evolution of the magnetic field of a neutron star. Interestingly, this investigation itself turned out to produce the strongest argument for the interpinning (see the concluding section of Chapter **3).**

If such an interpinning occurs, and is strong, then it is quite conceivable that it could have interesting consequences for the rotational history of the neutron star itself. In the standard literature it is assumed that the core superfluid is strongly coupled to the crust of the star because of the magnetic scattering of electrons of the vortices. It is explicitly assumed that the angular velocity of the core superfluid does not significantly differ from that of the crust. If the crust is suddenly spun up or spun down then it is assumed that the core superfluid will quickly respond through a re-arrangement of the vortices. It is this assumption that we have questioned in this chapter. *If vortices are strongly pinned to the fluxoids then a re-arrangement of the vortex lattice cannot be assumed to take place in arbitrarily short timescales, regardless of how quickly the information about sudden changes in the crustal angular velocity is communicated to the core superfluid.* To the best of our knowledge this question has not been investigated earlier. Our preliminary and exploratory investigation of this interesting question has already enabled us to draw some specific conclusions which we summarize below.

- We have argued that the core superfluid must be taking part in the longterm steady state spin-down of a neutron star. Any statement to the effect that the core superfluid should be considered as decoupled from the crust over timescales of years or so does not appear warranted. Therefore doubts expressed in the literature about the inferred surface magnetic fields in neutron stars (due to an over-estimation of the moment of inertia of the star which reacts to the radiation torque) are irrelevant.

- One of the major difficulties with the standard model of the glitch is in accounting for the large moment of inertia of the loosely coupled component of the superfluid that are implied by observations. Some of the recent giant glitches observed in the Vela and other puslars imply a moment of inertia of the superfluid involved as large as 20% of the total moment of inertia of the star. But according to standard models, the moment of inertia of the crustal superfluid cannot be more than $\sim 1\%$ of the total moment of inertia. This is the main difficulty with the standard theories of glitch that are currently in vogue. *It seems to us that this is a clear indication that the core superfluid must be an essential actor in the play.* Even if the concept of pinning between vortices and fluxoids is granted, it does not necessarily imply that the entire superfluid is affected. This depends on the relative orientation of the rotation axis and the magnetic axis of the star. For an arbitrary orientation only a fraction of the core superfluid will be constrained by pinning. This is because a certain fraction of vortices can *slide* along the fluxoids and therefore not experience any pinning barriers. While it is premature to attempt a detailed calculation of what fraction of the moment of inertia of the core superfluid is likely to be pinned, it seems quite reasonable to assert that *the fraction which is pinned will respond differently from the fraction that is unpinned.*

- **A** rather interesting consequence of the pinning of vortices'on fluxoids is the lag between the angular velocity of the core superfluid and the crust – unlike in the case where there is no pinning, the superfluid will be rototating faster than the crust in the steady state. As the magnetic field is expelled from the core, pinning force per unit length of the vortex will decrease. Consequently, the steady state lag between the core superfluid and the crust will decreate till eventualy it will practically vanish as the pinning force per unit length of the vortex becomes negligible. Thus, during the first 10^6 years or so one has a reservoir of angular momentum residing in the core superfluid. *We have argued that if the lag decreases in a series of discontinuous steps then it provides an alternative or additional mechanism for sudden spin-up of the crust as observed in glitches.* We have also argued that if at all relevant this is more likely to happen in older pulsars.

- In younger pulsars, such as the Crab pulsar, even a sudden decrease in the angular velocity of the core superfluid may only result in a slow increase in the angular velocity of the crust. The behaviour of the 1989 glitch has the right signature to be explained along these lines. To conclude, it has not been our intention to construct a detailed theory of the phenomenon of glitch involving the core superfluid. We merely wish to make the following observation. In our opinion the arguments for the interpinning between the vortices in the core superfluid and the fluxoids in the core superconductor are indeed compelling. If one grants this then there must be an interesting interplay between the magnetic evolution of a neutron star and its rotational dynamics. Our preliminary investigation reported here suggests a variety of rich and interesting possibilities.