CHAPTER VI

VERIFICATION OF THE LESLIE'S EQUATION FOR THE THRESHOLD FIELD OF A TWISTED NEMATIC CELL

Introduction

fa Chapter I we had considered the deformations in a thin film of nematic liquid crystal in three different geometries under the application of external fields. We shall now consider another geometry, viz., the deformation in a twisted nematic cell (TN cell) due to an external field acting along the, axis of twist. In the TN cell, the director is anchored parallel to the glass surfaces but the director axis is twisted on going from one plate to the other. Commonly the angle of twist is 90° (about an axis normal to the film) (Figure 6.1(a)). A TN cell acts as a retarder as well as a rotator, but because of the very large birefringence, the adiabatic theorem comes into force. In such a case a linearly polarised beam incident with its polarisation along (or perpendicular to) the director at the boundary travels with its polarisation always along (or perpendicular to) the local director (Chapter II).

Consider a TN cell with 90° twist. The light transmitted through this cell can be completely cut

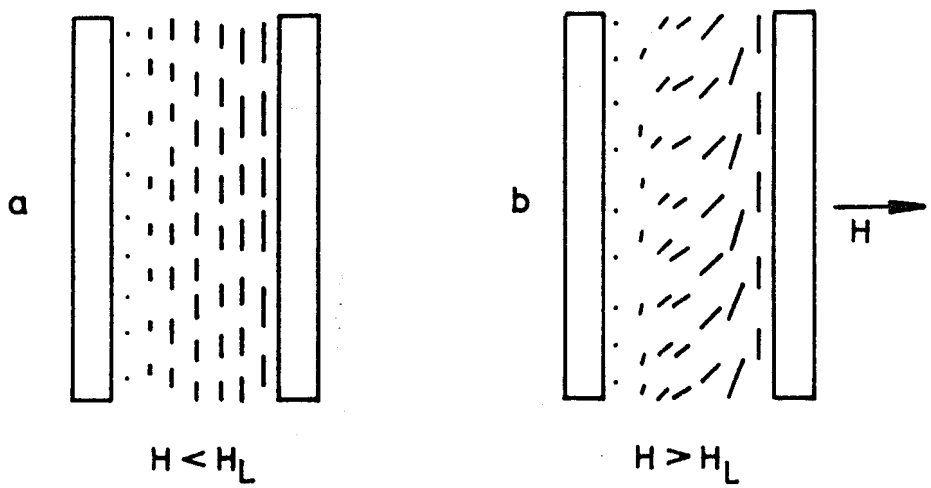

Figure 6.1

Schematic diagram showing the molecular alignment in a twisted nematic cell (a) when the magnetic field is less than Leslie threshold (H_L), and (b) when $H > H_L$.

off, if the cell is sandwiched between two parallel polarizers with polarization direction along $\vec{n}$ at either of the plates. Now if a sufficiently large magnetic field is applied along the twist axis, the deformation induced in the medium is such that the director tends to be parallel (except at She bounding surfaces) to $\vec{H}$ resulting in the transmission of light through the system (Figure 6.1(b)). Similar results could be obtained if the magnetic field is replaced by an electric field and the nematic liquid crystal has positive dielectric anisotropy. Schadt and Helfrich (1971) were the first to recognise the importance of such an electro-optic effect for display devices.

A general theory of the deformation in a twisted nematic sample has been given by Leslie (1970). He has derived an expression for a critical magnetic field required to distort the sample (Leslie threshold) when it is applied along the twist axis. The expression involves all the three elastic constants and as we mentioned in Chapter If this method can be used to determine k_{22} if we know k_{11} and k_{33}. Since we have measured all the elastic constants of a number of nematic compounds, we have made measurements on a TH sample in order to compare the experimental results

with the theory. We shall now briefly outline Leslie's theory.

Theory

Consider a twisted nematic sample contained between two plane glass plates. Let the rubbed directions in the two plates have an angle φ_0 between them. x_0 is the sample thickness. We choose the coordinate system such that $\vec{n}$ at the first plate is along X $\vec{n}$ rotates about Z axis so that at $z = x_0$, $\vec{n}$ makes an angle φ_0 with the X axis. Let a magnetic field be applied along Z axis, i.e., along the twist axis so that a weak distortion takes place in the medium, in addition to the initial twist. At any distance z, ($< x_0/2$ from the origin), $\vec{n}$ makes an angle θ with the xy plane (figure 6.2). The projection of $\vec{n}$ on xy plane makes an angle φ with x axis. Hence $\theta = \theta(z)$ a $= \varphi(z)$. In the absence of any external fields $\theta = 0$, $\varphi = \frac{\varphi_0}{2}(z - \frac{x_0}{2})/\frac{x_0}{2}$.

$$n_x = \cos\theta \cos\varphi, \quad n_y = \cos\theta \sin\varphi, \quad n_z = \sin\theta.$$

$$H_x = H_y = 0 \ , \qquad H_z = H.$$

Minimising the energy of the system (Leslie 1970) we get the following coupled equations of equilibrium.

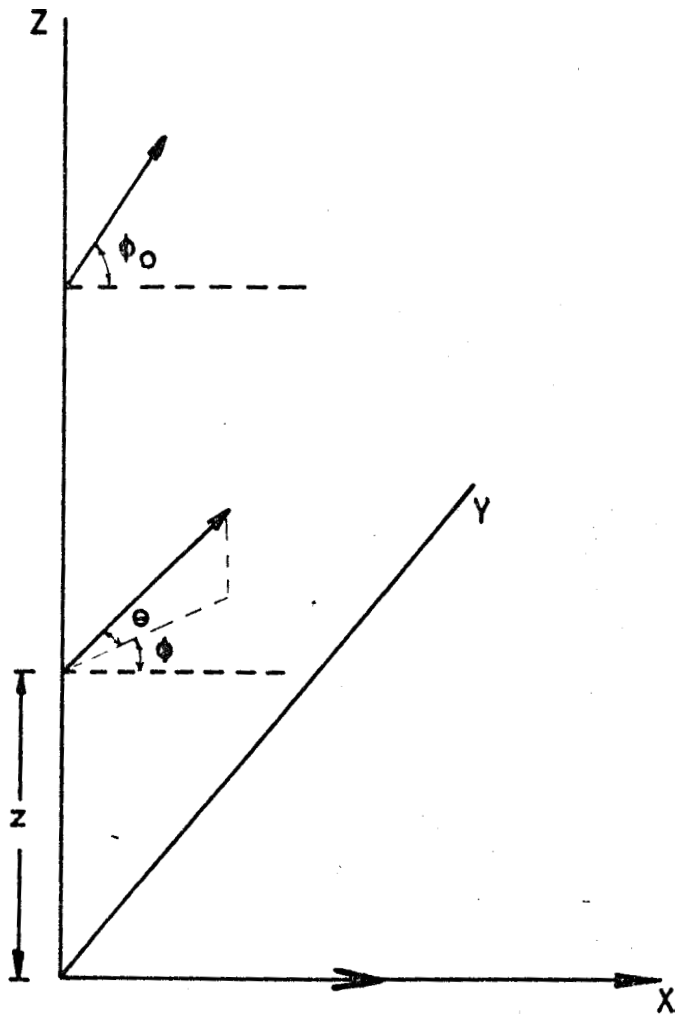

Figure 6.2

Diagram explaining the definitions of the angles $\theta(z)$ and $\varphi(z)$. The sample thickness is x_o. Magnetic field in applied along the twist axis (Z).

$$f(\theta)\frac{d^2\theta}{dz^2} + \frac{1}{2}\frac{d}{d\theta}[f(\theta)](\frac{d\theta}{dz})^2 - \frac{1}{2}\frac{d}{d\theta}[g(\theta)](\frac{d\varphi}{dz})^2 + \Delta\chi H^2 \sin\theta\cos\theta = 0 \quad (6.1)$$

and

$$g(\theta)\frac{d^2\varphi}{dz^2} + \frac{d}{d\theta}[g(\theta)]\frac{d\theta}{dz}\cdot\frac{d\varphi}{dz} = 0 \quad (6.2)$$

where

$$f(\theta) = k_{11}\cos^2\theta + k_{33}\sin^2\theta$$

$$g(\theta) = (k_{22}\cos^2\theta + k_{33}\sin^2\theta)\cos^2\theta$$

Integrating equation (6.2), we get

$$g(\theta)\frac{d\varphi}{dz} = \text{a constant } k \quad (6.3)$$

Now multiplying equation (6.2) by $2\frac{d\theta}{dz}$ and equation (6.3) by $2\frac{d\varphi}{dz}$, adding and integrating, we get

$$f(\theta)(\frac{d\theta}{dz})^2 + g(\theta)(\frac{d\varphi}{dz})^2 + \Delta\chi H^2\sin^2\theta = \text{a constant } c.$$

To find the value of c, we can apply the boundary conditions.

At the mid-plane of the sample,

$z = \frac{x_o}{2}$, $\frac{d\theta}{dz} = 0$, $\theta = \theta_m$ and $\frac{d\varphi}{dz} = \frac{k}{g(\theta_m)}$.

Hence

$$f(\theta)(\frac{d\theta}{dz})^2 + \frac{k^2}{g(\theta)} + \Delta\chi H^2\sin^2\theta = \frac{k^2}{g(\theta_m)} + \Delta\chi H^2\sin^2\theta_m ,$$

i.e.,

$$f(\theta)\left(\frac{d\theta}{dz}\right)^2 = k^2\left(\frac{1}{g(\theta_m)} - \frac{1}{g(\theta)}\right) + \Delta\chi\, H^2(\sin^2\theta_m - \sin^2\theta) \tag{6.4}$$

Integrating

$$\int_0^{\theta_m} \left[\frac{f(\theta)}{\Delta\chi H^2(\sin^2\theta_m - \sin^2\theta) + k^2\left(\frac{1}{g(\theta_m)} - \frac{1}{g(\theta)}\right)}\right]^{\frac{1}{2}} d\theta = \int_0^{\frac{x_o}{2}} dz = \frac{x_o}{2}$$

Writing $\sin\theta = \sin\theta_m \sin\psi$, we get

$$\int_0^{\pi/2} \left[\frac{f(\theta)}{\Delta\chi H^2 - k^2\left[\frac{g(\theta_m) - g(\theta)}{g(\theta_m)g(\theta)(\sin^2\theta_m - \sin^2\theta)}\right]}\right]^{\frac{1}{2}} \frac{d\psi}{\cos\theta} = \frac{x_o}{2} \tag{6.5}$$

Now as $\theta_m \to 0$, $\theta \longrightarrow 0$, and we *gat* an expression for a critical field H_L given by

$$\Delta\chi \cdot x_o^2 H_L^2 = \pi^2 k_{11} + (k_{33} - 2k_{22})\varphi_o^2 \tag{6.6}$$

Thus we see that the magnetic field-induced distortion starts only beyond a threshold field, which depends an the sample thickness, the angle of twist and the elastic constants of the compound.

As we mentioned earlier, the plane of polarization of the incident beam follows the director as it traverses the TN cell. To understand this consider a sample $\sim$ 20 μm thick. We can imagine the medium to be divided into a number of thin sections ($\sim 10^{-7}$ a thick) parallel to the bounding surfaces. For a 90° TN cell, the rotation per section is $\beta \sim 10^{-4}$ rad./cm. If the birefringence of the medium is 0.2, the phase retardation between ordinary and extraordinary vibrations per section is $\alpha \sim 2 \times 10^{-3}$. Hence α/β is quite large. In such a case, as was shown in Chapter I, the polarization follows the director. Therefore, if the incident polarization is along the initial director, even when a small field-induced distortion takes place in the medium, the state of polarization of the emergent light is practically unchanged. A significant change occurs only when a considerable distortion is induced in the medium such that β and α are comparable in their magnitudes. Thus the optical threshold (H_o) is higher than Leslie threshold (H_L). This lo fact has been observed by earlier workers. We must point out here, that as the thickness of the sample is reduced β increases and consequently H_o comes closer to H_L.

Previous work

Gerritsma et al. (1971) measured the capacitance of a twisted nematic cell of MBBA in the presence of a magnetic field applied along the twist axis. It was observed that the capacitance started decreasing beyond a critical field. The experiment was repeated for 5 thicknesses of the sample. The product $H_c x_o$ was found to be almost a constant except for $x_o < 10$ μm. This is to be expected since the capacitance depends on the alignment of n with respect to the field and is unaffected by the ratio β/α. However, it was found that when the observation was made optically the transmitted intensity started to change sharply at a field which was higher than H_c. (For example, for $x_o = 54$ μm, $H_o \sim$ 50% larger than H_c.)

Van Doorn (1973) made computer calculations for the intensity of light transmission through a TN cell using the theory of de Vries (1951) with slight modifications. The computation gave a thickness dependent values for the product $H_c x_o$. As we mentioned earlier, this can be easily understood in terms of the ratio β/α.

Berreman (1973) has also made computer calculations fur the transmittance of a TN cell between

parallel polarizers (parallel or perpendicular to rubbed directions) as a function of the angle of incidence of light. The calculations showed that the transmitted intensity changed only after the applied field is considerably larger than the Leslie threshold.

When a field induced distortion begins in the sample the birefringence changes. Hence if one has an optical set up which is sensitive to small changes in the birefringence rather than changes in the azimuth of the emergent light, Leslie threshold can be detected optically. We shall describe such a set up below.

Experimental

The experimental arrangement was the same as the one used for the measurement of k_{11} (Chapter III). A TN cell with twist $\varphi_0 = \pi/2$ was used. (The molecular alignment is shown in figure 6.1). The alignment of the director parallel to the glass surfaces was achieved by a thin film of silicon deposited at an oblique angle. The alignment of the sample with respect do the magnetic field and the measurement of the critical field, eta., were as explained in Chapter II. A light beam linearly polarised at 45°

to the initial director was allowed to enter the sample. Since the twist fa the sample is small compared to its phase retardation the ordinary and extraordinary vibrations follow the director and an elliptically polarised light emerges out of the sample. A quarter wave plate with appropriate orientation of its axis converts the light into a linearly polarised one which is then detected by a linear analyser.

To start with, both the quarter wave plate and the analyser were rotated to see a dark field of view. As the magnetic field is increased beyond the critical value, the field of view becomes bright.

In order to compare this threshold field with that when the polariser is parallel to the initial director, the experiment was repeated by appropriately rotating the polarizer, the analyser and the quarter wave plate. The threshold field in this case was found to be larger than that detected earlier.

Results and discussions

We have made measurements on 7CB for different thicknesses of the sample. Since all the three elastic constants are available (Chapter III) for this compound, Leslie's equation [Eqn.(6.6)] could be verified.

Table 6.1 contains values of $H_L x_o$ from five independent samples, for various temperatures. We have 3 sets of values for 25 μm samples and 2 for 50 μm samples. The maximum spread in the product $H_L x_o$ at any temperature is ±3.0% (figure 6.3). In general the values for the 50 μm sample are slightly 1 Since the error in the measurement of magnetic field is ~ ±25 gauss, the lower the critical field the larger is the error involved. Hence for tho 50 μm sample the error is larger (~ ±2%) compared to that for the 25 μm sample (~ ±1%). Moreover the procedure followed to detect H_L was to decrease the field in steps of 25 gauss and hence one tends to take slightly lower values (but within the error limit) for H_L. This is probably the reason why we get consistently lower values of $H_L x_o$ for 50 μm sample. We must mention here that the error involved in thickness measurement is small (± 0.1 μm). Allowing for these errors, the product $H_L x_o$ is a constant at any temperature. This indicates that the measured value of H_L is the Leslie threshold.

Using k_{11}, k_{22} and k_{33} values from Chapter III we have calculated the values of $H_L x_o$ using equation (6.6) for various temperatures. The calculated variation is shown as the solid curve in figure 6.3

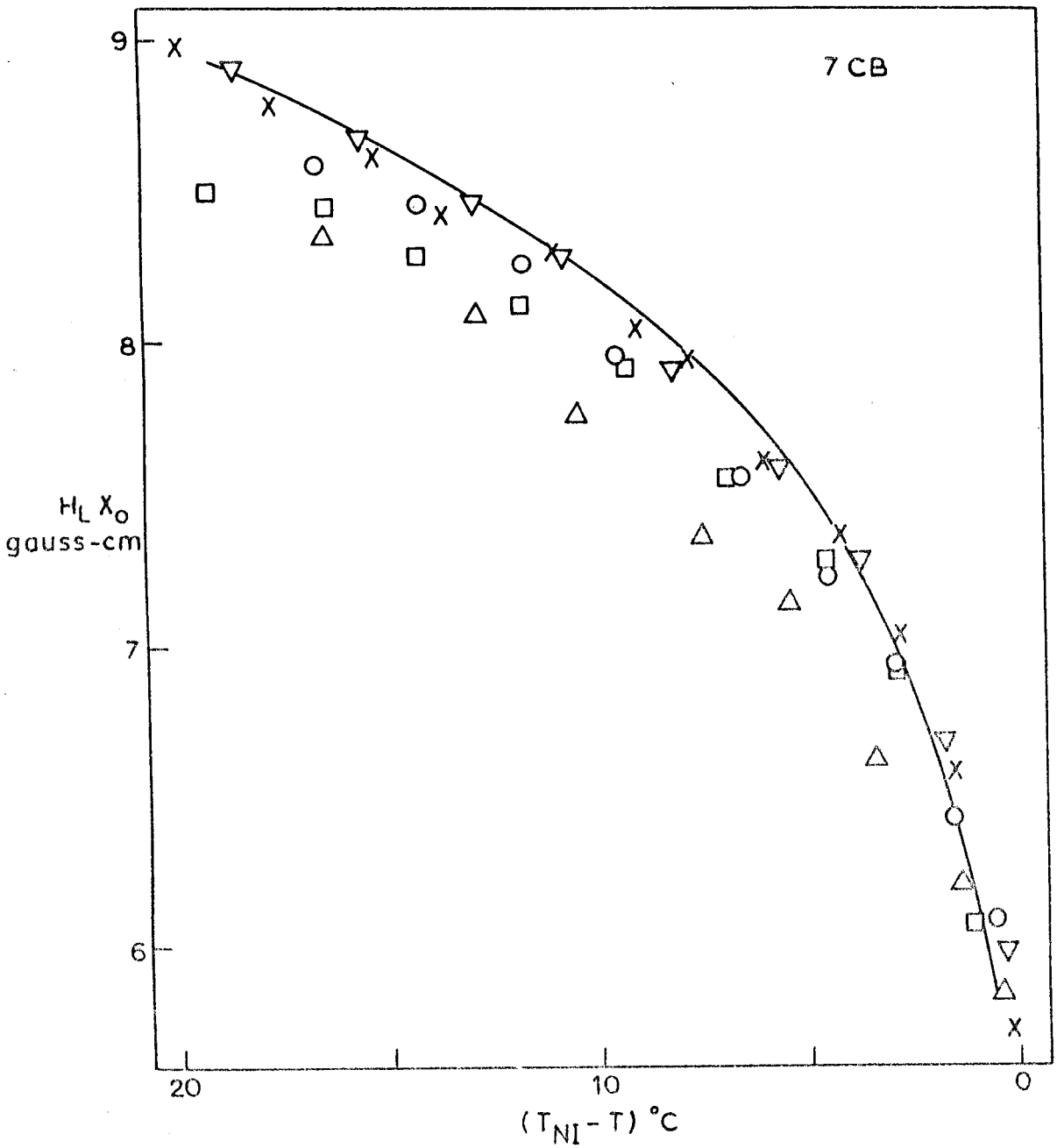

Figure 6.3

The product of the threshold field (H_L) and sample thickness (x_o) as a function of temperature. The different symbols represent the values obtained for different samples ; $\sim$ 25 μm (○, ▽, ×), $\sim$ 50 μm (□, △). The solid curve shows the variation according to Leslie's equation.

along with the experimental values. Now taking typical values of elastic constants, we have for 7CB at $T_{NI}-T = 10°C$, $k_{11} = 1.3 \times 10^{-6}$, $k_{22} = 0.7 \times 10^{-6}$, $k_{33} = 1.7 \times 10^{-6}$ we see that $(\frac{k_{33}-2k_{22}}{4})$ is small compared to k_{11}. Therefore in equation (6.6) the major contribution is from k_{11}. Hence the error in the theoretical estimate of $H_L x_o$ is $\sim \pm 1\%$. Considering the spread in the mean experimental value of $H_L x_o$ ($\pm 3\%$), we observe in the figure 6.3 that the theoretical and the mean experimental values agree within the limits of error. Thus Leslie's equation is experimentally verified.

We have also measured the optical threshold in the usual optical set up. The polariser axis was set parallel to the initial director. The $\lambda/4$ plate and analyzer were rotated to observe a dark field of view. As the magnetic field was increased, the field of view started brightening at a field H_o. For 25 μm sample at $T_{NI}-T = 1°C$ we found $\frac{H_o}{H_L} = 1.24$. For 50 μm sample at $T_{NI}-T = 3°C$, $\frac{H_o}{H_L} = 1.39$. Calculations of Van Doorn (1973) show that for $x_o = 54$ μm, the transmitted intensity for a TN cell containing MBBA changed for $\frac{H_o}{H_L} \sim 1.4$. This number agrees well with the experimental value. (We cannot really compare the two in a precise

quantitative way because the material constants in two compounds are different.) As we have explained earlier, and as is to be expected from computer calculations, we obtain a thickness dependent H_o.

Compared to the capacitance measurement method to detect H_L, the optical method described in this section is better. While in the former method the whole sample must have perfect alignment, to get a sharp decrease in the capacity at H_L, in the latter a small area with good alignment could be chosen for observation. Moreover this minimises any errors due to the non-uniformity of the sample thickness and temperature gradients. In addition, the measurement of thickness by optical method is quite accurate.

Conclusions

1 An optical set up consisting of a polarizer, $\lambda/4$ plate and analyzer sensitive to small changes in the birefringence of the sample has been used to get the Leslie threshold.

2 The experimental values of $H_L x_o$ agree with those calculated using Leslie's equation within the experimental errors, thus experimentally establishing Leslie's equation.

3 The optical threshold determined by the usual method is dependent on the, thickness of the sample. This la in accordance with the calculations of Van Doorn.

4 This optical method in which measurements could be confined to a small region of the sample is probably more accurate than the capacitance method which senses the, gross property of the whole cell.

References

Berreman, D.W., 1973 J. Opt. Soc. Am. 63, 1374.

de Vries, H. 1951 Acta Crystallogr. 4, 219.

Gerritsma, C.J., De Jeu, W.H. and Van Zanten, P. 1971 Phys. Lett. 36A, 389.

Leslie, F.M. 1970 Mol. Cryst. Liq. Cryst. 12, 57.

Schadt, M. and Helfrich, W. 1971 Appl. Phys. Lett. 18, 127.

Van Doorn, C.Z. 1973 Phys. Lett. 42A, 537.

Table 6.1

The product ($H_L x_o$) at different temperatures in 70B

$T_{NI}-T$ °C	$H_L x_o$ gauss cm.	$T_{NI}-T$ °C	$H_L x_o$ gauss cm	$T_{NI}-T$ °C	$H_L x_o$ gauss cm
19.3	8.50	16.5	8.35	16.7	8.[illegible]
16.4	8.45	12.9	8.09	14.3	8.46
14.3	8.29	10.5	7.77	11.7	8.27
11.8	8.13	7.3	7.55	9.5	7.96
9.3	7.92	5.4	7.14	6.6	7.57
6.8	7.55	3.3	6.62	4.5	7.23
4.5	7.29	1.3	6.20	2.9	6.94
2.9	6.92	0.5	5.83	1.6	6.42
1.1	6.07			0.6	6.08
x_o = 52.8 µm		x_o = 52.5 µm		x_o = 26.1 µm	
19.9	8.97	18.5	8.91	Calculated values	
17.8	8.79	15.6	8.66		
15.3	8.61	12.9	8.43		
13.6	8.43	10.7	8.28	19	8.94
10.9	8.31	8.1	7.91	17	8.80
9.0	8.05	5.6	7.58	15	8.66
7.7	7.95	3.7	7.28	13	8.52
5.9	7.62	1.7	6.68	11	8.32
4.2	7.37	0.3	5.97	9	8.15
2.8	7.04			7	7.90
1.4	6.58	x_o = 25.1 µm		5	7.58
0.1	5.72			3	7.11
x_o = 25.4 µm				1	6.26