CHAPTER 3

ON THE ORIGIN OF PULSAR MAGNETIC FIELDS
Fossil Fields vs. Field Growth after birth

There has been a recent suggestion that neutron stars may not be endowed with strong magnetic fields at birth, but that the observed magnetic fields of pulsars are built up gradually due to a thermally-driven battery process.

According to this picture, by the time a neutron star builds up a strong enough magnetic field to be able to function as a pulsar, the supernova remnant associated with it would have already faded away. This may be a possible explanation for the lack of observable associations between pulsars and supernova remnants.

We critically examine this suggestion in this chapter and find that because of the uncertainties in several physical parameters, it is not clear as to whether the field growth mechanism is likely to be efficient. It is also argued that even granting that the observed fields are built up after the birth, one cannot avoid the main conclusion of chapter 2, namely, that the majority of neutron stars are born as slow rotators.

CONTENTS

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3.1 INTRODUCTION

The discussions in the previous chapter have assumed that pulsars have field strengths in the range 10^{12} to $10^{13.5}$ Gauss **right** from **their birth.** Even before pulsars were discovered, Woltjer (1964) had argued that neutron stars will have strong fields. Ruderman and Sutherland (1973) suggested that just before the stellar core collapses to form a neutron star, convective processes in the core may lead to a tangling of any seed field till an equipartition field strength is established. This field gets amplified during the collapse due to flux conservation and leads to the observed magnetic field of pulsars. This will result in large neutron star fields at birth. More recently, however, it has been proposed that the neutron star fields may be generated after their birth, due to thermally-driven battery mechanisms (Woodward

1978; Blandford, Applegate and Hernquist 1983). According to Blandford et.al. (1983), the timescale for such a field build up may be as large as $\sim 10^5$ yr. Long before that the SNR would have dissipated in the interstellar medium. After the field builds up and the pulsar turns on, its relativistic wind will be essentially unconfined, giving too little synchrotron emission for them to be recognised as Crab-like nebulae (see, for example, Cheng 1983). Since this appears to provide an attractive resolution to the poor association between pulsars and supernova remnants we devote this chapter to a critical discussion of the proposed mechanisms for field growth. The first part of the discussion will be a critique of the theory and then we look at the observational evidence for such field growth in neutron stars.

Of the two different kinds of thermally-driven field growth that have been proposed in the context of neutron stars, one is the well known Battery effect proposed originally for normal stars by Biermann (1950), with the difference that in the present case the growth of the field is likely to be limited by the Hall effect (Woodward 1978). The second mechanism that has been proposed involves the thermoelectric effect (Blandford, Applegate and Hernquist 1983) - sufficiently hot, cooling neutron stars can amplify seed fields of $\gtrsim 10^8$ Gauss to $\sim 10^{12}$ Gauss, by an astrophysical analogue of what is known in Physics as the "Ettingshausen effect" (Ashcroft and Mermin 1976). We shall now discuss these two mechanisms.

3.2 THE BATTERY MECHANISM

The Basic Mechanism

The battery mechanism was originally proposed for rotating normal stars by Biermann (1950) and then reconsidered by Mestel and Roxburgh (1962) and Roxburgh (1966). Following their treatment, we outline below the physics involved in the process:

The material in the stellar interior is fully ionised. The short mean free paths ensure that the electron and ion partial pressures are of the same order. However, the gravitational force is felt almost solely by the ions. In a spherically symmetric star, dynamical equilibrium is reached by a very slight outward drift of the electrons, the resultant electrostatic field exerting on the electrons a force equal to their partial pressure gradient. In a rotating star, however, the centrifugal field also acts differentially on the two components, and again charge separation results. However, in this case, the electron partial pressure cannot be supported by an electrostatic field, since the centrifugal field is not derivable from a scalar potential (von Zeipel, 1924; Roxburgh 1966). The electron partial pressure then acts like a battery, in which non-electric forces continually drive electrons with respect to ions. This generates poloidal currents, leading to the build up of a toroidal magnetic field. In the two-fluid approximation, the equation of motion of the electron gas is given by (Cowling 1953)

$$\frac{\vec{j}}{\sigma} = \vec{E} + \frac{\vec{\nabla} p_e}{e n_e} - \frac{\vec{j} \times \vec{B}}{c e n_e} + \frac{\vec{v} \times \vec{B}}{c} \tag{3.1}$$

and the equation of hydrostatic equilibrium is

$$\frac{\vec{\nabla} p_{tot}}{\rho} = -\vec{\nabla}\Phi + \Omega^2 \vec{\varpi} + \frac{\vec{j} \times \vec{B}}{c \rho} \; . \tag{3.2}$$

Here $\vec{j}$ is the electric current density maintaining a magnetic field $\vec{B}$,

$\vec{E}$: the electric field

p_e : the pressure of the electron gas

n_e : the number density of the electron gas

e : the electronic charge,

c : the velocity of light

ρ : the density

p_{tot} : the total pressure

: the gravitational potential

Ω : the angular velocity

: the vectorial distance from the rotation axis and

σ : the electrical conductivity.

In a star supported by thermal gas pressure, p_e and n_e are related to p_{tot} and ρ as

$$p_e = \frac{Z}{Z+1} p_{tot} \quad ; \quad n_e = \frac{}{A m_p} \; {}' f$$

where m_p is the proton mass, A the atomic weight, and Z the atomic number of the ions. Using these and eliminating p_{tot} between (3.1) and (3.2) one obtains

$$\frac{\vec{j}}{\sigma} = \vec{E} + \frac{Am_p}{(Z+1)e}\left(-\vec{\nabla}\Phi + \Omega^2\vec{\varpi} + \frac{\vec{j}\times\vec{B}}{Zc\rho}\right) + \frac{\vec{v}\times\vec{B}}{c} . \qquad (3.3)$$

Using Maxwell's equation

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

and taking curl of (3.3), we find

$$\frac{1}{c}\frac{\partial\vec{B}}{\partial t} = \vec{\nabla}\times\left\{\left(\vec{v} - \frac{Am_p}{Z(Z+1)e\rho}\vec{j}\right)\times\frac{\vec{B}}{c}\right\} + \frac{Am_p}{(Z+1)e}\vec{\nabla}\times(\Omega^2\vec{\varpi}) - \vec{\nabla}\times\left(\vec{j}/\sigma\right) . \qquad (3.4)$$

As was originally pointed out by Biermann, $\vec{B}=0$ is a solution of (3.4) only if $\vec{\nabla}\times(\Omega^2\vec{\varpi})=0$, that is, if the centrifugal force term is derivable from a potential. Otherwise this term acts like a battery, generating a poloidal current $\vec{j}_p$ and a toroidal field B_t . Hence $\vec{\nabla}\times(\Omega^2\vec{\varpi})$ is called the "battery term" in the above equation.

3.2.1 Woodward's Hypothesis

Woodward (1978, 1984) has suggested that the above battery mechanism may be responsible for generating the observed magnetic fields of neutron stars. However, there is a serious difficulty with this suggestion. The matter in the neutron star is completely degenerate, and the material pressure is a function only of its density:

$$p = p(\rho) . \qquad (3.5)$$

The equation of hydrostatic equilibrium (3.2) in the absence of a magnetic field[1] reduces to

$$\frac{\vec{\nabla} p}{\rho} = - \vec{\nabla} \Phi + \Omega^2 \vec{\varpi} \quad . \tag{3.6}$$

With the pressure given by (3.5), the left hand side of (3.6) can be expressed as a gradient of a function of the density:

$$\frac{\vec{\nabla} p}{\rho} = \vec{\nabla} \pi \quad ; \qquad \pi = \pi(\rho) \tag{3.7}$$

and therefore,

$$\Omega^2 \vec{\varpi} = \vec{\nabla} (\pi + \Phi) \, . \tag{3.8}$$

We thus see that $\Omega^2 \vec{\varpi}$ is derivable from a potential. This is a restricted version[2] of a theorem due to Poincaré (1893). With the centrifugal field given by (3.8), its curl is zero, and hence **the battery term in (3.4) vanishes.** Thus a straightforward extension of the battery process in normal stars (Roxburgh 1966) to the case of neutron stars, as proposed by Woodward, is not possible. In particular, Woodward's suggestion that this battery works in the neutron star crust, which is in a state of rigid rotation, can be discounted since with Ω everywhere constant, the battery

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[1] In the case of a neutron star the third term in (3.2), which gives the magnetic pressure gradient, is $< 10^{-10}$ of the gravitational force, and hence can be neglected.

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[2] This theorem further proves that under these conditions the angular velocity can be a function only of the distance from the rotation axis, and that isobaric surfaces and constant density surfaces will coincide.

term in (3.4) is identically zero. It is most likely that such a state of rigid rotation prevails throughout the neutron star (e.g. Greenstein 1975) and this centrifugally driven battery mechanism is unlikely to be responsible for the generation of the observed magnetic fields of neutron stars.

However, the pressure-density relation (3.5) is strictly true at zero temperature. At a finite temperature, there is a small but non-zero additional thermal contribution to the pressure (of order $(T/T_F)^2$, where T_F is the Fermi temperature: Landau and Lifshitz, 1959). Typically this is $\sim 10^6$ times smaller than the degeneracy pressure (3.5). The temperature dependence of this additional pressure can be used to drive a different kind of battery process, proposed by Urpin and Yakovlev (1980) in the context of White Dwarf stars, and applied to the case of neutron stars by Blandford (1983) and Blandford, Applegate and Hernquist (1983). We shall discuss this now.

3.3 THE THERMOELECTRIC BATTERY

Blandford, Applegate and Hernquist (1983) have proposed a battery mechanism based on the thermoelectric effect for the generation of neutron star magnetic fields. The essence of the mechanism is as follows:

One has a cooling neutron star with a temperature gradient radially inwards, which generates a heat flux radially outwards. If one considers a small region of the crust, this heat flux is vertically upwards. Now, if there is a pre-existing horizontal magnetic field, it will deflect hot electrons from below, and cooler electrons from above slightly less in the opposite direction. The net effect is to produce a horizontal heat flux which, in turn, will generate a horizontal temperature gradient. This will create an additional pressure gradient which must be balanced by a thermoelectric field. This thermoelectric field has a non-vanishing curl, and under suitable conditions can lead to a growth of the initial seed field. Blandford et.al. (1983) argue that by this process neutron stars can generate their fields of $\sim 10^{12}$ gauss in $\lesssim 10^{5}$ yr, starting from an initial field of $\sim 10^{8}$ gauss. In the next few pages we shall reproduce some of the salient formulae (and notations) needed to discuss the physics of the mechanism. These are taken mainly from Blandford, Applegate and Hernquist (1983).

The electrons in the neutron star crust are degenerate and ultrarelativistic, and can be regarded as non-interacting. In the presence of electric fields, and gradients of chemical potential and temperature, the laws of charge and heat transport are given by (Landau and Lifshitz 1960; Ashcroft and Mermin 1976)

$$\vec{j} = \overleftrightarrow{\sigma} \cdot \vec{\epsilon} - \overleftrightarrow{\lambda} \cdot \vec{\nabla} T \tag{3.9}$$

and

$$\vec{F} = T \overleftrightarrow{\lambda} \cdot \vec{\epsilon} - \overleftrightarrow{\eta} \cdot \vec{\nabla} T \tag{3.10}$$

where the electrochemical field $\vec{\epsilon}$ is the sum of the electric field and the chemical potential gradient

$$\vec{\epsilon} = \vec{E} + \vec{\nabla}\mu/e. \tag{3.11}$$

In these expressions $\vec{j}$ is the electrical current, $\vec{F}$ is the heat flux and $\overleftrightarrow{\sigma}$ is the electrical conductivity. The thermal conductivity $\overleftrightarrow{\kappa}$ and thermopower $\overleftrightarrow{Q}$ are related to the coefficients $\overleftrightarrow{\lambda}$ and $\overleftrightarrow{\eta}$ by

$$\overleftrightarrow{Q} = (\overleftrightarrow{\sigma})^{-1} \cdot \overleftrightarrow{\lambda} \tag{3.12}$$

$$\overleftrightarrow{\kappa} = \overleftrightarrow{\eta} - T\overleftrightarrow{Q} \cdot \overleftrightarrow{\lambda} \tag{3.13}$$

$$\approx \overleftrightarrow{\eta} \text{ (upto first order in } T/\mu).$$

In these formulae, and in the rest of this section, the units $c=k=1$ are used. T is the temperature, μ is the chemical potential and e is the electronic charge. In the presence of a small magnetic field $\vec{B}$, for which the electron gyrofrequency is much less than the electron-phonon and electron-ion collision rates in the crust, the expression for the conductivities can be written, upto first order in T/μ, as

$$\overleftrightarrow{\sigma} = \sigma_0 \overleftrightarrow{\chi} = \frac{ne^2\tau}{\mu} \overleftrightarrow{\chi} \tag{3.14}$$

$$\overleftrightarrow{\kappa} = \kappa_0 \overleftrightarrow{\chi} = \frac{\pi^2 n_e T\tau}{3\mu} \overleftrightarrow{\chi} \tag{3.15}$$

where $\overleftrightarrow{\chi}$ is definedthrough

$$(\chi^{-1})_{ij} = \delta_{ij} + \epsilon_{ijk} \frac{e\tau}{\mu} B_k . \tag{3.16}$$

In the above, n_e is the electron density and τ the collisional relaxation time for the electron gas. In (3.16) δ_{ij} is the identity matrix and ϵ_{ijk} is the Levi-Civita symbol for 3 dimensions.

The coefficient $\overleftrightarrow{\lambda}$ is given by

$$\overleftrightarrow{\lambda} = -\frac{\pi^2 T}{3e}\frac{d\overleftrightarrow{\sigma}}{d\mu} . \tag{3.17}$$

Using expressions (3.14) through (3.17) in (3.9) and (3.10), and keeping terms only upto first order in T/μ , one obtains

$$-en_e\vec{E} + \vec{j}\times\vec{B} - n_e\vec{\nabla}\mu + \frac{\mu\vec{j}}{e\tau} - \vec{F}\,\frac{d(\mu/\tau)}{d\mu} = 0. \tag{3.18}$$

Then using Faraday's law, one gets after some algebra

$$\frac{\partial\vec{B}}{\partial t} = -\vec{\nabla}\times\vec{E}$$

$$= \vec{\nabla}\times(\vec{v}\times\vec{B}) - \vec{\nabla}Q_o\times\vec{\nabla}T - \vec{\nabla}\times\left[\frac{\vec{\nabla}\times\vec{B}_o}{4\pi\sigma_o}\right] \tag{3.19}$$

where

$$\vec{v} = \frac{\overleftrightarrow{\kappa}\cdot\vec{\nabla}T}{n_e\mu}\cdot\frac{d\ln(\mu/\tau)}{d\ln\mu} - \frac{\vec{j}}{en_e} \tag{3.20}$$

and

$$Q_o = -\frac{\pi^2 T}{3e\mu}\cdot\frac{d\ln\kappa_o}{d\ln\mu} . \tag{3.21}$$

Equation (3.19) is the basic equation describing the thermoelectric battery process. The three terms on the right hand side of (3.21) are similar to those in eq.(3.4), and can be interpreted as follows:

(i) $\vec{\nabla}\times(\vec{v}\times\vec{B})$ is a field convection tern where the convection velocity is the sum of the thermal diffusion velocity and the electron mobility.

(ii) The **second** term is **the battery term** and is proportional to $\vec{\nabla} n_e \times \vec{\nabla} T$; this describes the creation of field by thermoelectric currents. If $\vec{\nabla} T \cdot (\vec{B} \times \vec{F}) > 0$, this term contains a part proportional to $(-\vec{\nabla} n_e \cdot \vec{F}) \vec{B}$, leading to an exponential growth of the field if heat flows down the density gradient. Thus, in a neutron star, field growth will take place if the heat flux is radially outwards.

(iii) The third term is the ohmic decay term, and describes the dissipation of currents that are responsible for growth and maintenance of the magnetic field.

In the picture of Blandford **et.al.** (1983), the neutron star crust is assumed to be a crystalline solid covered by a layer of liquid metals. The interior is isothermal at a temperature $\sim 10^8$ K, and the temperature falls to $\sim 10^6$ K at the top of the liquid layer (fig. 3.1). The depth of the liquid layer is

$$Z_M \sim 4 \times 10^3 \, T_8 \, g_{14}^{-1} \ \text{cm} \tag{3.22}$$

where $10^8 T_8$ K is the temperature and $10^{14} g_{14}$ cm s^{-2} is the surface gravity.

, In the liquid, the isotherms and equipotentials coincide with constant density surfaces by requirement of mechanical equilibrium. Thus $\vec{\nabla} n_e \times \vec{\nabla} T = 0$, causing the battery term in (3.19) to vanish. This thermoelectric battery can, therefore, operate only in the solid crust where finite stresses in the lattice can prevent exact coincidence of isobaric and constant

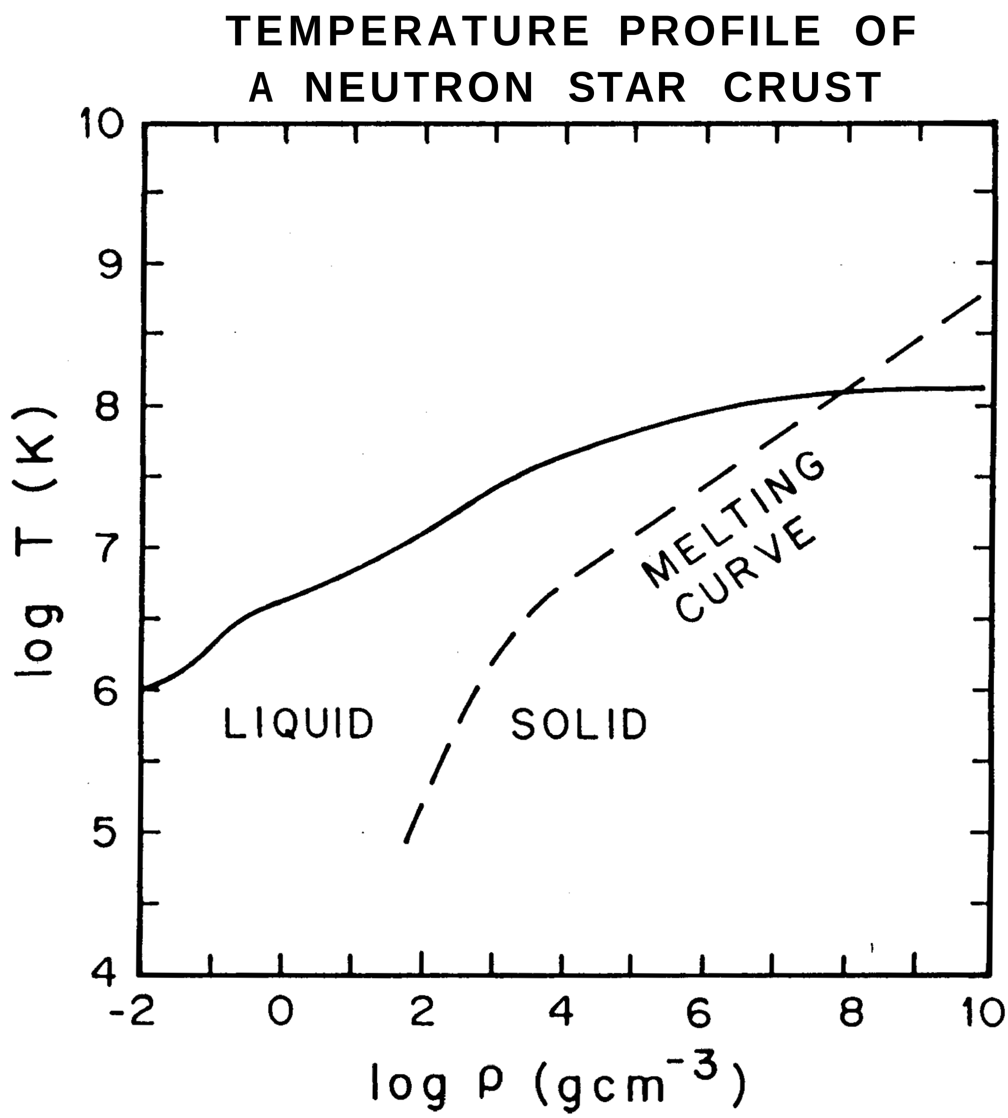

Fig. 3.1: The temperature profile of a neutron star crust (solid line) as computed by Gudmundsson et. al. (1982). The broken line shows the melting curve above which the crustal matter is in a liquid state.

density surfaces. Since the thermoelectric battery originates from the anisotropy of the transport coefficients introduced by a magnetic field (eq. 3.16) unlike the Biermann battery for normal stars (cf. section 3.2) it is not a "primary" mechanism, and needs a "seed" magnetic field which it can amplify. To follow the field evolution in the linear phase, Blandford et.al. (1983) impose a small magnetic field (for which the electron gyrofrequency << collision rate) and compute the resulting perturbation in the temperature gradient by using the steady state heat flow equation

$$\vec{\nabla} \cdot \vec{F} = 0 \tag{3.23}$$

where $\vec{F}$ is the net heat flux vector. This perturbed temperature distribution is then used to compute the growth rate of the magnetic field from eq. (3.19). Writing

$$B \sim B_0 \exp\left(\lambda t / t_M\right) \tag{3.24}$$

they obtain numerical solutions for the dimensionless growth rate λ. Here

$$t_M \equiv 4\pi \sigma_0 z^2 \Big|_{z = z_M} . \tag{3.25}$$

Since the battery process generates magnetic flux and the ohmic diffusion dissipates it, clearly the effectiveness of the mechanism depends on the relative timescales of these two processes. As expressed by Blandford et.al. (1983), the key parameter is the dimensionless quantity

$$\alpha \equiv t_M \Big/ \left(\frac{z_M}{F_0 / n_M \mu_M} \right) \tag{3.26}$$

where F_o is the unperturbed heat flux, and n_{ti}, μ_M are the electron density and electron Fermi energy at the top of the solid layer. z_M and t_M have been defined in eqs. (3.22) and (3.25) respectively. The quantity in parantheses is the time required to traverse a current loop of size z_M at the diffusion velocity associated with heat flux. This is typically the field growth timescale in the absence of ohmic decay. Evidently, the value of α must be large for the field to grow. Using (3.25) and (3.14) the above equation (3.26) can be rewritten as

$$\alpha = \frac{4\pi e^2 \tau_M z_M F_o}{\mu_M^2} \qquad (3.27)$$

where τ_M is the collision timescale at the top of the solid crust.

Blandford et.al. (1983) find that for a growing mode to exist at all (i.e. $\lambda > 0$) the value of α must be larger than 5. However, for the growth to be large enough to be interesting, λ must exceed 0.1, and this requires $\alpha \geqslant 22$.

The value of α is sensitive to the collision rates which determine τ_M, and the opacities, which determine F_o. The estimate of these parameters are somewhat uncertain and the results of different workers (e.g. Flowers and Itoh, 1976,1981; Yakovlev and Urpin, 1980) disagree to some extent. With the published values, α remains uncertain to within a factor of 6. Another additional source of uncertainty is the surface composition. Blandford et.al. (1983) argue that for a given internal temperature, the heat flux F_0 is inversely proportional to the atomic number Z of the ions in the liquid layer. Although it is commonly believed that the surface of the neutron star is made of iron (Baym, 1977), Blandford et.al. (1983) suggest that a substantial fraction may be helium (Z=2). This will increase the heat flux by an order of magnitude. Given these uncertainties one finds that the value of α will lie in the range

$$0.06\, T_8^{0.3} \lesssim \alpha \lesssim 4\, T_8^{0.3} \tag{3.28}$$

where $10^8\, T_8$ K is the internal temperature.

We thus see that the allowable values of α are far too small to permit field growths in the required timescale. Increasing the internal temperature would not help matters, since α is rather insensitive to it (see eq. 3.28), and anyway if the temperature is more than $\sim 10^9$ K, then neutrino emission, rather than conductive heat transport, would dominate the cooling process, and therefore it would not help in the present context.

Faced with this difficulty, Blandford et.al. (1983) have suggested that if most (> 90%) of the magnetic flux is produced in the overlying liquid layer, and is then "convected" into the solid, then even with small values of α, the required growth rate can be sustained. Since the above thermoelectric battery cannot operate in the liquid, one has to find an alternative mechanism. Blandford et.al. (1983) have suggested that in presence of a sufficiently strong seed magnetic field (> a few times 10^8 gauss), the horizontal heat flux generated by deflecting the vertical component will drive a circulation, which will amplify the field by a non-linear dynamo process. The detailed working of this dynamo has not been demonstrated, and it also remains uncertain whether the magnetic flux produced this way can be convected to the solid more rapidly than it is destroyed by either ohmic diffusion or buoyancy effects.

The final strength and configuration of the field depends also on the details of the evolution in the non-linear phase (electron gyrofrequency > collision frequency). In this phase the transport coefficients are greatly modified, and pending a detailed treatment, no clear picture of the field evolution is possible.

To summarize, although this thermoelectric battery mechanism is a very appealing mechanism for generating the observed magnetic fields of neutron stars, because of the difficulties in calculating transport coefficients from a microscopic theory, one is not able to say unequivocally whether this is a realistic mechanism.

3.4 OBSERVATIONS PERTAINING TO FIELD GROWTH

In this section we appeal to observations and ask whether the observed periods and fields of pulsars are consistent with their fields being generated after birth.

In the discussion to follow we shall need to know the evolution of the rotation period of a neutron star during the field growth phase. In the next section we shall derive a few general results which will be of use in later sections.

3.4.1 Evolution Of The Rotation Period During Field Growth Phase

A rotating, magnetized neutron star, even if it is not functioning as a pulsar, will slow down due to magnetic dipole radiation. When the magnetic field is strong, currents flowing through its magnetosphere will also provide a slowdown torque. The torque due to these currents and that due to the dipole radiation are expected to be of similar magnitude (Goldreich and Julian, 1969). We shall assume here that the spindown torque on the rotating neutron star equals that given by dipole radiation, when its magnetic axis and rotation axis are orthogonal to each other. The slowdown law can then be expressed as

$$-\frac{dE_{rot}}{dt} = -I\Omega\dot{\Omega} = \frac{2}{3c^3} R^6 B^2 \Omega^4$$

where B = dipole field strength at the surface of the star

R = radius of the star

I = moment of inertia of the star

Ω = angular velocity of rotation

$\dot{\Omega} = d\Omega/dt$

C = velocity of light

and E_{rot} = rotational energy of the neutron star.

For typical neutron star parameters, i.e. $I \sim 10^{45}$ gm cm^{2} and $R \sim 10^{6}$ cm, one gets

$$P(t)\dot{P}(t) = kB^{2} \simeq 10^{-15} B_{12}^{2} \text{ second} \qquad (3.29)$$

where P = spin period of the neutron star in seconds
$\dot{P}$ = dP/dt, and
10^{12} B_{12} gauss is the surface dipole field.

Let us now assume that the magnetic field of the star grows exponentially with time:

$$B(t) = B_{o} e^{t/\tau_m} \qquad (3.30)$$

τ_m is the growth timescale, and B_0 is the initial magnetic field. The period of the neutron star as a function of time can be obtained by using (3.30) in (3.29) and then integrating:

$$\boxed{\begin{aligned} P^{2} - P_{o}^{2} &= k\tau_{m}(B^{2} - B_{o}^{2}) \\ &= k\tau_{m} B^{2}(1 - e^{-2t/\tau_m}) \end{aligned}} \qquad (3.31)$$

where P_0 is the initial rotation period. The evolution of the rotation period is displayed in fig. 3.2(a). Eq. (3.31) will be used in several forms:

During the growth phase, the maximum change in spin period can be expressed in terms of the final magnetic field as

$$\boxed{P^{2} - P_{o}^{2} \leq k\tau_{m} B^{2}}. \qquad (3.32)$$

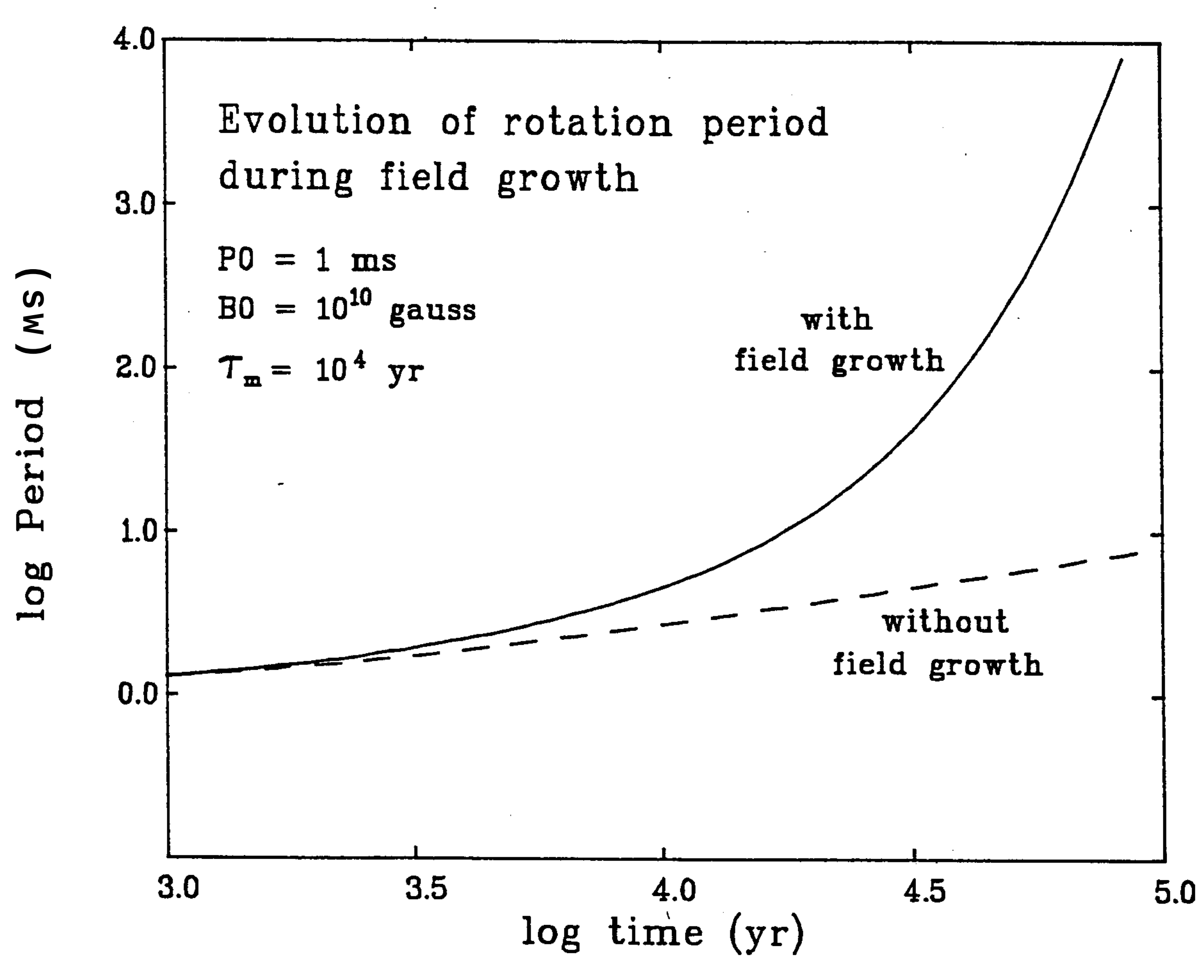

Fig. 3.2 (a) The evolution of the rotation period of a neutron star during the exponential growth of its magnetic field, compared to the evolution with a constant field. P0 is the initial spin period of the neutron star, B0 its initial magnetic field, and τ_m the field growth timescale.

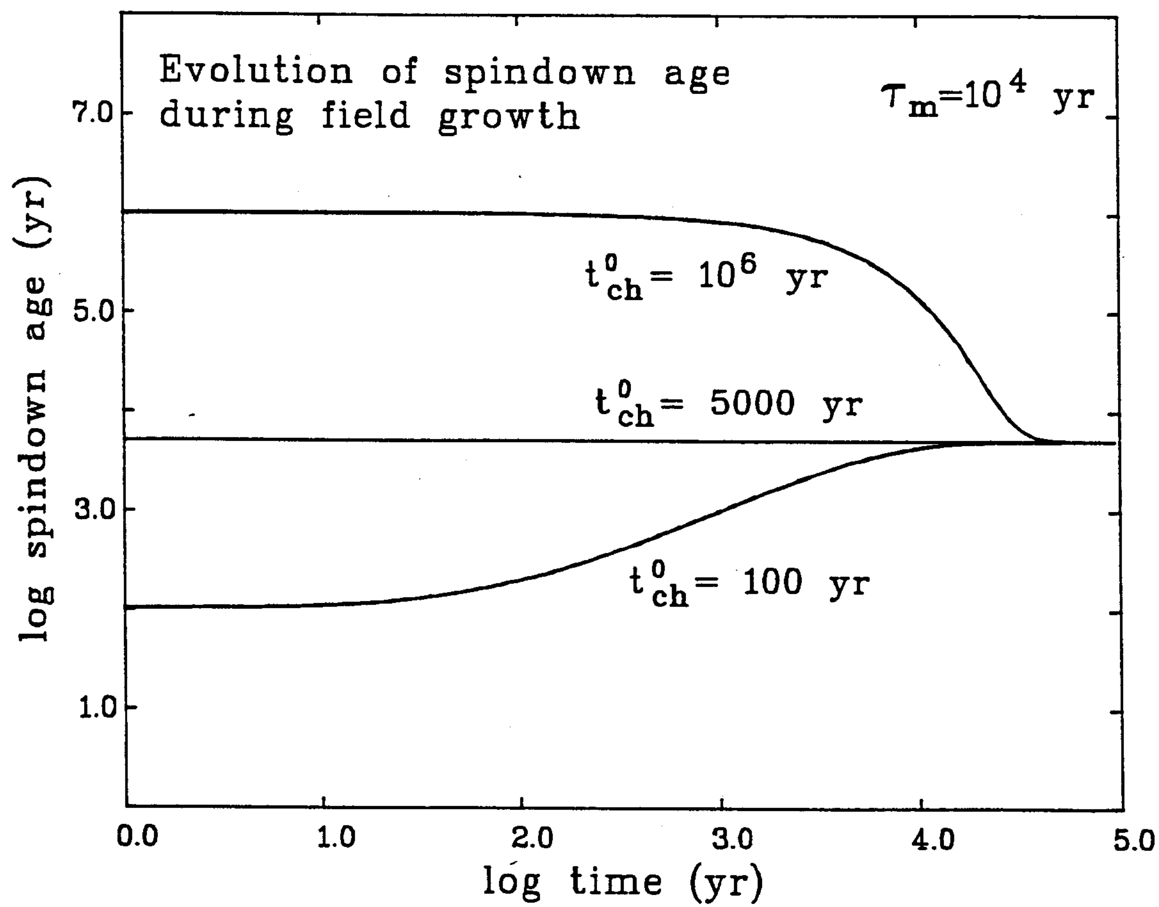

Fig. 3.2 (b): The evolution of the spindown age of a neutron star during an exponential growth of its magnetic field, plotted for three different values of initial spindown timescale t^o_{ch}. τ_m is the e-folding time for the magnetic field. As the field growth continues, the spindown age reaches the asymptotic value $\tau_m/2$.

If ξ $(\equiv \frac{B}{B_o})$ is the factor by which field has grown since the neutron star was born, a limit on ξ can be obtained in terms of the present magnetic field and rotation period of the neutron star. From (3.31)

$$P^2 = k\tau_m(B^2 - B_o^2) + P_o^2$$

$$\therefore \quad P^2 \geqslant k\tau_m B^2\left(1 - \frac{1}{\xi^2}\right)$$

$$\text{or,} \quad \frac{P^2}{} \geqslant \frac{t}{\ln \xi}\left(1 - \frac{1}{\xi^2}\right). \qquad \text{(using 3.30)}$$

The spindown age of a pulsar is defined as

$$t_{ch} = \frac{P}{2\dot{P}} = \frac{P^2}{2kB^2} \tag{3.33}$$

Hence the above inequality can be expressed as

$$\frac{\ln 4}{1 - 1/\xi^2} \geqslant \frac{t}{2t_{ch}}. \tag{3.34}$$

Finally using the definition (3.33) for t_{ch}, and (3.31), one obtains

$$t_{ch}(t) = \frac{\tau_m}{2} + \left(t_{ch}^o - \frac{\tau_m}{2}\right)e^{-2t/\tau_m} \tag{3.35}$$

where $t_{ch}^o = P_o^2/2kB_o^2$ is the initial value of the spin-down age. From (3.35) we find that as the exponential growth of the magnetic field continues, the spindown age reaches the asymptotic value $\tau_m/2$ (see fig. 3.2(b)).

If $t_{ch}^o > \tau_m/2$, then by (3.35),

$$t_{ch}(t) \leqslant t_{ch}^o \;; \quad \text{and} \quad \tau_m \leqslant 2t_{ch}. \tag{3.36a}$$

On the other hand if $t_{ch}^o < \tau_m/2$, then

$$t_{ch}(t) \geqslant t_{ch}^o \quad \text{and} \quad \tau_m \geqslant 2t_{ch} \tag{3.36b}$$

during the growth phase.

3.4.2 Fast Pulsars

One of the reasons why the mechanism of field growth and the timescale for the field growth suggested by Blandford et.al. (1983) is attractive is that it can explain the poor pulsar-SNR association. However, by the same token, the four observed pulsar-SNR associations pose a problem to the mechanism in the following sense. All these four remnants - the Crab Nebula, the Vela SNR, SNR MSH 15-52 and 0540-69.3 in the Large Magellanic Cloud - are fairly young and the pulsars in them have fields in excess of 10^{12} gauss. This would require extremely short growth timescale if the field was built up from a very small value $\sim 10^8$ to 10^9 gauss. In this section we shall comment on the required growth timescale for each one of these four pulsars.

The SNR MSH 15-52

This supernova remnant contains a 150 millisecond pulsar (PSR 1509-58) with a spindown age ($\equiv P/2\dot{P}$) of ~ 1600 years. However, according to the standard estimate, the age of the supernova remnant is $\sim 10^4$ yr. Barring the possibility of a chance coincidence (van den Bergh and Kamper 1984) or an error in the age estimate for the supernova remnant, this is by far the strongest evidence in favour of field growth. However, we shall show in chapter 6 that the observed properties of this SNR are consistent with those expected of a $\sim$ 1600 yr old remnant, if it is expanding in a low density bubble created by the strong wind of its progenitor. Blandford et.al. (1983)

have assumed that the supernova remnant is $\sim 10^4$ yr old, and have suggested that the neutron star is in fact as old as the SNR, but became a pulsar only $\sim$ 1600 years ago, when its field grew to sufficient strength. If this is true, then the ratio of present spindown age of the pulsar to its real age gives, using (3.34),

$$\ln \xi \gtrsim 3$$

where ξ is the ratio of final field to the initial field. This implies an initial magnetic field $B_0 \lesssim 7\times10^{11}$ gauss, and a growth timescale

$$\tau_m \lesssim 3200 \text{ yr}.$$

The upper limit to B_0 and τ_m correspond to a zero rotation period of the neutron star at birth. On the other hand, if the field growth has taken place from a very low initial value $\sim 10^8$ gauss, then the required growth timescale $\tau_m \lesssim 800$ yr, and by (3.32)

$$P_0 \gtrsim 130 \text{ ms}$$

that is, the neutron star must have been born spinning .fairly slowly. The upper limit to the growth timescale obtained above assumes that the field of this pulsar is still growing exponentially. If, instead, its field has stopped growing, or the growth has significantly slowed down on reaching a field of $\sim 10^{12}$ -10^{13} gauss, as Blandford et.al. suggest would happen, then one will require in the initial phase of growth a much shorter growth timescale than those mentioned above.

The Crab Pulsar

The Crab nebula is ~ 930 yr old and its pulsar (PSR 0531+21) has a spin period of 33 milliseconds and a spindown age of ~ 1200 yr. It is now well established that the pulsar is the energy source for the nebula. The energy in the relativistic particles and the magnetic field in the Crab nebula is more than $\sim 10^{49}$ ergs. The acceleration of the ejecta after the supernova explosion (Trimble 1971) requires another $\sim 10^{49}$ ergs, and a similar amount is necessary to account for the radiation from the nebula since its birth (Trimble and Rees 1970). Since the energy in all these forms must ultimately have come from the stored rotational energy, one can place strong constraints on the initial period and, therefore, the initial rotational energy. This argument tells one that the initial period of the Crab pulsar could not have been much longer than 20 milliseconds. If an appreciable amount of field growth has to have taken place for the Crab pulsar after its birth, then the growth timescale τ_m for the field must be much less than the present age of the nebula. If we impose the modest requirement that the field of the pulsar has grown by at least one order of magnitude since its birth, we find

$$\tau_m \lesssim 400 \text{ yr}.$$

Using this value of τ_m, and the present field $B \sim 3.7 \times 10^{12}$ gauss for the Crab pulsar, we see from (3.32) that

$$P^2 - P_0^2 \lesssim 1.7 \times 10^{-4} \text{ sec}^2$$

during the field growth phase. With $P \sim 33ms$, the initial period of the pulsar would be

$$P_o \gtrsim 30.3 \text{ ms}$$

much longer than the initial rotation period required by the energetics of the nebula. Or, in other words, with such a short growth timescale the rotation period hardly changes during the field growth (see fig. 3.3). To have an initial rotation period as small as ~ 20 ms, and yet to have a field growth by at least an order of magnitude, the only way then is the following: the field grows to its present value very quickly during the first $\lesssim$ 200 years, and then stops growing. The rotation period of the pulsar at the end of the growth phase is almost the same as what it started with - about 20 milliseconds. The pulsar then slows down to its present period during the rest of the time (fig.3.3). This demands a growth timescale

$$\tau_m \lesssim 80 \text{ yr}$$

for the field to grow by an order of magnitude. If, on the other hand, the field were to grow from an initial seed value of 10^8 gauss to its present value of $\sim 3.7 \times 10^{12}$ gauss, then the required timescale would be

$$\tau_m \lesssim 20 \text{ yr} .$$

Both these values of τ_m are extremely short ($\sim 10^{-2}$ times) compared to what Blandford et.al. (1983) suggest for typical pulsars.

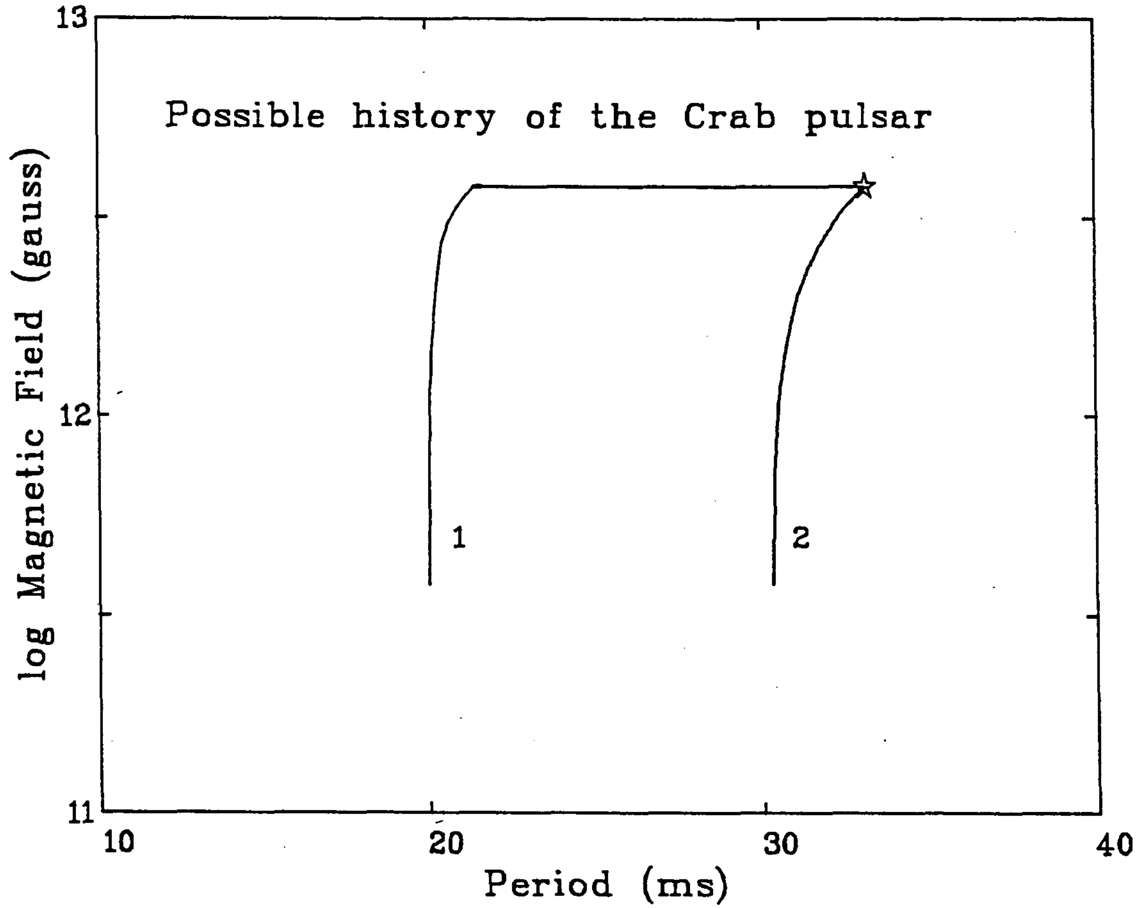

Fig. 3.3: Two possible evolutionary tracks with field growth leading to the observed magnetic field and rotation period of the Crab pulsar, marked by an asterik. Field growth by one order of magnitude, since its birth 930 years ago, has been assumed. Track 1 is generated using τ_m = 80 yr, B_{sat} = 3.8 x 10^{12} gauss and P_o = 20 ms. On track 2 τ_m = 400 yr and P_o = 30.3 ms. We argue in the text that track 2 is inconsistent with the energetics of the Crab nebula.

The Vela Supernova Remnant

The Vela supernova remnant, estimated to be $\sim 10^4$ years old, contains the pulsar PSR 0833-45 which has a spin period of 89 milliseconds and a spindown age of $\sim$ 11,000 years. The present energy content in relativistic particles and magnetic field in Vela X require an initial rotation period $\lesssim$ 50 ms for the pulsar. This, in turn, requires that the field growth should have taken place within the first $\sim$ 2000 years of its life. For the field to have grown by at least an order of magnitude, this will need a growth timescale $\lesssim$ 1000 years.

The LMC Supernova Remnant 0540-69.3

This SNR has several interesting features which will be discussed in chapter 5. Here it will suffice to mention that this remnant harbours a 50-ms pulsar with a spindown age of $\sim$ 1700 yr. The age of the remnant is not known, but is estimated to be $\sim$ 900-2000 years (see chapter 5 for a detailed discussion). Constraints similar to, but somewhat weaker than that for the Crab nebula may be obtained in this case also.

Conclusion

We thus see that if the magnetic field of pulsars are built up after their birth, then for the four known pulsars in SNRs the growth time for the field must have been extremely short, much shorter than that envisaged by Blandford et.al.

(1983).

Indeed faced with the difficulty of the four youngest pulsars having strong fields Blandford et.al. have suggested that if the newly born neutron star had very large angular momentum then the field growth might occur very rapidly. This may happen, for example, if part of the rotational energy can be converted to heat, or can be used to power the dynamo process more efficiently. But if this mechanism is realistic, then one is again forced to the conclusion that only a small fraction of neutron stars can be born spinning rapidly. For otherwise one will be left with the same problem - if the majority of neutron stars are born spinning rapidly, and by virtue of this their magnetic fields got built up very quickly, then one will end up predicting a very high birthrate for luminous pulsar-produced nebulae, which are not seen.

3.4.3 Accreting Neutron Stars

Blandford et.al. (1983) suggested in their paper that during the accretion phase of a neutron star in a mass transfer binary system, its polar cap regions will be heated up. The heat will penetrate the crust and raise the temperature of the interior. This will produce large outward flux through the cooler regions of the crust, which will lead to a generation of the magnetic field of the neutron stars. Neutron stars in accreting binaries would thus grow strong ($\sim 10^{12}$ gauss) fields in $\sim 10^{6}$ yr. This appears an attractive way to explain the high field observed in the very old neutron star in Her X-1. Evolutionary scenarios for this system suggest that this neutron star must be $\gtrsim 10^7$ yr old, and therefore its original field should have decayed (e.g. Sutantyo 1975; Sutantyo et.al. 1986). However, if the absorption (or emission?) line observed at 55 KeV is interpreted as a cyclotron line, then the derived field is $\sim 10^{12}$ gauss! But it should be remembered that even if this interpretation is correct, and the field near the surface is high, it may not be the dipole component but some higher multipole. The dipole component could have decayed as suggested by pulsar data.

A more serious difficulty with the accretion hypothesis for field growth is the following. Almost all the recently discovered binary radio pulsars have very low fields. Two of them, the "millisecond" pulsars, have a field of only $\sim 5\times10^8$ gauss. At first sight it might appear that during the accretion phase they would have built up high fields which could since then have decayed. But this is not in agreement with our current understanding of these millisecond pulsars, according to which these neutron stars were spun up to their present periods during an accretion phase (see van den Heuvel 1984 for a review). According to this scenario, in order for them to be spun up to such extremely short periods their magnetic field during the spin-up phase must have been close to the present value. We shall return to this question in chapter 9.

3.4.4 Is The γ-P Distribution Of Pulsars Evidence For Field Growth ?

Woodward (1978,1984) has made the following argument in favour of field growth in pulsars. He has plotted the magnetogyro ratio

$$\gamma \equiv \frac{\text{Magnetic moment}}{\text{Angular momentum}} \propto \left(P^3 \dot{P}\right)^{1/2} \qquad (3.37)$$

of known pulsars against their rotation periods (fig.3.4). He has then divided the pulsars in this plot into two groups - one with periods $P<0.5$ sec and the other with $P>0.5$ sec, and suggested that the distribution of pulsars in the γ-P plane shows two distinct features - an evolution with γ = constant for $P(0.5$ sec and a rapid enhancement of γ with period for $P>0.5$ sec. Such a behaviour has been interpreted by Woodward as evidence of field growth:

A Hall field limited growth (discussed in section 3.2) to explain the γ = constant evolution, and

an exponential growth to explain the rapid enhancement of γ.

In this section we wish to demonstrate that the γ-P distribution shown in fig. 3.4 can be understood satisfactorily without invoking field growth at all.

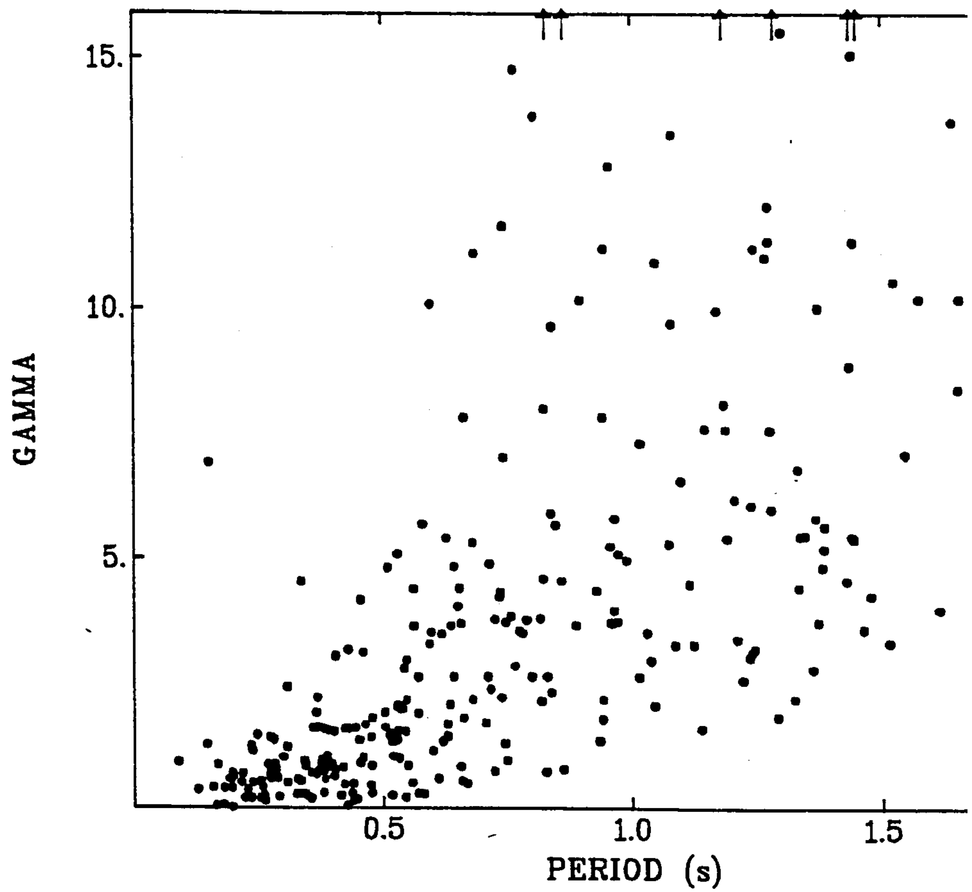

Fig. 3.4 The distribution of magnetogyro ratio vs. period of observed pulsars. GAMMA is proportional to magnetogyro ratio and is defined as $(P^3\dot{P})^{\frac{1}{2}} \times 10^8$. The arrows at the top of the figure indicate the P locations for which GAMMA exceeds the scale of the diagram (after Woodward, 1984).

The value of η, as defined by (3.371, is a product of the derived magnetic field $[\propto (P\dot{P})^{1/2}]$ and the rotation period of a pulsar. A pulsar will be born near the origin of this diagram, and during the course of its evolution, it will travel along a straight line with a positive slope as long as its magnetic field remains constant. The slope of the line will be directly proportional to its magnetic field. This is shown in fig. 3.5. As its magnetic field decays, it will deviate from the straight line motion and drop below it. A typical such trajectory is also shown in fig. 3.5. Another factor that will decide the distribution in the η-P diagram is the speed at which pulsars move along their tracks. Portions of the tracks which are at shorter periods are traversed more quickly, and as the pulsar progresses, its rate of advance is slowed down. While this is true of any particular track, the rate of advance along tracks belonging to different magnetic fields will be different. The higher the magnetic field, the faster will be the motion, and more sparsely populated will the track look. We have shown in fig. 3.5 two constant "characteristic age" ($\equiv P/2\dot{P}$) lines to give an idea of the differential motion along different tracks. If the pulsar magnetic fields do not decay, then the characteristic ages will be nearly equal to their true ages, and they will represent the position of the pulsar after the given amount of time has elapsed since its release at the origin. Finally, the rising lower envelope of the distribution at long periods can be understood in terms of pulsar "deaths", that is the pulsars stop functioning when the

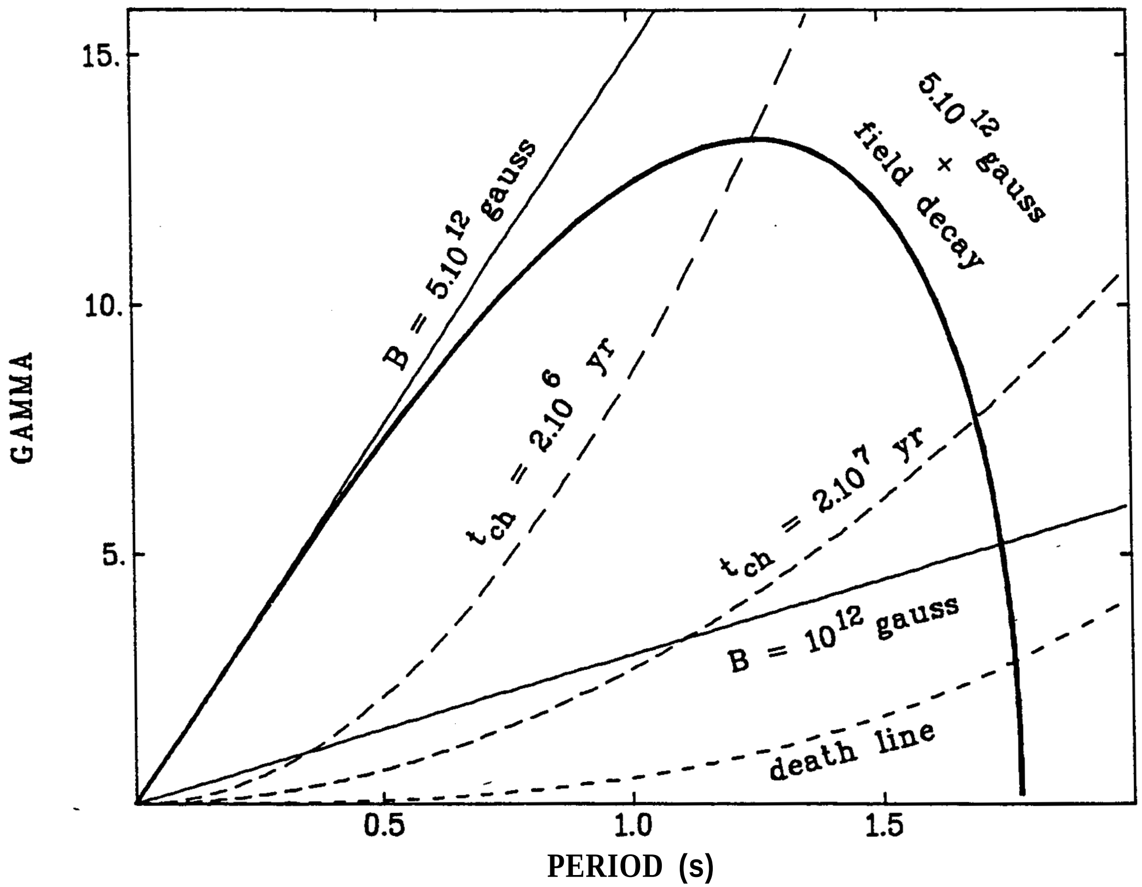

Fig. 3.5: Trajectories of pulsars in the GAMMA-period diagram. Thin solid lines represent evolution with constant magentic field, the thick line incorporates field decay in a timescale of 4 million years. Two constant characteristic age lines, and the death line are also shown.

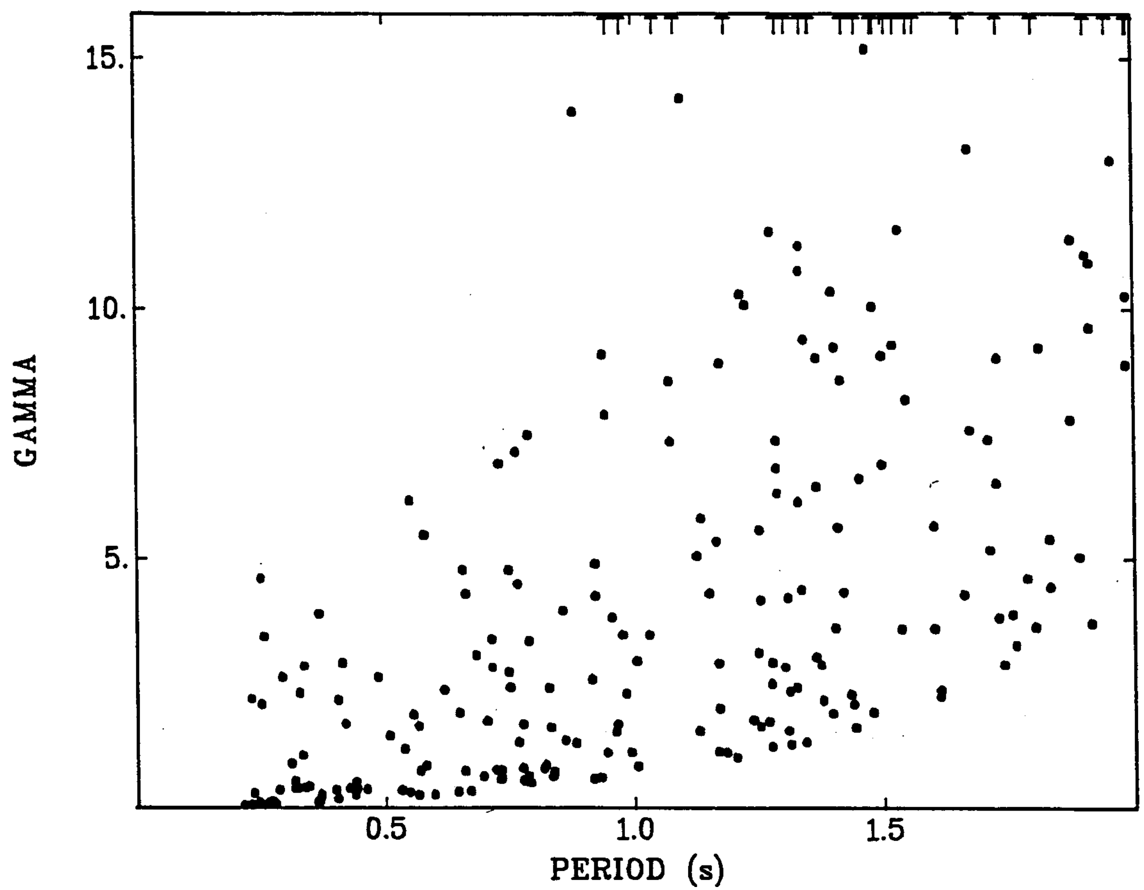

Fig. 3.6: GAMMA vs. period distribution of a simulated population of pulsars. The parameters used in the simulation are summarized in the text.

voltage $V \propto B/P^2$ [B = magnetic field] generated by the pulsar at its polar cap falls below a critical value of $\sim 10^{12}$ volts.

In figure 3.6 we have plotted a distribution of $\dot{P}$ vs P of a simulated population of pulsars. The pulsars in the diagram were generated at regular intervals and then allowed to evolve. Their spin periods were fixed to be 100 milliseconds at birth, but their magnetic fields were randomly assigned to have a value between 10^{12} to $10^{13.5}$ gauss, with a gaussian weight. Pulsars were made to disappear below the "death line" given by $B_{12} / P^2_{sec} = 0.15$. A field decay timescale of 4 million years was used. The "snapshot" of the population was taken 10 million years after the first pulsar was generated. It can be seen that a comparison of fig. 3.6 and fig. 3.4 reveals hardly any significant difference. It is therefore fairly clear that the gross features of $\dot{P}$ vs P diagram do not require the presence of field growth as claimed by Woodward. A more detailed comparison of the simulated distribution with the observed one is not straight forward, since the simulated distribution does not take into account various selection effects in pulsar surveys. However, one knows from the numerous detailed studies of pulsar statistics (see Narayan 1987 and references therein) that the pulsar data does not present any clear evidence of magnetic field growth.

Conclusion : The above analysis shows that one need not invoke field growth to understand the distribution of pulsars in the $\dot{P}-P$ diagram.

3.5 DOES FIELD GROWTH PROVIDE AN ALTERNATIVE TO SLOW INITIAL ROTATION ?

Before concluding this chapter, we wish to ask the following question. In chapter 2, we argued that the paucity of bright plerions could be understood either in terms of young pulsars being slow rotators, or pulsars "turning on" long after the supernova remnant has disappeared. It would thus appear that if one invoked field growth it would not be necessary to simultaneously require that the majority of pulsars are born as slow rotators. We now wish to argue that this is not consistent with the result of recent pulsar surveys, which show a distinct deficit of short-period pulsars i.e. with $P < 100$ ms (Stokes et.al. 1986). It will be argued below that even if the magnetic field of a pulsar grows substantially after its birth, one cannot get away from the conclusion that the majority of young pulsars are slow rotators.

Our present discussion will concern the period evolution of neutron stars during the phase of magnetic field growth. We shall consider the case where the magnetic field of the pulsar grows exponentially from 10^8 gauss to $10^{12.5}$ gauss in 10^5 years. The magnetic field and rotation period as functions of time are given by (3.30) and (3.31) respectively. In our specific example, since the magnetic field grows by 4.5 orders of magnitude in 10^5 years, the growth timescale

$$\tau_m = 9650 \text{ yr.}$$

In figure 3.7 we have plotted the trajectories of pulsars in the field-period diagram for different initial rotation periods. One feature is immediately noticeable - namely, most of the slowing down takes place at large fields. This is hardly surprising because the torque increases as square of the magnetic field. We see that the slowdown is almost insignificant till a field of $\sim 10^{11}$ gauss is reached. A neutron star starting out with a period of $\sim$10 ms slows down to at most $\sim$20 ms when the field is fully grown, and one with an initial spin period $\sim$100 ms hardly slows down at all during the field growth phase. It is clear therefore that if the majority of the observed pulsars evolved from fairly rapidly spinning neutron stars, then they would have had to go through the hatched region in fig. 3.7. But during this phase they should be detectable as pulsars. But as we see from the distribution of observed pulsars, there are very few pulsars in this region. Until quite recently the absence of pulsars in this region was attributed to selection effects of various kinds. However, from a recent sensitive survey designed specifically to look for fast pulsars, Stokes et.al. (1986) have concluded that there is no significant population of pulsars in our galaxy with periods between 10 and 100 milliseconds*.

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* It should be remembered in this context that the three "millisecond pulsars" discovered so far belong to a very different population of pulsars. Their magnetic fields are $\sim 5.10^8$ gauss.

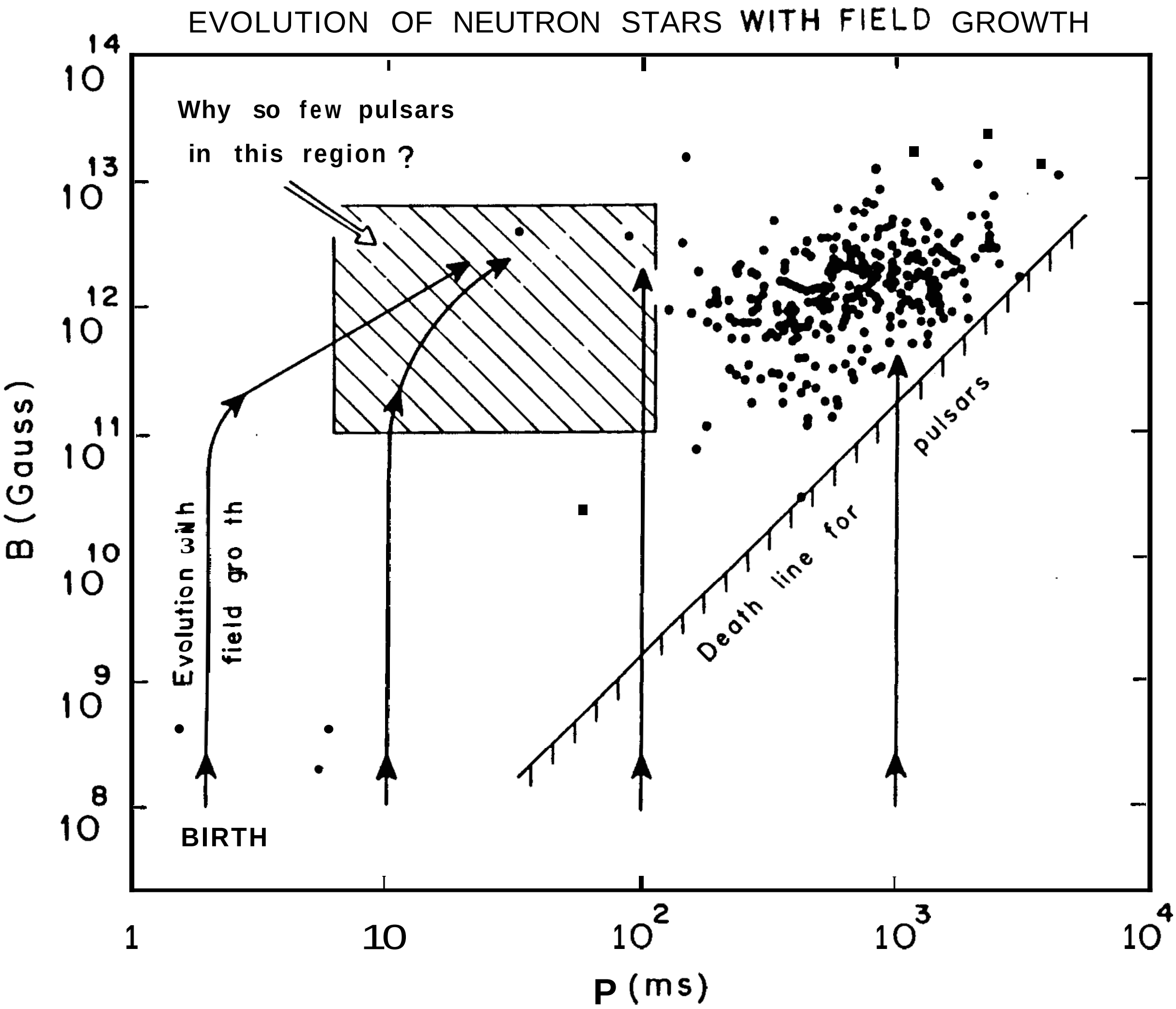

Fig. 3.7: The evolution of neutron stars during an exponential growth of their magnetic fields. The e-folding timescale for the mag— tic field has been assumed to be 9650 yr (see text). Trajectories corresponding to initial rotation periods of 2ms, 10ms, 100ms and 1s are shown. The periods and derived magnetic fields of observed pulsars are shown as dots. Neutron stars born with spin period less than 100ms must pass through the hatched region as functioning pulsars, but very few pulsars have actually been found in this region.

If we take this conclusion seriously, and we should, then it means that whether or not the magnetic fields of pulsars grow with time, the initial periods of the majority of them could not have been < 100 ms. This conclusion is in agreement with the analysis of Vivekanand and Narayan (1981) that most pulsars are "injected" with long periods.

Narayan (1987), has recently argued that the absence of short period pulsars could be understood in the field growth scenario without requiring long initial periods of pulsars. We disagree with this suggestion. The arguments presented in this section shows that field growth is not an alternative to long initial periods.

To summarize, we have seen in this section that no clear, unambiguous evidence for the field growth of neutron stars so far exists. Fast pulsars seem to suggest that rapid spin would generate magnetic field very quickly - yet the millisecond pulsars have not grown their fielda in 10^9 years. Finally, the scarcity of short period pulsars in the galaxy cannot be explained by field growth alone. No matter whether there is field growth or not, long spin periods of pulsars at birth seems an inevitable conclusion.

3.6 SUMMARY AND CONCLUSIONS

In this chapter we have taken a critical look at two mechanisms that have been suggested in the literature concerning the generation of magnetic fields of pulsars. It has been suggested that these mechanisms may provide a natural explanation for the poor association between pulsars and SNRs.

In section 3.2 we discussed the battery mechanism suggested by Woodward (1978, 1984) and came to the conclusion that in degenerate stars, such as a neutron star, this mechanism will not be operative.

The thermoelectric battery suggested by Blandford et.al. (1983) was outlined and discussed in section 3.3. Although this mechanism will work in principle and has several attractive features, the efficiency of the process depends critically on poorly understood details of fluid circulation and behaviour of transport coefficients in a strong magnetic field. As a result, it is difficult to say anything with confidence regarding, for example, the timescale for growth or the field geometry.

In section 3.4 we have examined the observational data for magnetic field growth and have arrived at the conclusion that no clear, unambiguous evidence exists for thermal generation of magnetic fields of neutron stars after their birth.

In section 3.5 we have shown that from the results of the recent pulsar surveys it is quite clear that field growth of neutron stars cannot be considered as an alternative to long rotation periods at birth. Irrespective of whether the magnetic field grows or not, slow rotation at birth remains an inevitable conclusion.

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