Wave propagation and diffusion in active and passive random media

by

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for the award of the degree of
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DECLARATION

I hereby declare that the work reported in this thesis has been independently carried out by me at the Raman Research Institute, Bangalore, under the supervision of Prof. N. Kumar. The subject matter presented in this thesis has not previously formed the basis of the award of any degree, diploma, associateship, fellowship or any other similar title of any other University or Institute.

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CERTIFICATE

This is to certify that the thesis entitled **Wave propagation and diffusion in active and passive media,** submitted by **S.** Anantha Ramakrishna for the award of the degree of DOCTOR OF PHILOSOPHY of Jawaharlal Nehru University, is his original work. This has not been published or submitted to any other university for any other degree or diploma.

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Preface

Our work reported in this thesis concerns wave propagation and transport in active and passive disordered media. The work can essentially be divided into two parts: The first deals with coherent wave transport and localization in spatially random media and the associated statistics of fluctuations, and the second is concerned with the development of stochastic models to describe the wave propagation in terms of an incoherent energy transport in stochastic media. In the first part, we also discuss the use of an imaginary potential as a quantum 'clock' for the sojourn time of a wave in a scattering potential.

In the first part (Part-A), consisting of Chapters-2, **3** and 4, we study wave propagation in spatially random media. It is well known that the interference associated with coherent multiple scattering of waves (for quenched or static disorder) causes physical quantities such as the reflection/transmission or the conductance/resistance of a disordered sample to become non-self-averaging (i.e., the fluctuations grow faster than the mean) regardless of the sample size. Thus, one needs the entire probability distribution over an ensemble to describe such quantities. In low dimensions. the effect of disorder on wave propagation is more drastic resulting in the localization of all states in one and two dimensions. This is particularly true for light, for which localization in higher dimensions can be accomplished only by a combination of strong Bragg scattering and large refractive disorder. Moreover, the Bosonic nature of light, which allows for coherent amplification and absorption, leads to new phenomena such as random lasers. In this part of the thesis, we study the statistics of the non-self-averaging quantities such as reflection and quantities describing the dynamical aspects such as the delay/dwell times of scattering from a random medium. One of the main ideas in this thesis is that the coherent amplification/absorption, which preserves the temporal coherence of the wave, will also cause a concomitant scattering of the wave. Thus, we have a synergetic interplay of Anderson localization and coherent amplification/absorption, where the localization could be caused by the very scattering concomitant with a spatially fluctuating amplification/absorption in a medium. In Chapter **3,** we define a 'clock' for the quantum-mechanical sojourn time in a scattering potential using coherent amplification/absorption as a mathematical artifice, but find that we have to correct for the extra scattering concomitant with it.

In the second part (Part-B), consisting of Chapters-5, 6 and 7, we develop new models for the problem of photon migration in turbid media, where we describe the wave propagation in a random medium as an incoherent energy transport in a stochastic (time varying random) medium. The underlying idea is that under conditions of weak, but multiple scattering, the transport of a wave becomes almost diffusive and the entire process can be described as a random walk of a particle. This approach, originally developed in the context of radiative transfer in stellar atmospheres, results in a Boltzmann transport equation for the specific intensity or the particle flux in the phase space. However, the general analytic solutions of this equation are unknown even for the simplest geometries, and the diffusion equation, which is a good approximation for the transport equation at long length scales ($L \gg l^*$, where l^* is the transport mean free path) is most-often used. But, the diffusion approximation, which is a Wiener process for the spatial co-ordinates of a particle. is physically unrealistic, and accounts neither for a finite mean free path nor for a finite and constant speed(c) of the particle, which is characteristic of light propagation in a random medium. It has also been shown experimentally to fail to describe phenomena at short length-scales ($L < 10l^*$) and short time scales ($t < 10t^*$ where $t^* = l^*/c$). It is precisely these length- and time-scales that are involved in medical imaging and diagnostics using laser light, and it is of importance to develop better and alternative schemes to the diffusion approximation. We develop here simple models of photon migration in a stochastic medium, where the photon propagates with constant speed in between the scattering events. This imposition of the constraint of constant speed effectively incorporates a finite mean free path into the problem as well as persistence in the phase space.

Below, we present a summary of the problems studied and the results obtained chapterwise. Each chapter has a self-contained introduction, which sets the background for the material presented in that chapter.

Chapter 1: This is an introduction to the thesis, pertinent to the work reported in it. We review the important ideas and previous literature that are directly related to the problems addressed in the thesis.

Chapter 2: A mismatch in the imaginary part of the refractive index (imaginary potential) always causes a concomitant scattering in addition to amplification or absorption. Pure imaginary (amplifying) potentials can cause resonant enhancement of the scattering coefficients, which is important for Anderson localization of light in higher dimensions. The probability distribution of the reflection coefficient for light reflected from a one-dimensional random amplifying medium with cross-correlated spatial disorder in the real and the imaginary parts of the refractive index is derived using the method of invariant imbedding. The statistics of fluctuations have been obtained for both the correlated telegraph noise and the Gaussian white noise models for the disorder. In both cases, an enhanced backscattering (with a reflection coefficient greater than unity) results because of coherent feedback due to Anderson localization and coherent amplification in the medium. The results indicate that the effect of randomness in the imaginary part of the refractive index on localization and reflection is qualitatively different.

Chapter 3: The delay time for scattering is the most important quantity regarding the dynamical aspect of scattering in quantum mechanics, and one of the common measures for this quantity is the Wigner phase(ϕ) delay time ($\hbar d\phi/dE$). This quantity, however, has certain deficiencies and alternative clocks such as precession of a spin in a magnetic field have been proposed. Here, we discuss a non-unitary clock, involving absorption/amplification by an added infinitesimal imaginary potential(iV_i) and find it not to preserve the positivity of the conditional sojourn times, in general. The sojourn time is found to be affected by the scattering concomitant with the mismatch, however weak, due to the very clock potential(iV_i) introduced for this purpose. We propose a formal procedure, separately for the cases of wave propagation (non-tunneling at above-the-barrier energy) and tunneling (at below-the-barrier energy), by which the sojourn time can be clocked ideally using the non-unitary counter by correcting for these spurious scattering effects. We further find that the conditional sojourn time for reflection is positive definite only if we agree to consider only those partial waves that have traversed the region of interest. This is justified in that the sojourn time should causally relate to the region of interest. The resulting time is then positive definite for an arbitrary potential and has the proper high- and low-energy limits. We also discuss why the spurious effects effectively cancel out for a random potential.

Chapter 4: In this Chapter, we consider the distribution of sojourn/delay times for wave reflection from a one-dimensional random potential. We show that the sojourn time distribution for the reflection is related directly to that of the reflection coefficient, derived with an arbitrarily small but uniform imaginary part added to the random potential. The sojourn time distribution in the weak disorder-high energy limit then follows straightforwardly from the earlier results for the reflection coefficient, and coincides with the distribution for the Wigner delay time obtained recently by other workers. All the moments of the distribution are divergent. The sojourn time distribution for a random amplifying medium is then derived. In this case, however, all the moments work out to be finite. We also correct an earlier calculation of the distribution of the Wigner delay time, where a slightly different form of the distribution had been obtained. Further, our numerical simulations using the Tight Binding Hamiltonian indicate that the probability distribution obtained for the sojourn time using the imaginary potential and the distribution for the Wigner delay times coincide for both weak and strong disorder. For energies very close to the bandedge, however, the Wigner delay time distribution begins to differ and becomes non-zero for negative delay times indicating a strong deformation of the incident wave packet. The sojourn time distribution obtained from the imaginary potential method displays no such behaviour, indicating that this clocks the 'literal' sojourn time of the wave in the potential.

Chapter 5: Here we adapt use of the Ornstein-Uhlenbeck process of Brownian motion to describe photon migration in turbid media. The Ornstein-Uhlenbeck process of Brownian motion is able to incorporate the finiteness of the mean free path and a well defined root-mean-squared (rms) velocity but assuming, of course, a distribution of speeds. We show by a path integral approach that the finite r.m.s speed defined by the fluctuation-dissipation theorem for this process is a stronger global constraint than a weaker average constraint implemented recently by others. We have developed approximate analytic solutions based on the mirror-image method for absorbing boundaries for this process. The results have been compared to Monte-Carlo simulations and good agreement is found even at short time-scales. The simplicity of the solution for absorbing boundaries makes it an important and useful alternative to the diffusion approximation.

Chapter 6: Here, we develop a model in which the propagation of light in a scattering medium is described as the motion of a special kind of a Brownian particle on which the fluctuating forces act only perpendicular to its velocity. This strictly and dynamically enforces the constraint of constant speed of the photon in the medium. A Fokker-Planck equation is derived for the probability distribution in the phase space assuming the transverse fluctuating forces to be a white noise. Analytic expressions for the moments of the displacement $< x^n >$ along with an approximate expression for the marginal probability distribution function $P(x,t)$ are obtained. Exact numerical solutions for the phase space probability distribution have been obtained for infinite media, semi infinite media and slab geometries with absorbing boundary conditions. The results show that the velocity distribution randomizes in a time of about eight times the mean free time $(8t^*)$, only after which the diffusion approximation becomes valid. This factor of eight is a well known experimental fact. A persistence exponent of $0.4350 \pm .005$ has been calculated for this process in two dimensions by numerically studying the survival probability of the particle in a semi-infinite medium. We also study the case of a stochastic amplifying medium.

Chapter 7: In this Chapter, we discuss a generalization of the Telegrapher process to higher dimensions, which describes the diffusion of inertial particles in a model phase space. In one-dimension, it is long since known that the Telegrapher equation describes exactly the probability distribution function in one dimension for a particle undergoing random scattering events and moving with constant speed between the scattering events. Here only two values of the velocity $\pm c$ are allowed. We have generalized this process to higher dimensions $(d \geq 2)$ rigorously, where the particle can move only along the 2^d directions of the diagonals of a d dimensional hypercube. Using a stochastic approach, a closed set of 2^d coupled, linear, first-order, partial differential equations for the probability distribution function is obtained. This admittedly artificial phase space incorporates considerable persistence in the photon random walks. We discuss several aspects of this model including the effects of the angular non-symmetry of the model that has been missed out in similiar other studies.

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