Chapter 4

Dynamics of defects with periodic structures

4.1 Introduction

So far we have dwelt upon the elastic properties of static solitons and soliton lattices. As described in chapter I, a static soliton becomes dynamic, i.e, it starts moving if there appears a difference in the base state potential energies or if there is an asymmetry in the potential energy associated with the core of the soliton. These dynamic solitons are actually solitary waves, since they preserve their structure during motion. However, they do not preserve their structure after a pairwise collision. So even though, they are not true solitons, in liquid crystal literature they are referred to still as solitons. In liquid crystals these dynamic solitons exhibit many unique features not found in solitons of other condensed matter systems.

We study in this chapter the structural and dynamical features of two kinds of dynamic solitons which are associated lattice structures. They are:

- Soliton with a tapered lattice. This is a dynamic single soliton with an uniform orientation at one end and a nearly periodic director distortion at the other end. The distortions decay exponentially to reach another uniform state.

- Multi-soliton lattices. This is a periodic stack of single solitons.

These defects have been studied both in nematic (N) and ferronematic (FN) liquid crystals.

4.2 A particle analogy

A mechanical analogy is often useful in understanding these soliton structures. We illustrate this with an example of an FN in a rotating magnetic field. In this geometry, both the field and director are confined to the same plane ($x-$y plane say). The differential equation governing the director motion has already been discussed in the introduction and is given by

$$K\phi_{XX} = \eta\phi_X + \beta \sin 2\phi + \alpha \sin\phi - F \quad (4.1)$$

where a = MH, $\beta = \frac{\chi_a H^2}{2}$, $F = \gamma_1\omega$, $\eta = \gamma_1 u$ and $X = x -$ ut is a moving coordinate frame moving with a velocity u. Often F is referred to as the driving force. Equation (4.1) [1], can be looked upon as representing the motion of a particle with mass K, moving in a potential $g(\phi) = -\alpha\cos\phi - (\beta/2)\cos 2\phi + F\phi$ with a damping coefficient η. The maxima and minima of the negative of potential energy $g(\phi)$ is brought out in Figure 4.1. Needless to say, the peaks here refer to stable or metastable states and valleys unstable states.

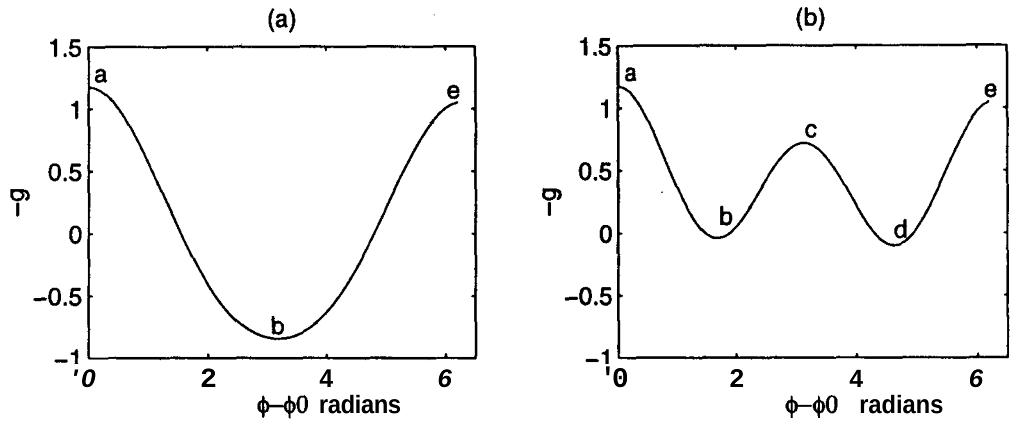

Figure 4.1: The potential energy $g(\phi)$ of an FN (a) for $\beta < a$. With $a = 50cm^{-2}$, $\beta = 10cm^{-2}$, $F = -1cm^{-2}$ and $\phi_0 = \phi' = -0.0142$ radians *(b)* for $\beta > a$. $a = 10cm^{-2}$, $\beta = 50cm^{-2}$, $F = -1cm^{-2}$ and $\phi_0 = \phi'' = -0,0090$ radians.

The potential curve is drawn for both $\beta < a$ and $\beta > a$. The particle analogy helps us to understand the structure and dynamics of single solitons and multi-soliton states. The salient features of this analogy are:

(i) **A** dynamic single soliton moving with a uniform velocity u and connecting base states of the same magnetic potential energies corresponds to a particle motion under such a friction η that, starting from rest at a hill top 'a' it comes to rest on the next hill top 'e'. In other words, the particle goes from one stable equilibrium point to another stable point. Here, the damping experienced by the particle for this motion is said to be critical. Some new properties of such soliton moving due to the core asymmetry in the potential energy were discussed in the introductory chapter.

(ii) **A** dynamic single soliton connecting states of different magnetic potential energy corresponds to a particle starting from rest from a hill top 'a' and coming to rest in a valley 'b' or 'd'. In the highly damped case the particle slides down monotonically from rest at a hill top down to the adjacent valley. When the damping is not high enough, the particle settles down at the bottom of the valley after going through damped periodic oscillation about the bottom of the valley. This oscillatory motion corresponds to the periodic distortion accompanying the soliton. Since the amplitude continuously decreases. This is a dynamic soliton with an associated tapered lattice. This type of soliton is present both in nematics and ferronematics. If the unstable state is at 'b' then the soliton is called the ***A*** soliton and if it is at 'd' the soliton we call it a E soliton.

(iii) Another type of particle motion is possible only in the case of ferronematics which has a metastable peak at 'c' whenever $\beta > \alpha$ as shown in Figure 4.1(b). In this type, the particle starts from rest at 'a' and goes to 'c' and falls back to the valley at 'b' where it settles down after damped oscillations about 'b'. We call this the G soliton.

(iv) **A** particle starting from rest at a hill top 'a' scales several hill tops before coming to rest on one of them. This corresponds to a periodic array of single solitons i.e., a dynamic soliton lattice. The velocity of the particle is non-zero at any intermediate hill top and zero at the last hill top where the particle comes to rest.

We discuss in this chapter the last three types of solitons. Incidentally, the E and G solitons are new states not discussed so far in literature.

4.3 Single solitons with a tapered lattice

We consider first these single solitons in ferronematic liquid crystals in a rotating magnetic field. In all the cases, the soliton is driven by an asymmetry in the magnetic potential energy of the base states. This is shown schematically for the case of ferronematics in Figure **4.2.** This motion is to be contrasted

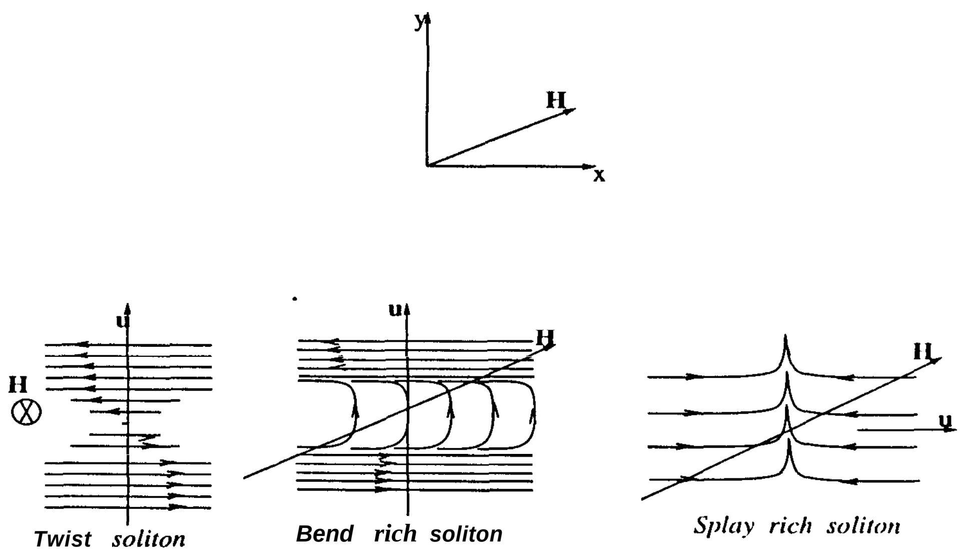

Figure **4.2:** *Dynamic single solitons in a ferronematic in a rotating field (H)*

with the motion of single solitons connecting base states of the same magnetic potential energy and which are driven by the asymmetry in the potential energy inside the core of the soliton. As discussed in chapter I, for a single soliton connecting the base states of the same magnetic potential energy, the velocity is a function of the driving force $\mathbf{F}$ and is zero at $\mathbf{F} = 0$ and diverges at $\mathbf{F} = \mathbf{f}$ [1]. It is easy to conclude that in the present case, even when the field is non-rotating ($F \neq 0$) the soliton will be in motion.

4.3.1 Dynamics

These solitons are driven primarily by the base state asymmetry in its magnetic potential energies [4]. However, apart from the base state asymmetry there is also present a core asymmetry in the magnetic potential energy, since in the centre of the soliton the director field is not symmetric with respect to the magnetic field. These two together give rise to a dynamic single soliton with an associated periodic distortion.

Some of the general properties of such lattices can be summarised by considering a specific example - that of a ferronematic in a rotating magnetic field. Referring to Figure 4.1(b), we can see that in this case, the magnetic potential energy $g(\phi)$ has two stable states at 'a' and 'e', one metastable state at 'c' and two unstable states at 'b' and 'd'. As already stated, depending on the amount of damping, the particle will either slide down from 'a' to one of the unstable states at 'b' or 'd' or will settle down to these states with an exponentially damped oscillation. Since the damping of the particle corresponds to the velocity of the soliton, we either get a very fast moving single soliton without periodic distortions or a slowly moving soliton with an associated periodic distortion, the amplitude of which decreases exponentially. This can be qualitatively understood by solving the linearised form of equation (4.1). This corresponds to the tail region where the amplitude of oscillations are small. In this limit, this equation is a damped harmonic oscillator equation whose analytical solutions are well known. For example, below a certain η we get a periodic damped oscillation. Beyond a certain value of the friction $\eta = u * \gamma_1$

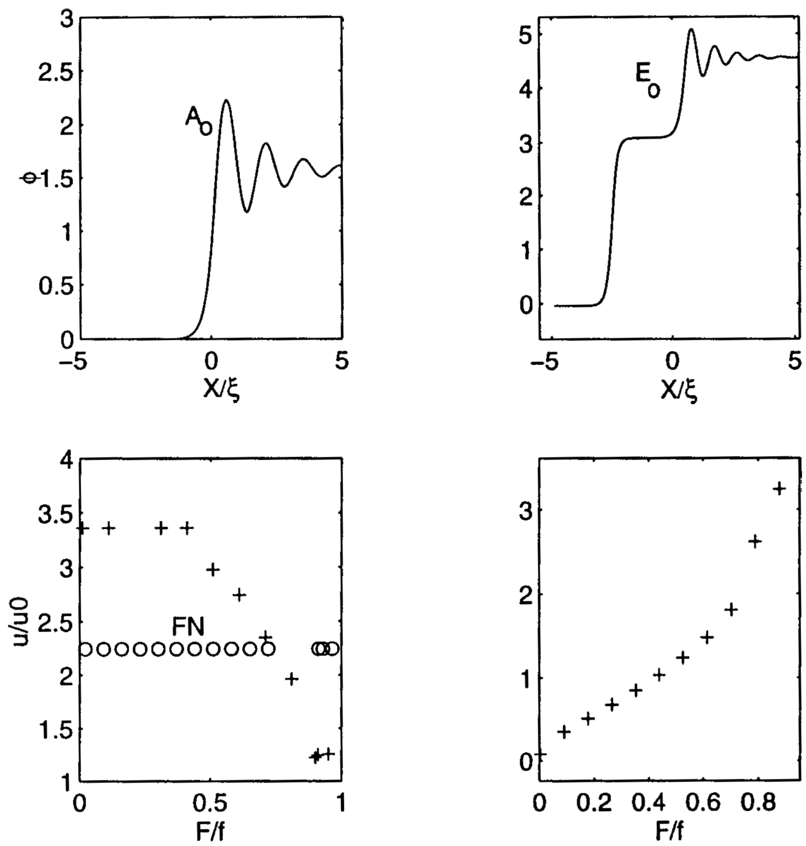

Figure **4.3:** *Solitons with a tapered lattice. (a) The A_0 type of soliton (b) E_0 type of soliton. Scaled velocity u of a soliton as a function of scaled force for (c) A_0 soliton and (d) for E_0 soliton.*

there is no oscillatory motion but a monotonic decay. Also, while the driving force F solely affects the intial amplitude of the periodic distortion, the α and β coefficients affect the intial amplitude and period as well. In our problem these parameters have to be numerically evaluated by integrating equation (4.1). We get the following answers for three distinct *types* of single solitons with a tapered lattice.

- <u>The $\boldsymbol{A}$ soliton</u>

  In a $\boldsymbol{A}$ type of soliton, as said earlier the particle slides down from a to b. This motion from the stable to the unstable state occurs over a range of values of η i.e., $\eta' < \eta < \eta''$. That is, in the case of the soliton this occurs over a range of velocity values. Depending on the precise value of η, we get a soliton with an oscillatory tail or one without any oscillations. The former is known as the A_0 type of soliton and the later the $\boldsymbol{A}$ type [5]. We consider here only the A_0 type of soliton. We have computed numerically, the soliton velocity as a function of F. Since for every F there is a range in the velocity, we have chosen that velocity in the range beyond which the oscillations in the soliton profile is absent. The computations were done for nematics also $(\alpha = 0)$ as this soliton is permitted both in nematics and ferronematics. The results are shown in Figure 4.3(a). The scaled velocity $u_0 = 2\sqrt{(\alpha^2) + (\beta^2)}/\gamma_1$. In the case of nematics, the velocity at small values of F remains a constant and decreases to zero as $F \to$ f. However, in the case of FN the velocity of the soliton remains more or less constant at all of $F/$f. Also both in the case of N and FN the A_0 soliton has a non-zero velocity at $\mathrm{F} = 0$. This is to be expected since the soliton connects a stable state to an unstable state. Incidentally, in the case of both nematics and FN the A_0 type of soliton cannot be stabilised very near $F =$ f.

- <u>The E soliton</u>

  This dynamic soliton connects a stable state at 'a' to an unstable state at 'd' of Figure 4.1(b). This soliton which we call the E type is peculiar to FN. This soliton is also permitted over a range of η i.e., velocity values.

Again we can have a soliton with or without the periodic distortions. The soliton with an oscillatory tail is called the E_0 soliton. The velocity-force characteristics of this soliton is very different from the A_0 type of soliton. As seen in Figure 4.3(b), the velocity of this soliton continuosly increases and diverges near the asynchronous regime but at $F < f$.

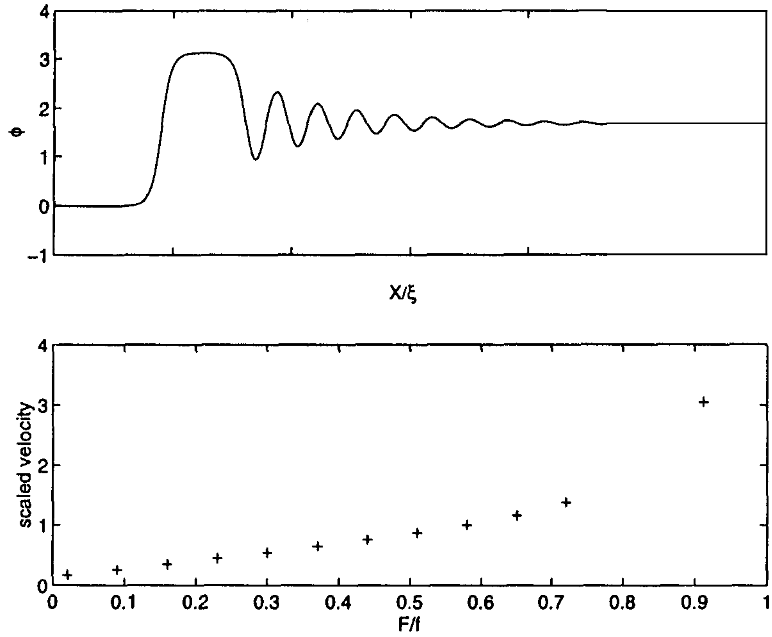

Figure 4.4: *A* G_0 soliton with its velocity-force characteristics.

- <u>The G soliton</u>

A G soliton is present in the case of an FN whenever $\alpha < \beta$. This soliton connects a stable state at 'a' to the unstable state at 'b' through the metastable state at 'c'. The presence of the metastable peak at 'c' is crucial to the existence of such a soliton. This soliton is only possible whenever the excess elastic energy expended in having a higher distortion (going upto 'c' instead of going directly to 'b') is comparable to the gain in energy due to smaller friction η. Here again we can have a soliton with or without tapered lattice. The former is called the G_0 soliton. The structure and the velocity force characteristic of this soliton is shown in Figure 4.4. The velocity, as in the case of an E_0 soliton diverges at a $F < f$.

4.3.2 Structure

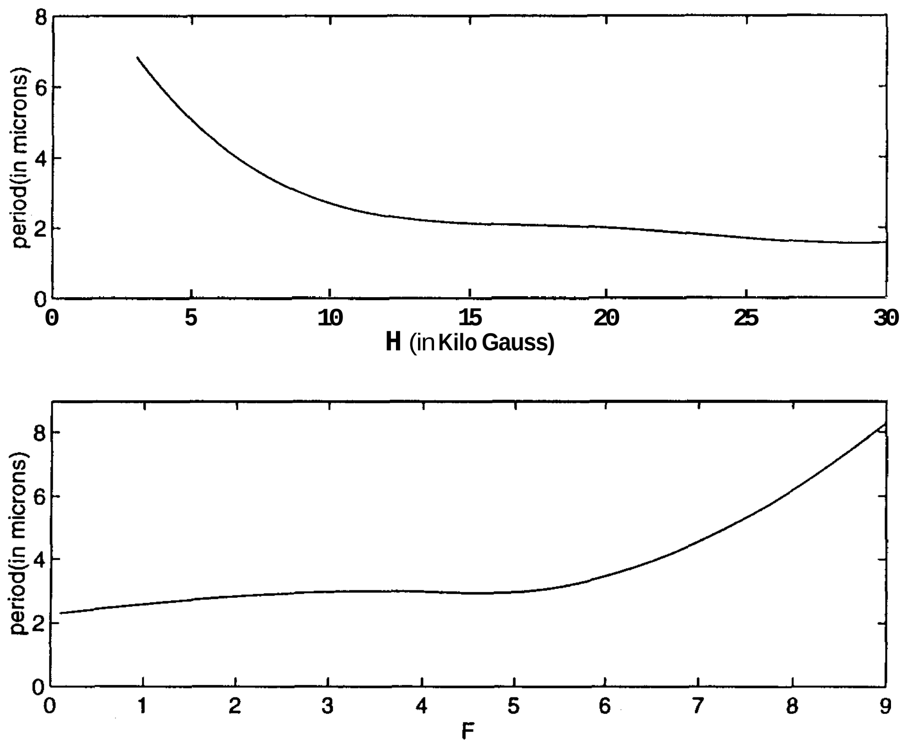

Figure 4.5: *Period dependance of the tapered lattice on* **(a)** *F and* (b) *on H. For $H < H_c$* **we** *have asynchronous regime*

In both the A_0 and E_0 type of solitons, the periodic distortions associated with the soliton show exponential decay of their amplitudes. But the period of these distortions is not a constant all through the lattice. Figure 4.5, shows the

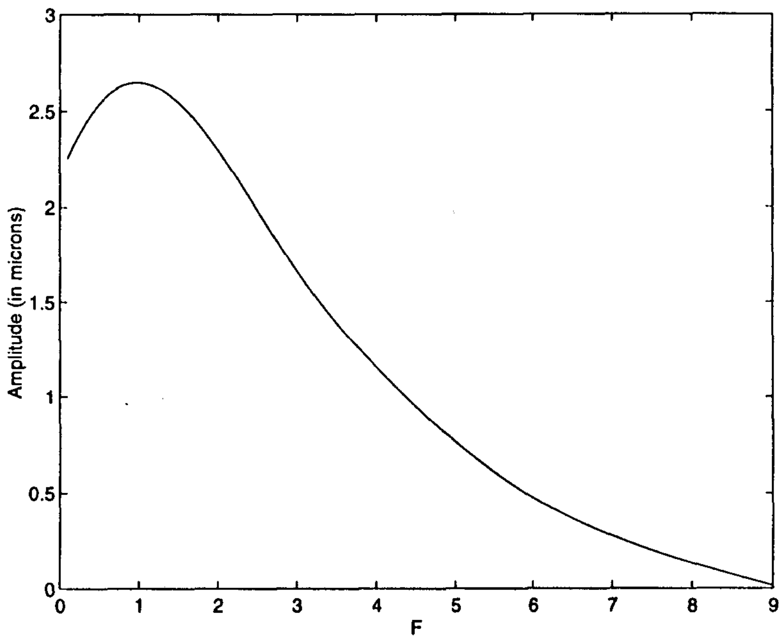

Figure 4.6: Dependance of initial amplitude of the tapered lattice on F

dependence of the average period on both H and F. Interestingly, in contrast to the behaviour of static soliton lattices discussed in the previous chapters, the period of this soliton decreases on increasing H. The initial amplitude of the oscillations, on the other hand after an intial increase, decreases with increasing F and is not affected to any appreciable degree by changes in H. The amplitude variation with F is shown in Figure 4.6.

It is to be mentioned in this context that the existence of **a** range of velocities in the case of solitons connecting the stable and unstable states is a well known feature of front propagation from stable to unstable states [6]. The actual velocity selected by the medium will be based on a marginal stability criterion. We have not addressed ourselves to this problem here.

4.4 Soliton lattices

An array of single solitons will constitute a soliton lattice. From an experimental point of view, a multisoliton structure can possibly be obtained as follows. A nematic with $\chi_a > 0$ is homeotropically aligned along the z direction and a magnetic field in the x – y plane. Then it is known that, beyond a threshold field, the director in the central region of the sample, tilts towards the x – y plane. In the process planar solitons are formed in many places in the x – y plane. Two like solitons which are formed close to each other will repel, in a static situation. But due to the rotation of the field about the z axis, such two or more solitons can coexist and form what is called as a multisoliton lattice. The single solitons forming the lattice will all be moving in the same direction with the same velocity. Hence the multisoliton structure as a whole will move with this single velocity. The multisolitons that we discuss here are not infinite in extent i.e., the number of single solitons in an array is finite. In the particle analogy, a multisoliton lattice with n single solitons corresponds to a particle having such an initial velocity that it traverses n peaks before coming to rest on a peak.

We give here an analysis of multisolitons based on particle analogy. For soliton lattices also, the governing partial differential equation is the same as the case of single solitons discussed so far. That is, the director orientation in a lattice and the velocity of the lattice can be obtained by solving

$$\frac{\partial\theta}{\partial t} = K\frac{\partial^2\theta}{\partial x^2} - \alpha\sin\theta - \beta\sin 2\theta + F \qquad (4.2)$$

We solve this equation, as in the case of single solitons, by assuming that a traveling wave multi-soliton exists. Then the corresponding ordinary differential equation obtained in a frame moving with a velocity u, is solved. It must be mentioned no two succesive solitons in a multisoliton lattice are exactly identical. Also, the lattice parameter is not a constant all through the structure. We have worked out the structure and propagation velocities of such multisolitons both in N and FN. We find the effects of magnetic field (H) and F to be very similar in both the cases. Here we present results obtained for nematics in a rotating magnetic field.

4.4.1 Dynamics

Theoretically computed velocity-force characteristic for multisolitons in nematics is shown in Figure 4.7 for two different values of n the number of single soliton units in the soliton lattice. Interestingly, the velocity of a multisoliton

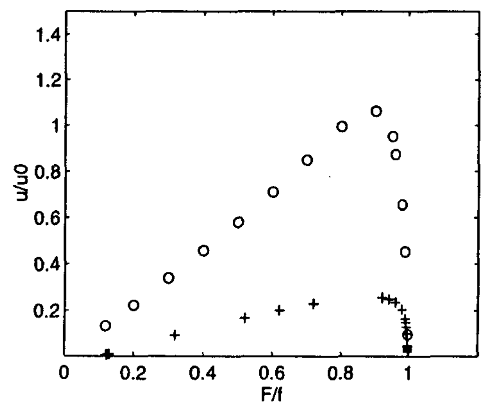

Figure 4.7: *A typical velocity profile of a multisoliton lattice. The curve with (o) is for $n = 10$ and the curve with (+) is for $n = 25$.*

does not diverge at any value of F. The velocity starts from zero, exhibits a peak in the range $0 < F < f$ and then rapidly decreases actually going to zero as $F \rightarrow f$. This behaviour is entirely different from what we find in the case of single solitons. As n increases the overall velocity of the multisoliton lattice decreases but the characteristic remains qualitatively the same.

4.4.2 Structure

Effect of force (F)

The structure of a multisoliton undergoes considerable modifications as F increases, This is depicted in Figure 4.8(a) for $n = 10$. As F increases, the multisoliton structure undergoes a gradual transition from an array of well spaced solitons to an array of closely spaced solitons. At high F, we find a nearly uniform periodic structure i.e., $\phi_x = const$. Further the average bend or splay or twist in the structure increases as F increases.

The number n of single solitons that can be stabilized at a given velocity is also decided by F. Interestingly this number increases from unity at $F = 0$ reaches a peak value before it drops to unity at $F = f$. In other words multi-solitons cannot exist at $F = 0$ and $F = f$. This aspect is depicted in Figure 4.8(b).

The period of the soliton lattice as a function of F is shown in Figure 4.8(c) for different values of n. The important result is that the period diverges as $F \rightarrow 0$. This is understandable since in this limit, the differential equation permits only a single static soliton. In this context, we may recall the experimental results of Migler and Meyer [7]. In their observations, the distance between any two neighbouring solitons in a spiral soliton lattice obtained in a rotating field, diverged as the rotational frequency which is propotional to F is decreased.

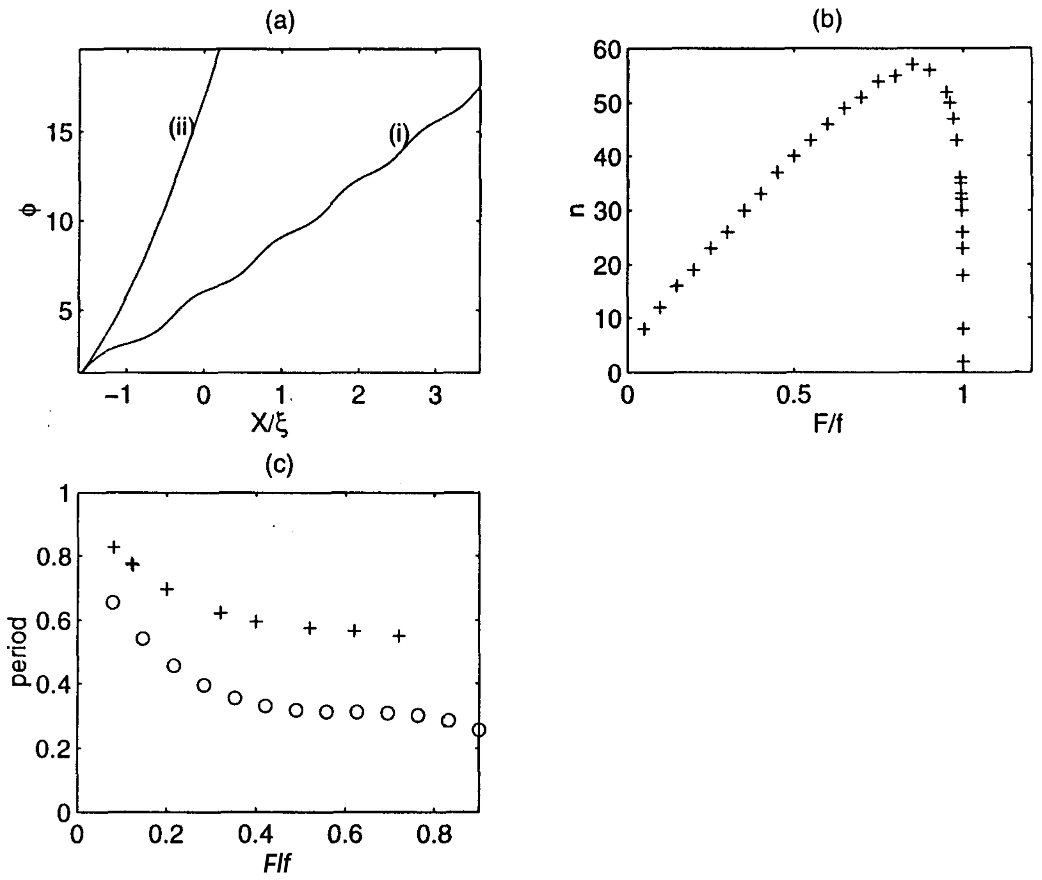

Figure 4.8: *(a) The profile of a soliton lattice with $n = 10$ for (i) $F/f = 0.1$ (ii) $F/f = 0.9$ (b) The number n of solitons in a soliton lattice with $\eta = 1.2cm^{-1}$ as a function of F/f. (c) The variation in the period p (in μm) of a soliton lattice as a function of F/f. The curve with (o) is for $n = 10$ and the curve with (+) is for $n = 25$.*

Effect of field (H)

The period of a multisoliton lattice is sensitive to H both in N and FN. The structure of the soliton lattice undergoes drastic modifications as the field is increased. From a uniformly distorted structure at low fields the lattice develops into a well formed soliton train at higher fields in the case of an N. There is also a decrease in the average bend or splay at higher fields. These features

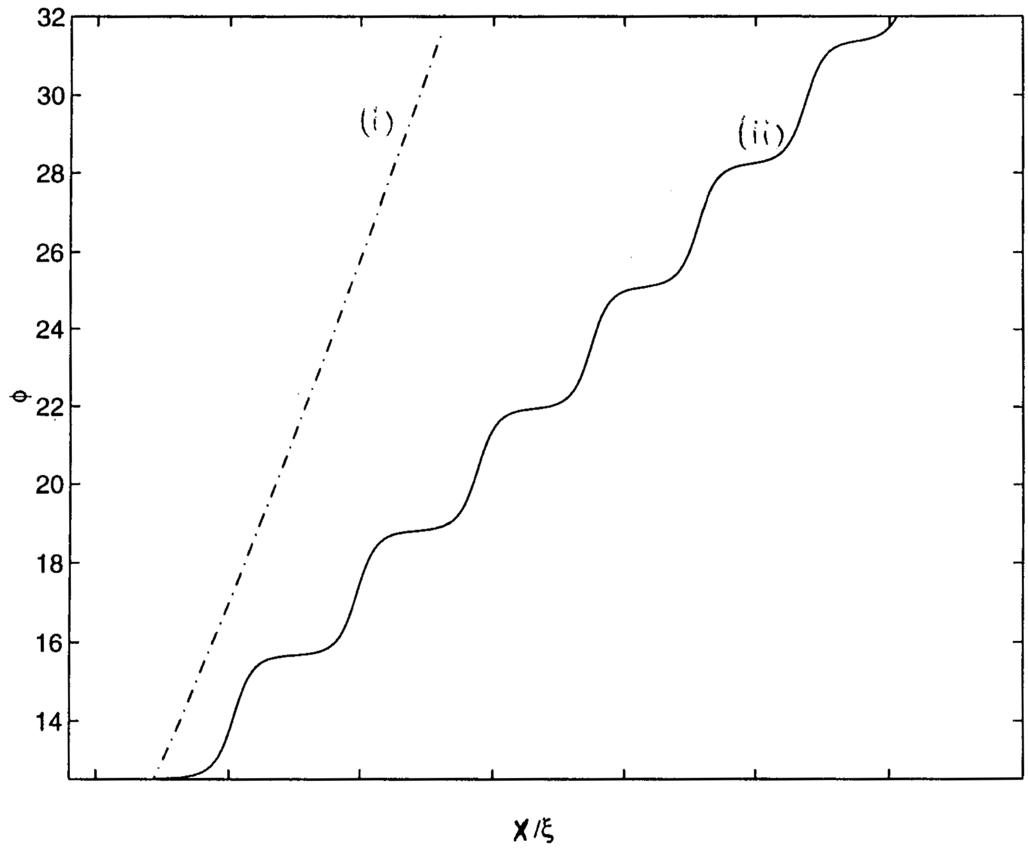

Figure **4.9:** *The profile of a soliton lattice in an N with n = 10 at two different magnetic field values (i) H = 2000 Gauss (ii) H = 9000 Gauss*

are shown in Figure **4.9.** In the case of FN however, the average splay or bend more or less remains a constant. It should be remarked that in FN, very high field values cannot be considered due to the possibility of grain migration [8].

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