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High Angular Resolution Optical Interferometry

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Angular resolution is exceptional by its extremely uneven character in various spectral ranges. Contrary to a simple minded diffraction analysis, the highest resolutions are currently obtained in radio-astronomy, at the longest wavelength end of the spectrum (Fig.1). Studies of extragalactic radiosources with a few milliarcseconds resolution are now available from radio observations. If the criterion of more or less uniform angular resolution was adopted as desirable, a major effort at optical wavelengths should certainly be undertaken.

Conventional ground based optical astronomy has been restricted to an angular resolution of about one arc second by the perturbing effect of the earth's atmosphere. Atmospheric turbulence produces inhomogeneities in the refractive index of the air. A plane light wave of uniform amplitude launched into such a medium acquires amplitude and phase fluctuations. Consequently, an optical image formed by focussing such a wave exhibits fluctuations in intensity, sharpness and position. This results in the degradation of angular resolution of an image with a sufficiently long exposure (orders of 1 sec and more) to a value much lower than the diffraction limit of the telescope. Thus for example a 5 m telescope which should have been capable of resolving angular detail as small as 0.025 arcsec gets degraded by atmospheric turbulence by a factor of nearly 50. In other words, a 5 meter telescope capable of gathering one million times the light of a single human eye can outperform the eye by a factor of only 60 in angular resolution which is no better than a 10 cm amateur astronomer's telescope.

Although observations of the effects of seeing date back to ancient times it is comparatively recently that progress has been made in understanding the phenomenon of obtaining high angular resolution pictures in the presence of the turbulent atmosphere.

Around 1970, a young French astronomer named Antoine Labeyrie developed a novel but not particularly difficult method of observation and analysis to surpass the atmospheric seeing limit and to reach the full diffraction limited resolution expected from theory. He gave the new approach the name speckle interferometry. This works by recording images using exposure times between 1/30 and 1/100 sec. During these brief instants the distribution of turbulence can change by only a fraction of r_0, called Fried's parameter,* so that the pattern of blurring is effectively frozen. Computation of the auto-correlation of these pictures gives some information about finer details in the original object which would have been erased in longer exposure.

Another promising approach to obtain high angular resolution images is to combine the well known Michelson Stellar interferometer (Fig.2) with techniques employed by radio astronomers. One of them is the principle of synthesising an image of a self-luminous object by making measurements of its mutual coherence function (Fourier transform of the

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* Fried's parameter, r_0 characterises the atmospheric disturbance and is numerically equal to the size of an aperture across which the r.m.s. phase fluctuation is one radian.

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brightness distribution also called visibilities) rather than by direct imaging. It is also not necessary to measure all the Fourier components simultaneously. One can build up an image gradually by recording with different parts of the aperture at different times. In the extreme case this can be done by using no more than two small apertures one of which is moved about on the ground to occupy each part of the desired aperture in turn (Fig. 3). Since the phase errors are induced by the atmosphere scale with the telescope apertures, one can build images which are less degraded by atmospheric turbulence by using telescopes whose apertures are smaller than r_0 which is normally around 10 cm and using short exposures to record the fringes. Such a two-element interferometer can only measure the fringe amplitude and not the fringe phase in the presence of atmospheric noise.

Another technique used by radioastronomers to make images from data which are badly corrupted by different phase errors at each telescope is the principle of closure phase (Fig.4).

A closure phase is derived from simultaneous measurements on a triangle of three baselines formed by three apertures 1, 2 and 3. In ideal conditions with no atmosphere the observed phases on the three baselines would be ϕ_{12}, ϕ_{23}, and ϕ_{31}. These phases are properties only of the source and its position in the sky relative to the interferometer. In practice the phase are affected by the changing atmospheric phase paths above each aperture giving phases α_1, α_2, α_3. The observed phases will be ($\phi_{12}+\alpha_1-\alpha_2$), ($\phi_{23}+\alpha_2-\alpha_3$), $\phi_{31}+\alpha_3-\alpha_1$). Each contains wholly unknown quantities, but the sum of the three, the closure phase $\phi_{123} = \phi_{12} + \phi_{23} + \phi_{31}$ is independent of the atmosphere. When 4 baselines are measured simultaneously there is another combination of degraded observations that is independent of telescope and atmospheric effects -the closure amplitude $(A_{12} \times A_{34})/(A_{14} \times A_{23})$.

A large body of experience and technique has been accumulated by radio astronomers by operating synthesis telescopes over the last thirty years. This can be extended in principle for efficient processing of visibility data at optical wavelengths. However, the important differences between the optical and radio situations which should be borne in mind are :

1. The fundamental noise limit in optical observations is Poisson photon noise. In radio astronomy it is Gaussian receiver noise.
2. At optical wavelengths the atmospheric coherence

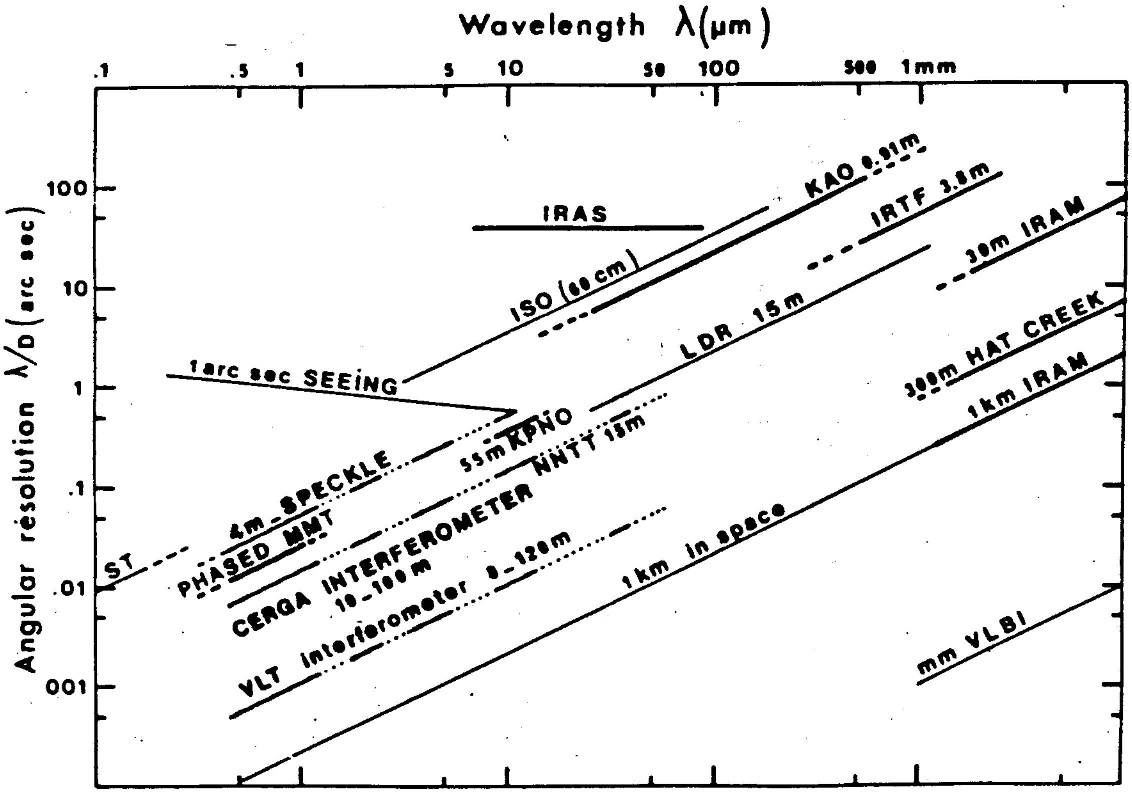

Fig. 1. Angular resolution in the wavelength range 100 nm - 1 cm obtainable in 1985 (heavy lines) and foreseen for the year 2000 (lighter lines). (Ref. 7).

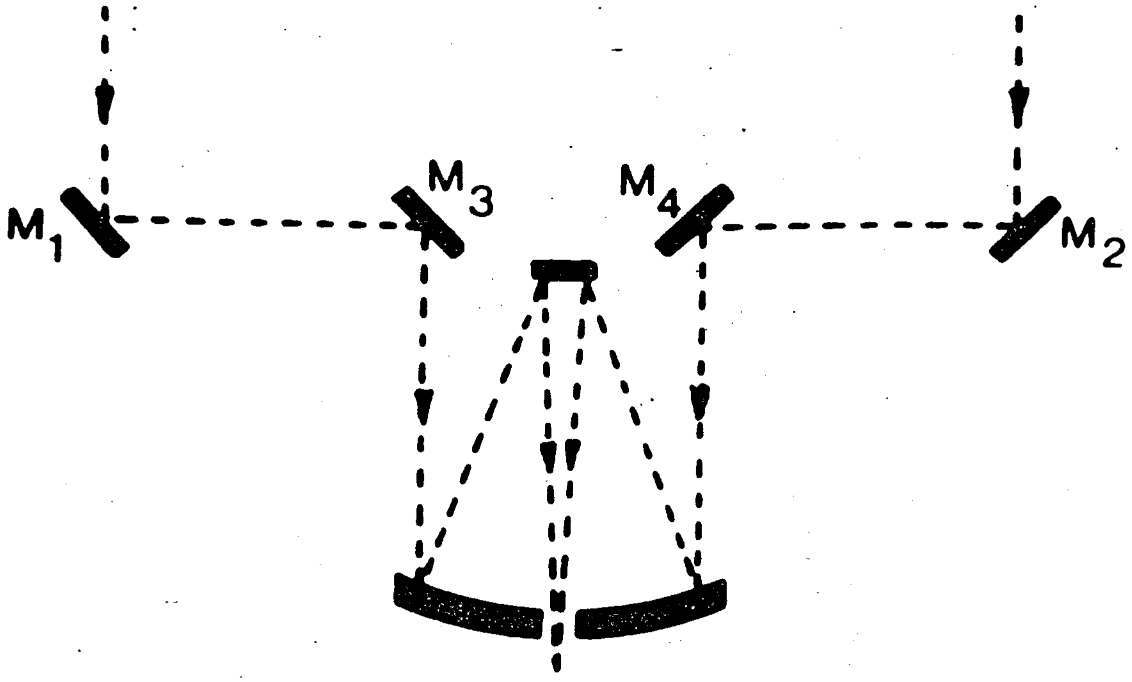

Fig. 2(a). Michelson interferometer

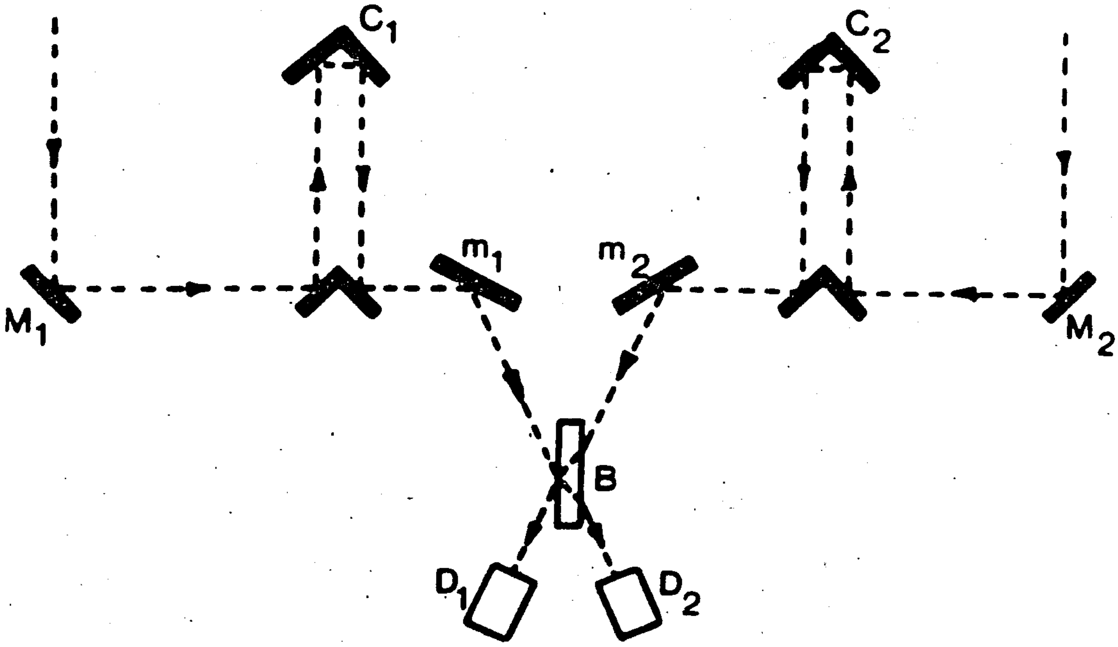

Fig. 2(b). Modern Michelson interferometer (Ref. 3)

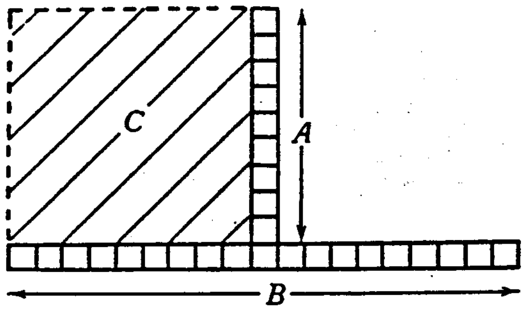

Fig. 3(a). The spacings between each of the elements of a strip A and each of the elements of strip B represent all the spacings found in the rectangle C.

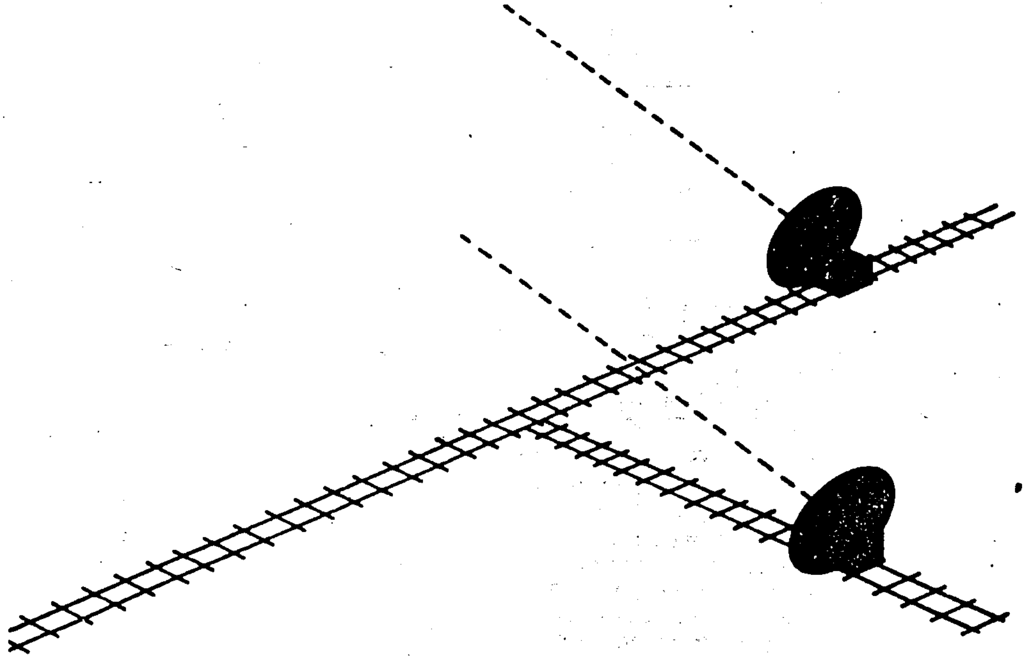

Fig. 3(b). A variable spacing interferometer for two-dimensional aperture synthesis. (Ref. 8)

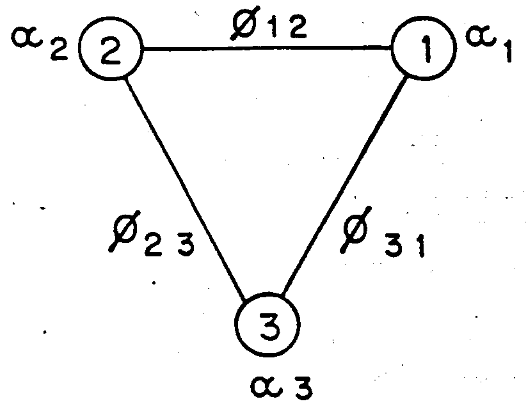

Fig. 4.Closure phase is derived from simultaneous measurements on a triangle of three baselines formed by three apertures 1, 2 and 3.

time, T_{coh} in which the r.m.s. phase fluctuation at a stationary point is one radian and the lateral coherence length, r_{coh} over which the instantaneous r.m.s. phase deviation is one radian, are much smaller than at radio wavelengths. Because of this photon noise increases and a large number (10^3) of frames are needed to determine closure phases to within a few degrees.

3. Coherent low noise amplifiers are not possible at optical wavelengths.
4. At optical wavelengths photon counting devices are available.

Developments in Optical Interferometry

In 1974, A. Labeyrie showed that two separate telescopes could be made to operate as a coherent interferometer. A few years later, in 1977 the angular diameters of Capella A and B were measured with a modest two telescope interferometer. Now-a-days, multiple aperture interferometry is emerging as the most promising instrumental development in optical astronomy. Several proposals involving four apertures or more and baselines as long as 600 m have been put forward by groups at the Universities of Sydney and Cambridge, Cerga in France, European Southern Observatory and many other astronomical centres.

University of Sydney's astronomy department is proposing the construction of a powerful instrument to obtain high angular resolution. They hope to achieve angular resolutions down to 7.5 x 10^{-5} arc seconds at 500 nm with a limiting sensitivity given by magnitude m = + 7.5. They have chosen to make the aperture diameter of the interferometer equal to r_0 which is roughly equal to 10 cm. The array is linear extending to a length of 600 meters along the N-S direction to simplify the active path compensation. Since only two telescopes are used at present, principle of optical closure phase cannot be employed in image formation.

A group of radio astronomers from Cambridge have proposed a system called 'COAST' (Cambridge Optical Aperture Synthesis Telescope) to exploit and adopt techniques of aperture synthesis developed in radio astronomy; particularly the crucial feature of the measurement of closure phases. Their system consists of four telescopes mounted on three radial arms in the form of a 'Y', the maximum length of each arm being 60 m. The telescopes will each consist of a 500 mm diameter flat siderostat mirror. This instrument is intended to produce pictures with a resolution of 0.001 arc sec with a limiting magnitude of +14.

In what follows I will describe some important factors which have to be accounted for in any design.

Size of the Telescope

The aperture size and coherent integration time that can be employed in a ground based optical interferometer are limited by atmospheric perturbations to the optical phase. The short coherent integration times (T_0 = 10 msec) and small apertures (r_0 = 10 cm dia), imposed by the small scales of atmospheric perturbations to the optical phase, mean that the amplitude and closure phase measurements are seriously affected by photon noise. If we try to gather more light by increasing the aperture size or exposure time, there will be fluctuations of many radians in the optical phase across the apertures and during the exposure causing — (1) the measured amplitude and closure phases to fluctuate because there is no longer a single phase error associated with each aperture; (2) the fringe visibility to decrease, deteriorating the SNR (signal to noise ratio). Clearly there is an optimum trade off between collecting more light and these atmospheric effects. It has been shown that to minimise the photon noise at low light levels one can use apertures approximately 30 cm or 3 times the atmospheric coherence length (r_{coh}) and 20 m sec integration time or twice the atmospheric coherence time (T_{coh}). This assumes that we know the values of r_{coh} and T_{coh}, but since they both vary with time, some thought will have to go into developing methods for determining these parameters in real time and into making the aperture sizes and exposure times adjustable.

Number of Telescopes Required in the Array

The minimum number (n) of the telescopes needed in an imaging array is central to its design.

A large number of telescopes is desirable for many reasons

1. The number of independent closure phases is (n-1) (n-2)/2 and the simultaneous baselines is n (n-2)/2. Thus the fraction of the total phase information available is (1-2/n). Thus fewer the elements in the array the more we are dependent on source modelling and the less control we have over systematic errors in the observation, which usually determine the dynamic range of observation.
2. The sensitivity in the final image improves as n increases.

To combine the beams for each baseline using beam splitters, the signal from each mirror must be split (n-1) ways. This increases the observing time to reach a given S/N ratio even in a loss-free system.

It is a matter of judgement to balance the conflicting advantages of large or small n. Clearly it should not be brought below four, which are required to derive the amplitude and the phase closure relations.

Configuration of the Array

1. Since large number of telescopes can't be used (reasons outlined earlier) it is essential to use a combination of earth rotation and changes of the station position to obtain all the baselines necessary for observations of a particular source. One should remember that atmospheric seeing, limits

our ability to use earth rotation for more than ± 3 hours about the meridian.

2. Any configuration chosen should not overlook the convenience of arranging optical paths from the telescopes to the central building.
3. It is not necessary that the configuration which gives a uniform coverage of all the possible base lines (uniform U, V coverage) is best under all circumstances. Rather it is more appropriate to tie down the astrophysical problems that one is going to tackle and decide on a configuration that gives the best SNR for the objects that one is interested in.
4. A set of n apertures may be arranged to provide n(n-1)/2 nonredundant measurements of the object Fourier transform. But for an n telescope array there are only (n-1) (n-2)/2 independent closure phases and (n-2) (n-3)/2 independent amplitude closure relations. If one considers only phase, since it is always the more important of the two parts, amplitude and phase, of the Fourier transform, one realises that there are (n-1) more unknowns than the number of constraints imposed by the phase closure relations. Since there are more unknowns than the number of equations, there exist infinite solutions even in the absence of measurement noise. Under such conditions only parametric solution to the imaging problem is feasible. Parameters determined using algorithms amount to model building. Modifications in the configuration of the array are possible which allow model-independent image reconstruction. This uses the fact that if two aperture pairs have the same vector spacing they represent measurement of the same object parameter, thus reducing by one, the number of unknowns in the problem. In addition to this, specifying an arbitrary value for two of the Fourier transform phases is equivalent to fixing an arbitrary position for the object in the sky. If one is interested in the morphology of the object this is inconsequential. Thus if we drop (n-3) visibilities, by choosing an array with (n-3) redundant spacings, one may expect to obtain a solution without recourse to apriori models. For example, in an array with 18 telescopes, (n-3)/[n(n-1)/2] is 10% of the available spacings. Apart from giving a model independent solution such a redundant spacing calibration is also known to result in a very high dynamic range map.

This still leaves the requirement of computer simulations to assess the quality of images obtained using different aperture arrangements and under different noise conditions. This combined with more structural information is required to determine whether such a configuration is likely to be an effective means of addressing the pertinent astronomical questions.

Design of the Telescopes

The most important criteria for the telescopes are mechanical stability, reliability and pupil rotation. This requirement is met if the beam leaves the telescope and enters the correlator vertically. Polarisation effects are minimized by high reflectivity coatings and by requiring all reflections to be at angle of incidence of 45° or less. Overcoating can reduce polarisation effects though often at the expense of light throughput. It is important to assess the advantages and disadvantages of siderostat configurations, alt-az mount, alt-alt mount for the telescopes in the light of :

1. Number of reflections required for the light to reach a fixed laboratory;
2. Telescope control and drive circuitry requirements to achieve absolute pointing accuracies better than 1". A facility should exist to coalign the images at the beginning of each exposure so that they overlap well enough to allow interference.
3. Ease of telescope transportation to obtain more baselines.

It is important to carry out experiments to determine the differences between free atmosphere and suitably insulated pipes to transfer light from telescope to central station. It is worthwhile investigating the possibilities of using optical fibres for the same.

Path Compensators

Elementary theory tells us that we shall not observe correlation or fringes, if the path lengths from the source to the point where interference takes place differ by more than $C/\Delta\gamma$ where $\Delta\gamma$ is the optical bandwidth. Differential path lengths due to the change in angle between the baseline vector and the direction of the source, has to be maintained below a staggeringly low value of 2 micron for a bandwidth of 10 nm and must also be capable of changing at speeds of 2 mm/sec to keep the loss of the fringe visibiity below 2%. Additional factors which cause differential path lengths may be classified under two categories namely instrumental and non-instrumental.

For example, differential changes in the two arms of the instrument due to thermal expansion, can be classified as instrumental. It may be possible to eliminate them with auxilliary interferometers using laser metreology and by calibrating the instrument frequently on bright stars.

Two sources of path difference which lie outside the instrument can be classified under the two headings, namely, - astrometric and atmospheric. Astrometric path differences correspond to uncertainties in the direction of the star. An inspection of the fundamental star catalogues and of short term variations in refraction suggests that the uncertainty in the absolute direction of a star is never much less than 0.25 seconds of arc. The corresponding uncertainty in the differential path length increases with the base-line and is shown in figure 5 for stars at low angles of elevation.

Atmospheric path differences correspond to fluctuations in the arrival time of the light at the two ends of the baseline due to turbulence in the atmosphere. For long baselines these fluctuations are largely uncorrelated and theory suggests that we may write

$\Delta l_{rms} = 0.416 \times (d/r_0)^{5/6}$

Δl = fluctuating differential path length

d= baseline

r_0 = Atmospheric coherence length

Fig. 5 shows Δl_{rms} vs. d. This suggests that for d = 100 m, Δl = 100 micron and Δl increases to about 1 mm at baselines as long as 1 km. If these pathlengths are left uncorrected, the usable optical bandwidth gets restricted. Fig 5 shows the maximum possible optical bandwidth one can safely use since they cause loss of fringe visibility less than 1%. Thus for a 100 m baseline the usable bandwidth is 2 A. For baselines much greater than 100 m the permissible bandwidth is less than 1 A which is technically unattractive. Thus for longer baselines one has to correct for unknown and varying path differences. For bright stars an obvious solution is to maximise the observed value of fringe visibility by optimising the delay in one arm of the interferometer. But for observing stars near the limiting magnitude one must develop some other technique.

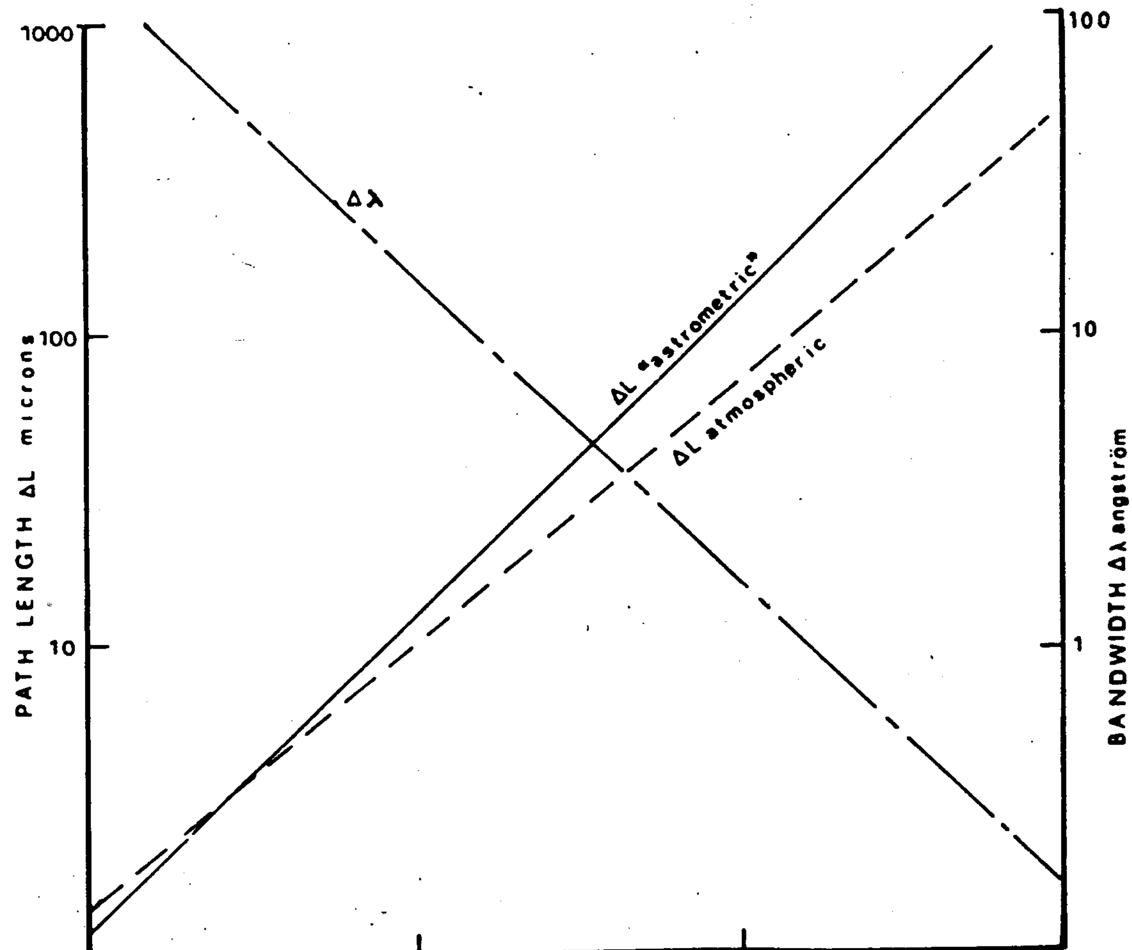

Fig. 5. The differential path length Δl due to uncertainities in the direction of a star (astrometric) and to turbulence in the atmosphere (atmospheric). The corresponding maximum optical bandwidth $\Delta \lambda$ for a loss of fringe visibility of<1%. (Ref. 3).

Aberrations of the Wavefront due to Atmosphere

In general the wavefront of the light at the primary will not be plane nor will it be normal to the direction of the star. It will be tilted and curved and will vary rapidly in both phase and amplitude.

The measured fringe visibility will be reduced by fluctuations in the relative phase of the light reaching the telescopes. An R.M.S. fluctuation in the relative phase during the sampling time of around 10 m sec can cause a loss of 1% in fringe visibility. For wind speeds of a few meters per second and $r_0 = 10$ cm it follows that the sampling time T must be no greater than a few milliseconds. The associated fluctuations in the intensity of the light at the two telescopes can be removed by normalising each elementary observation by the total number of photons received in that interval.

Fluctuations in the mean tilt of the wavefront causes random changes in the apparent direction of the light arriving at the telescopes. Due to these changes the two combining beams do not enter the beam splitter at precisely the correct angles and reduces the correlation so that

Correlation observed = True correlation $[1-1.3(D/r_0)]$

where D = Diameter of individual telescopes

r_0 = Fried's parameter

This effect is shown in Fig 6. Thus the measurements are critically dependent on the atmospheric scintillation. This loss can be corrected to a large extent by an angle tracking system. In such a system one of the primary mirrors is servo-controlled to maintain the two beams of light at the correct relative angle within a fraction of a second of arc. To control the primary mirrors one can take all the light incident on the primary in one polarisation and focus it on to quadrant detectors which provide the necessary error signals. Fig. 6 shows that the loss in fringe visibility can be greatly reduced by angle tracking even though not completely eliminated because the wavefront is not flat.

Further the measured values of fringe visibility can be corrected by software if the causes for the loss can be computed.

For example, the effect of an imperfect angle

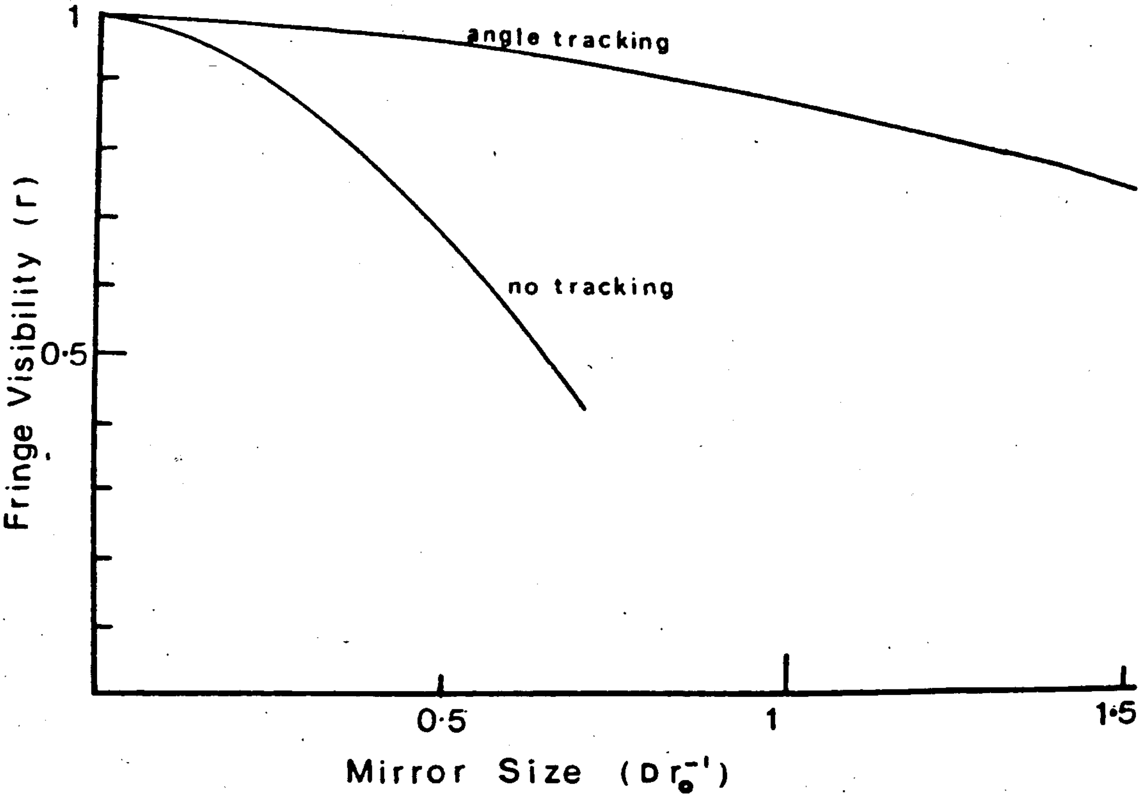

Fig. 6. The loss of fringe visibility due to atmospheric turbulence with and without angle-tracking. For a mirror size D and scintillation size r_0. (Ref. 3).

tracking system can be corrected by measuring the noise power output of the quadrant detector which is proportional to the residual uncompensated fluctuations in the angle of arrival of light. One can calibrate the system for the ratio of the noise power to the loss in fringe visibility. This still leaves the correction for the loss of fringe visibility due to wavefront curvature. Since this loss depends on r_0, one obvious way is to have an instrument which estimates r_0 in real time. Since the wavefront curvature decides the sharpness of the image produced by each telescope, one can also estimate the loss by measuring the image sharpness.

The performance of the angle tracking system and the losses due to the curvature are important factors in the performance of an optical array. Perhaps the current development of active optics may be able to help optical arrays perform better. Another approach would be to use double Fourier interferometry which

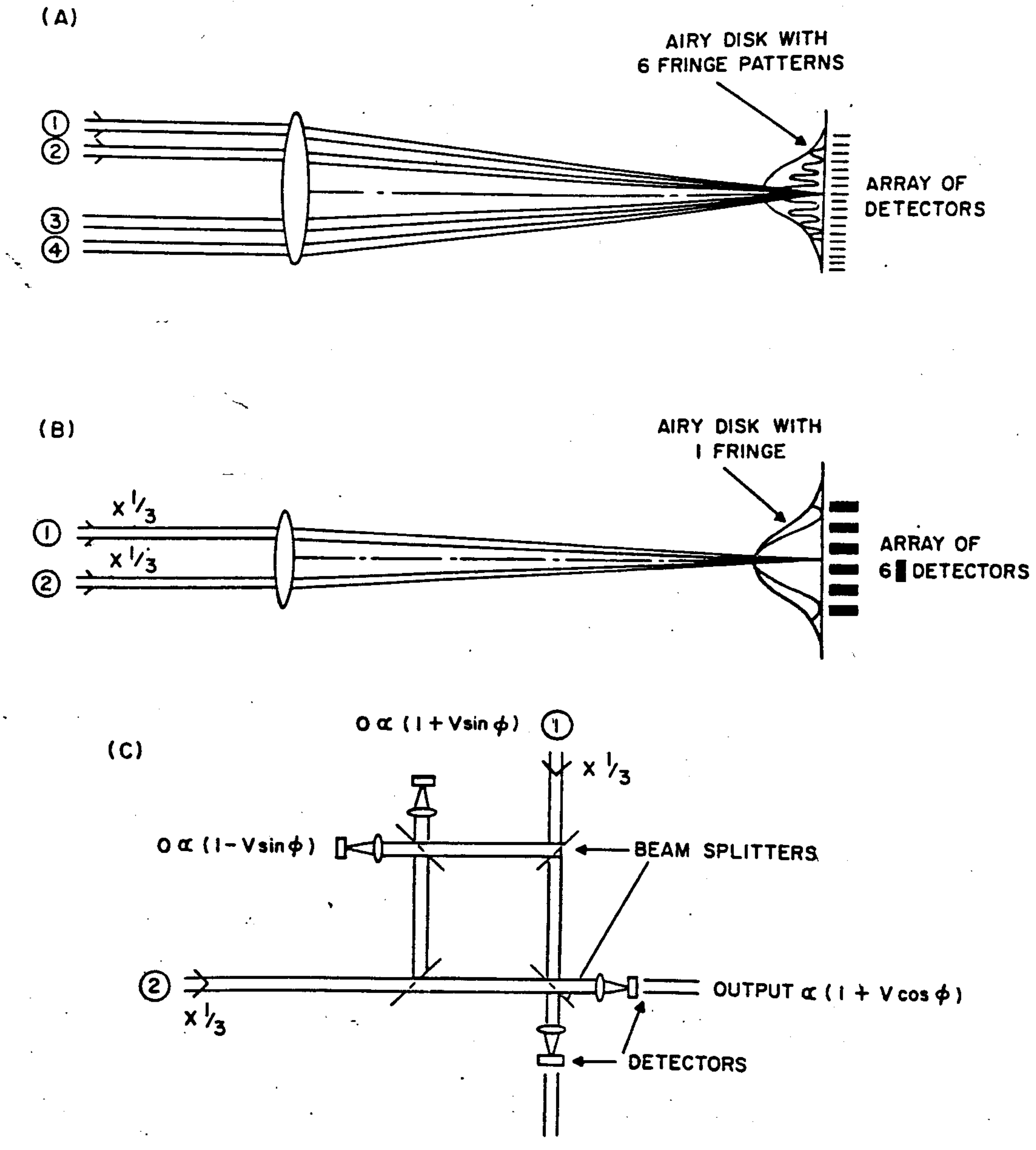

Fig. 7. Beam combining techniques. (Ref. 1)

gives the combination of high spatial and spectral resolution. Apart from providing more detailed and specific information concerning the structure of stellar atmospheres this reduces the limitation introduced by delay tracking errors due to reduced bandwidth. Even though spectral line synthesis is not new to radio astronomy, the availability of holographic gratings reduces the complexity of such a system at optical wavelengths.

Beam Combining

Various methods have been proposed for interfering the beams coming from the telescopes in order to measure the complex degree of coherence (Fig. 7).

The beams from all the telescopes could be brought to a common focus giving an Airy disk crossed by fringes corresponding to the many baselines. The optical arrangement is simple but a disadvantage is that it requires, for each short exposure a Fourier transform of the outputs of the array of detectors to give the amplitudes and phases of the fringes and requires a large number of detectors. An alternative arrangement is to split the beam from each telescope and combine each with a corresponding split beam from one of the other telescopes. The result is a single fringe pattern crossing the Airy disk which can be sampled with a minimum of four to six detectors for each baseline. No requirement for Fourier transforms arises in this case. A third alternative is to recombine the beams for each baseline with different phases using beamsplitters. At low light levels the combination of all the beams into one pattern is better and it can be similarly shown that the same holds true for the determination of closure phases at low light levels. But a redundant array poses a different kind of problem while combining all the beams together. Interferometric pairs which contribute to the same spatial frequency but have uncorrelated atmospheric phase errors don't add coherently and increases the noise in the system. Further the information from the redundant aperture pairs is inextricably mixed together and provides only one measurement. To circumvent this difficulty, proposals have been made to use specially designed interferometers that re-map the information and permit one to unscramble the interference between aperture pairs with the same spacing. Switching techniques used in radio astronomy, especially Walsh function switching may play a very prominent role in these types of applications.

Detectors

Photon detectors are commonly used for sensing the optical radiation. Here the individual photons comprising the optical flux interact directly with the electrons in the detector. This direct interaction provides a very fast detector making these detectors suitable even for short integration times, but the effect is also wavelength dependent. Photon detectors in common use are the photomultipliers, Avalanche photo diodes, Micro channel plates, etc. The important parameters to evaluate the suitability of a detector system to a given application are spectral response, speed of response and sensitivity. At very low light levels the ultimate limit to the accuracy of image recording is set by Poissonian statistics of the incident photons. Practical detectors invariably contribute additional noise principally because a fraction of the photons goes completely undetected, uneven gain processes accord detected photons unequal statistical weighting. Read-out noise, amplifier noise and the addition of background noise further reduce the sensitivity. A clear trend in the use of detectors in astronomy in recent years has been the increased application of solid state imagers such as C.C.D.'s and the associated electronic data processing and computer techniques, many of them applied on line at the telescope.

Data Processing

From the integrated values of amplitude and closure phase it will be possible to produce and display images. One approach is to use the usual radio astronomical map making algorithms such as self-cal. They involve first interpolation of observed visibilities on to a rectangular grid and then iterative estimation of phases which must be consistent with the closure phases and a non-negative sky.

Another approach which also uses closure phase in an indirect way is the technique of speckle masking or bispectrum analysis developed in optical interferometry. The bispectrum function is a third moment of the complex visibility of the observed source distribution. Specifically, triple products of all visibility elements which can be mapped on to a triangle of discrete interferometer elements are included in the bispectrum. Computational aspects of bispectral analysis in interferometric imaging and its SNR analysis, are getting a great deal of attention these days.

To conclude, the main challenges in the installation of a multi element synthesis array are :

1. Design of massive supports for mechanical and vibrational stability and very careful design of moving components essential to ensure accurate fringe visibility measurements.
2. Accurate path length compensation to measure fringe visibilities without any loss. For example, for a bandwidth of 100 A path length equality has to be better than 10 micron.
3. The optical beam alignment of each mirror should be extremely good. (better than 0.2"). This has to be achieved by active tilting of mirrors controlled by image motion sensors.
4. There are challenging aspects of optics required to minimize the light loss and retain polarisation before being combined in an optimal way.
5. Detectors with very high quantum efficiency and fast read-out time are needed. The read-out times should be short enough to freeze the atmospheric turbulence.

Scientific Goals

There is very little information so far available on the structure of objects in the optical band of wavelengths on angular scales much less than one arc second. The areas waiting to be explored are asteroids and small solar system bodies, close binaries, stellar diameters and surfaces, circumstellar matter, and nuclei of globular clusters and galaxies (to name only a few). It should be mentioned that even the relatively bright objects to which these techniques are applicable have not been systematically and extensively studied so there is much astrophysically significant information to be obtained by these high resolution techniques in the coming decade. Few more applications of high angular resolution are :

1. Measurement of surface fluxes and effective temperatures of single stars. This can be determined directly only if the angular size is measured.
2. Binary Stars :— Under favourable conditions, it is possible to determine all the physical parameters of a binary system. (e.g., inclination of orbit, distance, masses.)
3. Variable Stars :— Study of Cepheid variables can lead to their distance determination and hence can contribute to the foundations of the cosmological distance scale.
4. Stellar atmosphere studies will provide data on mass loss from stars, a key factor in theories of stellar evolution.

As mentioned earlier, with little information available on these, it is important to initiate activity in this area at an early stage.

Much of the article is based on the following references.

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General References

A recent article — "Interferometric imaging in Optical Astronomy" by F. Roddier published in Physics Reports Vol 170, No. 2, Nov 1988, gives a very exhaustive list of references on the subject of optical interferometry.