STUDIES IN
HIGHER DIMENSIONAL AND HIGHER ORDER GRAVITY

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DOCTOR OF PHILOSOPHY'

by

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CERTIFICATE

It is certified that the work contained in the thesis entitled STUDIES IN HIGHER DIMENSIONAL AND HIGHER ORDER GRAVITY, by BIPLAB BHAWAL, has been carried out under our supervision and that this work has not been submitted elsewhere for a degree.

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SYNOPSIS

In the last few years the search for a consistent theory of quantum gravity and the quest for a unification of gravity with other forces have both led to a spurt of interest in theories with extra spatial dimensions incorporated in them, e.g., Kaluza-Klein theory, Superstring, Supergravity etc. Another subtle aim of studying higher dimensions is to explain why the observed universe is so specific to choose **1+3** dimensions although always there exists a greater generality in the statements and mathematics describing the laws of nature.

It can be shown that the Lagrangian of a higher dimensional theory may naturally incorporate some additional higher order terms. These terms also appear in the low frequency limit of the superstring theory. In the absence of the knowledge of the complete theory of quantum gravity, from time to time, attempts have been made to gain further insight by studying models which include only leading order corrections.

This thesis aims to study various physical processes associated with some models of such theories with a view to isolating signatures of higher dimensional and higher order terms. We essentially concentrate our study on higher dimensional Einstein action modified by the Gauss–Bonnet combination, the leading second order correction. It highlights especially two distinctive features of higher dimensional gravity — the nonexistence of bound states in central mass problems and the semiclassical decay of the ground state. Several related works have been done with the aim of probing the geometry and realising the essential differences between models of such theories and those in ordinary four dimensional general relativity.

In chapter I, we first set the motivation for studying higher dimensional theories and present a chronology and brief review of the developments in this line. Next we go on to introduce Lovelock gravity that naturally incorporates higher order terms in the Lagrangian of higher dimensional theories and mention the implication of such terms with a special reference to superstring theory.

One can show that all such terms can be identified with the dimensionally extended Euler forms corresponding to each of the even number of dimensions. An analysis (based on the existing literature) of such terms in the language of differential forms has also been presented. The last section enlists and summarizes some works already done on some models of these theories.

In chapter II, we concentrate our study on a simple model of such a theory — a static spherically symmetric solution — the Boulware Deser Black Hole (BDBH). We specifically studied two cases first — the geodesic motion and the Hawking radiation.

In the former case, we compared the results with those in higher dimensional Schwarzschild spacetime solution of the Einstein–Hilbert Lagrangian. An interesting result of this study is the nonexistence of any stable bound orbit in higher dimensional black hole spacetime. This can be thought to be a general relativistic analogue of the consequences of the Bertrand's theorem in Newtonian Mechanics. This is an important distinctive feature of the higher dimensional theory. We also observed that the presence of higher order terms in the action does not significantly affect the nature of geodesic orbits, except changing the position of the horizon and the maximum point of the effective potential. Associated with this section is Appendix A where we present generalized expressions for the deflection of null ray in higher dimensional black hole spacetime.

Our study of the Hawking radiation from BDBH is based on the solution of the scalar field in such a spacetime. One may see that the techniques used in the four dimensional case can be extended to any higher dimensional static spherically symmetric black hole solution in a straight forward manner. The case of five dimensional BDBH is unique in the sense that only in that case one can identify the cosmic censorship hypothesis with the third law of black hole thermodynamics. The last section contains some comments on the back reaction problem. Since the detailed back reaction problem is, in general, notoriously difficult to untangle, we attempt to make some guesses on this and the singularity formation process.

Chapters III, IV and V concentrate on another very important unique feature of higher dimensional gravity — the semiclassical decay of the ground state. Such a process can never occur in four dimensional general relativity because the positive energy theorem ensures the uniqueness of the ground state (four dimensional flat Minkowski spacetime).

Chapter III is a review of the works done in this line. We first introduce the concept and salient features of the semiclassical decay process of false vacuum in ordinary quantum field theory (without gravitation). Then we summarize how the situation becomes highly nontrivial when one applies these concepts to the gravitational field.

We describe important steps in Witten's proof of the positive energy theorem based on spinor algebra. This proof cannot be fully generalized to higher dimensional gravity. The difficulties arise when one considers multiply connected spacetimes like $M^4 \times S^1$. We describe how Witten proved the semiclassical instability of the flat spacetime of such a topology. The spacetime into which the ground state decays is known as Witten bubble.

In chapter IV, we study massless scalar waves in the Witten bubble background. Such a study is essential for understanding the classical properties and the evolution behaviour of such a spacetime. We could write the timelike and angular parts of the separated Klein Gordon equation in terms of hyperbolic harmonics characterized by the generalized frequency w. The radial equation is cast into the Schrodinger form. The coordinate transformation used for this purpose has a special meaning which has been discussed in Appendix C. The above mathematical formulation is applied to study the scattering problem, the bound states and the corresponding stability criteria. The results confirm the concept of the bubble wall as a perfectly reflecting expanding sphere. It has been found that bound states as well as quasi-normal modes are absent. It has been shown that the bubble spacetime is stable with respect to any arbitrary scalar perturbation. In Appendix B, we also present an alternative scalar wave solution in this spacetime.

In chapter V, we study the semiclassical decay of the $M^4 \times S^1$ ground state in higher order gravity. We also address the question of the validity of the positive energy theorem in this theory. We obtained two instanton solutions representing the decay. Correspondingly, two alternative Lorentzian spacetimes (into which the ground state decays) have been obtained. The first solution, in the limit of vanishing coupling constant for the Gauss–Bonnet combination, approaches the Witten bubble solution, whereas the second solution is an entirely new one. Both solutions have similar qualitative features as those of the Witten Bubble.

Chapter VI, the last chapter, mentions some important works done by other people and highlights some prospects as well as problems in higher order gravity. It summarizes and reviews our work in this broader perspective, and proposes logical extensions and improvements thereof.

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* Boulware Deser Black Hole

NOTATION

(1) Indices : Latin indices represent all the spatial coordinates $(1, 2, 3, \ldots$ etc. upto $\mathrm{D}-1$ for D dimensional spacetime), whereas Greek indices denote all coordinates including both temporal (0) and spatial ones $(1, 2, 3, \ldots$ etc.). While using the language of differential forms, we will represent the coordinates of the orthonormal basis by the capital letter Latin indices $(A, \mathrm{B}, \mathrm{C}, \ldots$ etc.).

(2) Signature : We choose signature $(- \; + \; + \ldots +)$ for the spacetime metric. The corresponding rule is that timelike intervals are imaginary and spacelike ones are real.

(3) **Summation Convention** : Einstein Summation Convention is always followed, i.e. repeated index will be summed over all the values it may take.

(4) Units : Depending on the convenience and judging the relevance of the actual values to our discussion, we shall frequently set one or more of the following four fundamental constants to unity— c, the speed of light, G, the gravitational constant, k_B, the Boltzmann constant, $\hbar$, the Planck constant$[/(2\pi)]$. We indicate our choice in every case in our discussion.