Lecture III

Coronae, haloes and glories

IN the present lecture, we shall consider the phenomena which arise from the diffraction of light simultaneously by a great many particles or obstacles, the size of these being sufficiently large to permit of an elementary approach to their explanation. Many such phenomena are known, and it is of advantage to consider them together in a general survey, so that the common principles underlying all such cases may be brought into relief. The optical character of the particles, their size, shape and number, the manner in which they are disposed and orientated in space, and the particular circumstances of observation may all influence the results. Included within the survey are some natural phenomena which may be observed in the earth's atmosphere when particles of water or ice are present in it and are suitably illuminated by the rays of the sun or the moon.

Diffraction by a cloud of particles: Secondary radiations derived from the same primary source, and therefore having specifiable phase-relations with it and with each other, would evidently be capable of interference. Hence, when a cloud of particles is present in a light-field and the radiations diffracted by the individual particles are superposed at any given point of observation, interferences would arise. Their character would be determined by the phase-differences, in other words, by the optical paths traversed from the original source to the individual diffracting particles and thence to the point of observation. Considering first a case in which the line joining the primary light source with the point of observation passes *through* a cloud of particles, it is evident that the optical paths would differ infinitely little for all particles lying on this line or in its immediate vicinity. On the other hand, the optical path would alter in a rapidly increasing measure with the actual position of the particle as it lies further and further away from this line. Thus, in general, *except along the direction of propagation of the* *light rays from the original source*, the distribution of the individual diffracting particles in space is a controlling factor in determining the optical effect produced by a cloud of such particles.

Considering the effects produced by the cloud in any direction other than that of the primary rays, we shall assume that the particles are distributed at random and execute *rapid uncorrelated movements* within the cloud. It is obvious that in such circumstances, the interferences between the effects of the individual particles would be unobservable. We may then assume the observed intensity in the field to be a summation of the intensities of the individual effects. If all the n particles were similar and produced similar effects at any point of the field, the total observed intensity would be n times the effect of an individual particle. On the other hand, *if the particles occupied stationary positions* within the cloud, the situation would be entirely different. However numerous the particles might be, and howsoever they might be distributed within the cloud, the phase-relations between them would be determinate, and hence the interferences between the individual effects should be observable. We have to evaluate the result of such interferences to find the optical effect due to the entire cloud of particles.

The problem which thus arises of finding the effect of n superposed radiations of equal amplitude but of differing phase may be dealt with graphically by means of a two-dimensional diagram. Choosing a given point O as origin, we draw a set of n *radii vectores* of equal length A representing the amplitudes of the n superposed radiations; their relative phases would be given by the angles which these make with each other. It is evident that the resultant obtained by the summation of the vectors so drawn would depend on the manner in which the terminal points of the radii are distributed around the circle on which they lie. If, for example, the phases are all identical, the vectors would all be superposed, and the resultant amplitude would be nA and the resultant intensity n^2A^2. If, on the other hand, the n vectors divide the circle into n equal arcs, the resultant amplitude and intensity would both be zero. If now, we consider the case in which the n phases are distributed at random, it is obviously impossible to specify what either the amplitude or the phase of the resultant would be *in any particular trial*. The diagram, however, gives some indications of a general character regarding what we may expect to find. If the number n be sufficiently large, the most probable location of the points on the circle in a random distribution would evidently be a sensibly uniform one. Hence, the most probable resultant intensity would be the same as for a perfectly regular distribution, namely, zero. It is evident also that the resultant intensity averaged over a large number of trials would be nA^2. This follows immediately, if we suppose that the phases vary continuously and rapidly with time, so that the n intensities, each of which is A^2, become additive.

Thus, *for a random distribution of phases, the most probable resultant intensity is zero, while the average intensity in a large number of trials would be* nA^2. For a more complete description of the case, we have to find an expression for the probability that the intensity has a specified value I in different trials. It is easily verified that this is given by the exponential probability formula

$$dW = \exp(-I/nA^2)\cdot d(I/nA^2).$$

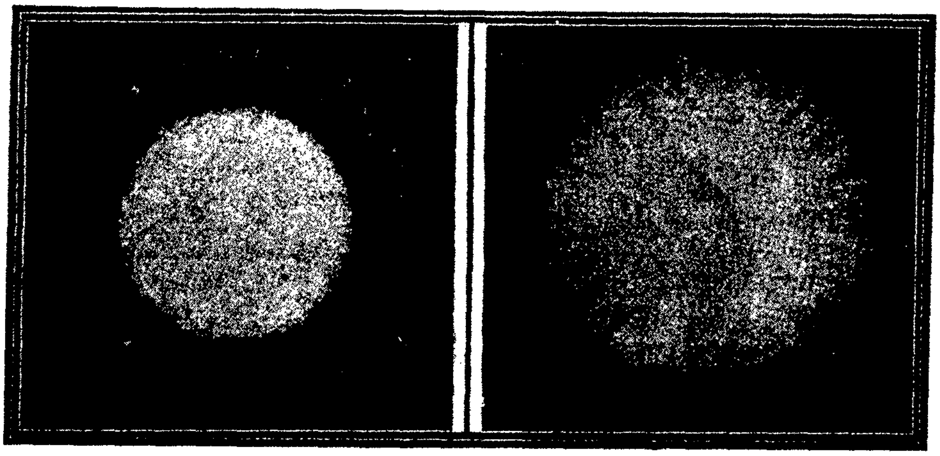
The formula agrees with what the graphical treatment suggests; it shows that the probability is a maximum for zero intensity and that it diminishes continuously and ultimately vanishes with increasing values of the intensity. Further, the integration of dW over all possible values of I gives unity as it should, and the average intensity found by integrating $I\,dW$ over all possible values of I is nA^2, as already found. Hence, the formula $dW = \exp(-f)df$ correctly gives the probability of any given value of the observed intensity expressed as a fraction f of the average intensity. We note also that the formula agrees with that given by a detailed consideration of the problem on the basis of the general theory of probability.* It is important to notice that the chance of finding any particular resultant intensity decreases continuously as it increases, and that the average intensity is very far indeed from being the most probable intensity. Indeed, the average is determined entirely by the cases in which the resultant intensity is greater than the most probable value which is zero.

A point source of monochromatic light viewed through a cloud of particles would appear surrounded by a corona or halo due to diffraction by the particles. The radiations diffracted by the particles and reaching the retina of the eye and focussed thereon are superposed and would thus be capable of interfering with each other. The foregoing discussion shows that if the particles are all similar and are disposed at random in space, the intensity in the corona would only *statistically* be a summation of the intensities of the diffraction patterns produced by the individual particles. While the general features of the pattern due to each separate particle would be recognisable in the aggregate effect, the latter is essentially different in detail. Instead of a continuous distribution of intensity, we have a violently fluctuating one which, in general terms, may be described as a dark field on which appear a great many points of illumination irregularly distributed and of varying brightness. The illumination at such points arises from the accidental agreements of phase of the effects of the diffracting particles, while the dark field results from the general cancellation of their effects by mutual interference. *Each such point in the corona exhibiting an observable intensity is, therefore, essentially an optical image of the original source produced by the entire*

*Rayleigh, I, *Philos. Mag.*, 1880, **10**, 73, *Scientific Papers*, **1**, p. 491.

*cloud of particles functioning as a randomly distributed set of secondary sources of light.**

As we shall show later, the theoretical conclusions set out above are fully supported by the experimental results (see figure 66a). It is important to remark that, in practice, cases may also arise in which the diffracting particles are not distributed at random in space. The distribution may either present a closer approach to uniformity, or may tend in the opposite direction, the individual particles clustering together to form large groups. The optical effects would in either case differ from those observed with a random spacing of the particles. In the limiting case of a perfectly uniform distribution, the particles would in effect constitute a diffraction grating. We would then get sharply-defined and intense diffraction spectra located at regular intervals in a dark field. The transitional cases, where the distribution of the particles in space is neither completely random nor completely uniform, are of particular interest. The phenomena observed in all such cases may be included under the general descriptive term of "diffraction haloes", the expression "corona" being reserved for the case of randomly spaced particles. As examples of such haloes, we may turn to figure 23 on page 432 of the second lecture, in which the effect of viewing a source of light through a thin piece of mother-of-pearl was illustrated. As was remarked on



(a) (b)

Figure 66. Diffraction corona due to lycopodium spores showing (a) granular structure in monochromatic light, and (b) radial streaks in white light.

*G N Ramachandran, *Proc. Indian Acad. Sci.*, 1943, **A18**, 190.

page 432, the size and shape of the crystallites of aragonite, and their spacing and orientation within the mother-of-pearl determine the character of these haloes, and as will be evident from the three examples reproduced, these features and the resulting haloes are very different in the three great classes of mollusca. We shall meet with other cases of the production of diffraction haloes later in the present lecture.

Coronae due to water droplets: The well known coronae or disks of light with marginal coloured rings seen surrounding the sun or the moon when viewed through thin clouds are amongst the most familiar phenomena of meteorological optics. What we see in such cases is evidently the cloud itself which becomes visible by reason of the light incident on it and diffracted through various angles by the particles of which it is composed. The optical character of the phenomena, as well as the form and level of the clouds exhibiting them, make it clear that the coronae with vividly coloured rings owe their origin to minute spherical droplets of water contained in the clouds. Thin clouds consisting of small particles of crystalline ice do exhibit observable disks surrounding the sun or moon when seen through them. But these are usually of smaller size and have a quite different and characteristic distribution of intensity. They are also much less vividly coloured than the coronae arising from water clouds. Indeed, a cirrus haze can just as readily be recognised by the diffuse illumination observed in the vicinity of the sun or the moon as by the familiar halo due to refraction by the ice-crystals seen at an angle of 22° from the luminary. It should be remarked also that clouds do sometimes display marked iridescence in circumstances which indicate that their temperature must be well below the freezing point of water. Such iridescence is often observed at quite large angles with the sun, though, of course, a complete corona is not usually then seen. Whether such iridescent clouds consist of crystalline particles of ice is a debatable question. The vividness of the colours suggests that the particles are probably supercooled droplets of water, or possibly even an amorphous form of solidified water. The retention of an amorphous structure and of non-crystalline shape by droplets of water when supercooled is a well-established fact of observation under laboratory conditions, and it is permissible, therefore, to suppose that it can also occur in nature.

Coronae can also be artificially produced and observed in the laboratory over a wide range of droplet size, and they are actually more striking than the coronae seen in nature, the colours of which are somewhat diluted by the finite angular dimensions of the sun or the moon. As is well known, a sudden expansion of moist air, if of sufficient magnitude, results in the formation of a cloud consisting of minute droplets of water. The condensation usually occurs around "nuclei" of some sort, and the number of droplets formed and therefore also their size depends on the number of such nuclei present. The size of the droplets as indicated by their rate of free fall, as also by the optical effects which we shall presently consider exhibits a remarkable uniformity. It may be regulated within wide limits by varying the amount of the expansion and the number of nuclei present. Very beautiful and interesting effects are observed when such clouds are viewed under strong illumination, or if a bright source of light is seen through such a cloud. By using an electric arc as the source of light and projecting an image of it as seen through the cloud chamber on the screen, the coronae due to water droplets can be shown as a beautiful lecture demonstration. Using a red glass as a monochromatising filter, or with a quartz mercury lamp and single ray filter, the coronae may be readily photographed.

The particles of water in a cloud are very numerous and yet not so numerous as to occupy an appreciable fraction of the volume of the air. They are obviously distributed at random in the space, and presumably execute small irregular movements. It follows that though the droplets are all illuminated by the same original source of light, we may nevertheless regard them as practically independent sources of diffracted radiation. The justification for this is that the phases of the diffracted radiations in any assigned direction from the different droplets are totally unrelated. An exception must, however, be made in considering the rays diffracted in the same direction as the rays incident on the drops; for, in this direction the optical paths for all the drops are identical, and hence their amplitudes must be added to find their resultant effect. In all other directions, we may add the intensities of the diffracted radiations from the drops and expect the results to be in accord with the facts.

The appearance of the coronae in the experiments is found to vary in a remarkable manner with the size of the droplets. The central disk of the corona as seen with the finest droplets is not white but shows vivid colour varying with their size; as the drop size is altered progressively, there is a recognisable cycle of changes in the colour observed. The sequence of changes observed with increasing size of the droplets is not a mere progressive diminution in the angular diameter of the corona as seen in monochromatic light. A periodic alteration in the diameter and intensity of the coronal disk is noticeable, while from the published photographs* it is evident that the relative intensities and positions of the outer rings vary notably when the drop size is altered.

The experimental facts thus compel us to reject the usual explanation of coronae which is based on the assumption that the droplets may be regarded as opaque spheres. The starting point for a more satisfactory theory is a consideration of the phase-changes resulting from the passage of plane waves through a transparent sphere of liquid. In the limiting case when the refractive index of the liquid μ is only slightly greater than unity, the waves pass through the sphere without any change of amplitude but with a change of phase $\xi \cos \varepsilon$ where ξ is $4\pi a(\mu - 1)/\lambda$; a is the radius of the sphere and ε is half the supplement of the

*M N Mitra, *Indian J. Phys.*, 1928, **3**, 175. The photographs reproduced in figure 67 are due to Mr H Ramachandran.

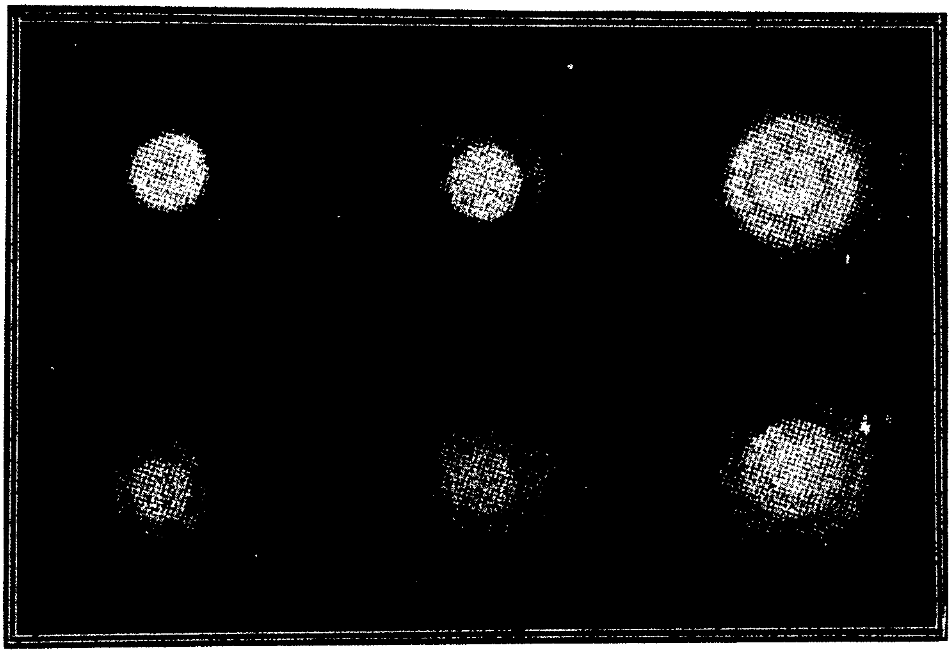


Figure 67. Coronae due to water droplets of different sizes.

angle subtended at the centre by the path inside the sphere, being zero for a ray passing centrally and $\pi/2$ for a marginal ray grazing the surface. The wave-front on emergence would thus exhibit *a dimple* having the same radius as the drop and a depth equal to the maximum retardation it produces. If $(\mu - 1)$ be not small, this simple picture would not be accurate, as the wave-front on emergence from the drop would exhibit both amplitude and phase changes. We may, however, adopt it as the basis for an approximate theory which, though it could scarcely be expected to give a complete account of the facts, should nevertheless go far towards doing so.

If the dimples in the wave-front be removed, and the resulting holes filled up, the diffracted radiations would disappear. It follows that the effect of a drop may be found by *subtracting* from the optical effect of the dimple in the wave-front, the effect produced by plane waves of light passing through a circular aperture of the same radius in an opaque screen. The relation between the amplitude and phases of the two effects which are thus superposed determines the observed phenomena, and it is evident that the interference between them is responsible for the observed cycle of changes in the appearance of the corona with increasing drop size.

The detailed calculations are made on much the same lines as for a simple circular aperture. Besides the phase-change $\xi \cos \varepsilon$, we have also to consider the phase-difference between the different parts of the wave-front introduced by the observation of their resultant at a great distance d and at an angle β with the incident rays. This may be written as $\eta \sin\varepsilon \cos\alpha$, where ε is the angle already introduced, and α is the azimuthal angle defining the position of an element of area, viz., $a^2 \sin\varepsilon \cos\varepsilon \, d\varepsilon \, d\alpha$, in the wave-front emerging from the drop. η stands for $2\pi a \sin\beta/\lambda$. The disturbance in the direction β due to the light which has traversed the drop is given by the integral

$$\int_0^{\pi/2}\int_0^{2\pi} \frac{a^2}{\lambda d} \sin(Z - \xi\cos\varepsilon + \eta\sin\varepsilon\cos\alpha)\sin\varepsilon\cos\varepsilon \, d\varepsilon \, d\alpha.$$

On integration with respect to α, this yields

$$\frac{2\pi a^2}{\lambda d}\int_0^{\pi/2} J_0(\eta\sin\varepsilon)\sin(Z - \xi\cos\varepsilon)\cos\varepsilon\sin\varepsilon \, d\varepsilon.$$

If we put $\mu = 1$, ξ vanishes, and the integral reduces, as it should, to the effect of a simple circular aperture of radius a, namely,

$$\frac{2\pi a^2}{\lambda d}\sin Z \cdot \frac{J_1(\eta)}{\eta},$$

and, as already remarked, the contribution of the drop to the corona is found by deducting this from the foregoing integral.

In the exact forward direction, β is zero and η vanishes. The foregoing integral can then be completely evaluated, and after the deduction indicated is made, it gives for the amplitude the expression*

$$\frac{2\pi a^2}{\lambda d}\sin Z\left(\frac{\cos\xi}{\xi^2} + \frac{\sin\xi}{\xi} - \frac{1}{2} - \frac{1}{\xi^2}\right) + \frac{2\pi a^2}{\lambda d}\cos Z\left(\frac{\cos\xi}{\xi} - \frac{\sin\xi}{\xi^2}\right).$$

The intensity of forward scattering is thus

$$I_f = \frac{4\pi^2 a^4}{\lambda^2 d^2} F(\xi),$$

where

$$F(\xi) = \left[\left(\frac{\cos\xi}{\xi^3} + \frac{\sin\xi}{\xi} - \frac{1}{2} - \frac{1}{\xi^2}\right)^2 + \left(\frac{\cos\xi}{\xi} - \frac{\sin\xi}{\xi^2}\right)^2\right].$$

The discussion hereafter follows, in the main, two papers by G N Ramachandran.† $F(\xi)$ is plotted against ξ in figure 68 exhibiting the manner in which the forward intensity varies with the size of the droplets. The curve starts from the origin, increases in direct proportion to ξ^2, reaches a maximum and then oscillates, finally tending to a value 1/4. For very small particles, the intensity

*T A S Balakrishnan, *Proc. Indian Acad. Sci.*, 1941, A13, 188.

†G N Ramachandran, *Proc. Indian Acad. Sci.*, 1943, A17, 171 and 202.

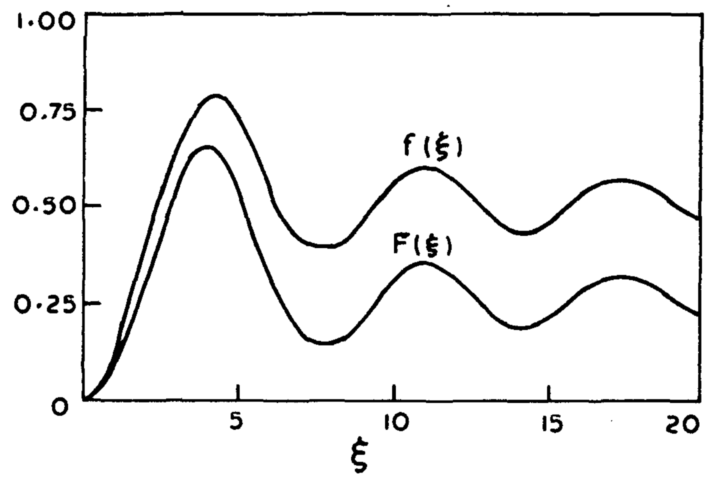


Figure 68. Graph showing the variation of $F(\xi)$ and $f(\xi)$ with ξ.

reduces to the expression

$$\frac{64\pi^4}{9d^2}(\mu - 1)^2\frac{a^6}{\lambda^4}.$$

This is identical with Rayleigh's well-known formula for the blue of the sky, except that in our formula we have a factor $4(\mu - 1)^2$ instead of $(\mu^2 - 1)^2$ to which it is nearly equal if μ does not differ much from unity as is assumed in the theory. Thus, in the initial stages, for very small droplets, the theory predicts a preferential scattering of the smallest wavelengths, which is a readily observable phenomenon. For larger particles, the intensity reaches a maximum and then oscillates. Over this range, we may neglect terms in $F(\xi)$ involving higher powers of $1/\xi$ than the first and write

$$I_f = (4\pi^2 a^4/\lambda^2 d^2)(1/4 - \sin\xi/\xi).$$

It is evident from this that the light would show a cyclic change of colours with increasing particle size. Finally, for very large particles, the expression becomes equal to $(\pi^2 a^4/\lambda^2 d^2)$, agreeing with the intensity at the centre of the diffraction pattern due to a circular aperture of the same radius.

It is evident also that the superposition upon the primary waves of the radiations scattered forward by the particles in a thin cloud of thickness dl must result in the alteration of both the amplitude and the phase of the latter in its passage through the layer. The former would depend upon the coefficient of $\sin z$ in the expression for the forward scattering and the latter on the coefficient of $\cos z$. Writing the diminution of the amplitude due to the particles in the layer as

kdl, and the retardation as $(n-1)dl$, we deduce that

$$k = 2\pi Na^2\left(\frac{1}{2}+\frac{1}{\xi^2}-\frac{\sin\xi}{\xi}-\frac{\cos\xi}{\xi^2}\right) = 2\pi Na^2 f(\xi) \quad \text{(say)}$$

$$n-1 = N\lambda a^2\left(\frac{\sin\xi}{\xi^2}-\frac{\cos\xi}{\xi}\right),$$

where N is the number of particles per unit volume.

The intensity of the incident beam falls off in its passage through the cloud, and after passing through a length l it may be represented by

$$I_l = I_0 \exp(-2kl).$$

I_0 being the intensity of the incident beam. The attenuation coefficient $2k$ is thus proportional to $f(\xi)$. The course of this function with increasing ξ is also plotted in figure 66, and is seen to be similar to that of the forward intensity, giving rise to periodic changes in the colour of the transmitted beam also. When ξ is not small, higher powers of $1/\xi$ may be neglected, and the attenuation coefficient becomes $4\pi Na^2 l(1/2 - \sin\xi/\xi)$, finally tending to a value $2\pi Na^2$ which is the same as if the particles were opaque. *Also, the value of the attenuation coefficient is double what would be given by simple geometric considerations, and this may be explained as due to diffraction which introduces an extra loss of energy.*

Since the attenuation coefficient and the intensity of forward scattering undergo similar variations as ξ increases, it is clear that the colour of the source as seen through the cloud must be complementary to that of the light scattered forward as can be observed, for instance, when the sun is seen through the puffs of steam emitted by a locomotive. However, with thick and long columns of cloud, the scattered light itself will be attenuated, and the phenomena thereby modified.

The refractive index of the cloud also undergoes oscillations, alternately becoming greater and less than unity, as the size of the particle steadily increases. In the limiting case of large particles, it becomes practically unity. This is readily explained by the fact that large droplets transmit little light, and such opaque particles can produce change in refractive index.

So far, we have been considering only the light coming out in the forward direction. When we turn to the diffraction in other directions, it is found that the integral for the amplitude cannot be evaluated completely; but it can be expressed in the form of a series. The method to be adopted depends upon whether ξ is small or large. In the case where coronas are observed, ξ is sufficiently large, and the evaluation may be done by writing $x = \xi\cos\varepsilon$, and by repeatedly integrating by parts with respect to x. We then obtain a series, which, on omitting terms containing higher powers of $1/\xi$ than the first, reduces to

$$\frac{2\pi a^2}{\lambda d}\left[\sin Z\left(\frac{\sin\xi}{\xi}-\frac{J_1(\eta)}{\eta}\right)+\cos Z\frac{\cos\xi}{\xi}\right].$$

The contribution of the drop to the intensity is therefore

$$\frac{4\pi^2 a^4}{\lambda^2 d^2}\left[\frac{J_1^2(\eta)}{\eta^2}-\frac{2\sin\xi}{\xi}\cdot\frac{J_1(\eta)}{\eta}+\frac{1}{\xi^2}\right].$$

The intensity depends both on ξ and η, and hence on the size of the droplets and on the angle of diffraction. Since the function $\sin\xi/\xi$ oscillates and diminishes progressively as ξ increases, the intensity of the corona would fluctuate as a/λ alters, the fluctuations diminishing in extent as the size of the droplets increases. The coronal disc would, in consequence, exhibit colours which are most vivid with the smallest drops, and go through cycles with their saturation progressively diminishing as the drops become larger. The ratio of $\sin\xi/\xi$ to $J_1(\eta)/\eta$ increases rapidly as we move away from the centre of the corona. Hence, the colours would be more prominent towards the margin of the central disc than at its centre. The presence of the function $J_1(\eta)/\eta$ gives rise to alternate bright and dark rings, which can be observed in monochromatic light, but the positions of these rings would be greatly influenced by the value of $\sin\xi/\xi$ and the whole appearance of the corona would be different from that of the diffraction pattern of an opaque circular disc. The cyclic changes in this function $\sin\xi/\xi$ also give rise to an alternate contraction and expansion of the ring system. In the limit when ξ is sufficiently large, the corona, or at least the central part of it for which η is not too great, tends to become similar to that given by a set of opaque spheres.

Colours of mixed plates: Very beautiful phenomena are shown by the heterogeneous films known as "mixed plates". Though they differ essentially from the coronae due to water-droplets discussed in the foregoing pages in their nature and origin, there are, nevertheless, some features common to the two cases which justify their being considered in this lecture: To obtain the "mixed plates", a few drops of egg albumen are spread between two plates of glass about ten centimetres square in size and a centimetre thick. The plates are then separated and put back together a few times and slid over each other with a circular movement. The material is thus worked up into a film of uniform thickness which, when seen under the microscope, appears as a thin layer of liquid enclosing a large number of air-bubbles. These vary in size and are irregularly arranged and often depart considerably from a circular shape, but except in special circumstances, show no bias towards elongation in any particular direction. Gorgeous colours are shown by such films when they are freshly prepared and are not too thick. On being allowed to stand, the albumen in the film begins to dry up and forms hexagonal networks between the two plates. The character of the optical phenomena then completely alters.

The colours of mixed plates may be studied in two distinct ways which are roughly analogous to the Haidinger and Newtonian methods of viewing the interferences of transparent plates. The first method is to prepare a film of uniform thickness between flat plates and to view the source of light through the films with the eye placed close behind it and adjusted for distant vision. The second method is to form a mixed plate between two glass lenses in the manner of Newton's rings and to view the illuminated film with the eye placed behind the plate at a suitable distance. As the effects alter with the angles of incidence and observation, the source of light should in either case be of small angular area. An aperture in a screen backed by a filament lamp or a mercury arc may be employed and a dark field of observation should be provided around it. In the first method of observation, the eye observes the light diffracted by the film simultaneously over a wide range of angles. In the second method, films of different thickness are seen simultaneously at nearly the same angle of diffraction; this angle, of course, may be varied by moving the eye laterally. In either case, the angle of incidence of the light on the film may be varied by tilting the plate with reference to the direction of the source.

The characteristic effect* of which the explanation largely covers the whole theory of mixed plates is the diffraction halo seen around a bright source of light viewed normally through the film (see figure 69). This halo consists, in monochromatic light, of a series of circular rings, alternately bright and dark, which concentrically surround the source. The rings are narrowest nearest the centre of the halo and widen as we proceed towards its outer margin. If white light is used, the outermost ring is practically achromatic and is followed within by coloured rings. A thick plate shows numerous close rings, while a thin plate shows fewer rings which are wide apart. The rings move inwards when the thickness of the film

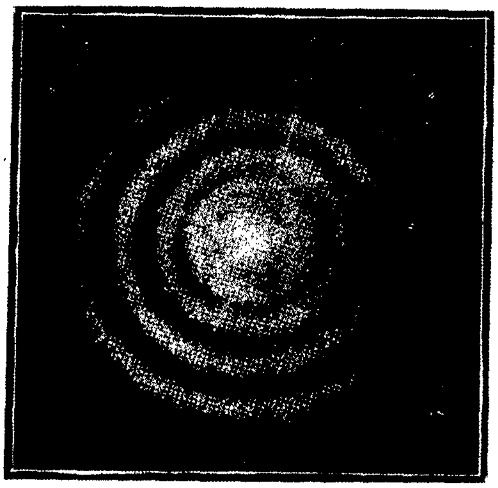


Figure 69. Diffraction halo of mixed plates.

*C V Raman and B Banerji, *Philos. Mag.*, 1921, **41**, 338.

is reduced. Thus, the thinner the plate, the more striking are the colours shown by the rings nearest the centre of the halo. A source of white light viewed through the film appears dimmed in intensity and exhibits a hue complementary to the colour of the part of the halo actually overlying it. A monochromatic light source fluctuates in intensity when the thickness of the films through which it is viewed is altered, appearing brightest when the halo has a dark ring at the centre, and feeblest when the source is overlaid by a bright ring. The rings near the centre of the halo show peculiar variations in their visibility depending on the thickness of the plate through which the source is viewed, sometimes being scarcely observable and sometimes very vivid and clear. Such fluctuations are not shown by the outer rings in the halo. The observations indicate that there is a second ring-system of small angular extension superimposed upon the main system and affecting its visibility when the two sets of rings are not in coincidence in any particular direction.

It is clear from the facts already stated, that the character of the halo is determined by the thickness of the liquid-air film and not by the size or shape of the air bubbles in it. It is also evident that the halo registers the characteristics of the diffracted radiation from the laminar edges in the film.* Each line element of the edge diffracts light principally in a plane normal to its own direction; the part which proceeds towards the air-side of the boundary may be referred to as exterior diffraction, and the parts towards the liquid side as interior diffraction. The existence of both types of diffraction in equal intensity but with opposite phases at small angles with the incident beam is shown by the Foucalt test. As in the case of the striae in mica, the laminar boundaries in mixed plates appear as brilliantly coloured *double lines* when the light is blocked out at the focus, the colour being complementary to that of the central fringe in the Fresnel diffraction patterns (figure 70).

Though the interior and exterior diffractions by the laminar boundary are symmetric *at small angles*, they cease to be so at larger angles. The interior diffraction is much more intense and is visible over a wide range of angles, whereas the exterior diffraction rapidly diminishes in intensity and vanishes when the angle of diffraction exceeds a few degrees. This is readily seen on illuminating the film and viewing it obliquely through a microscope. The two halves of the edge of each bubble appear of very different intensities and indeed one half very quickly vanishes, while the other half remains visible but shortens into a crescent as the obliquity is increased.† The reason for these facts is obvious when we consider the form of the laminar boundaries which owing to the action of surface tension have a specific shape independent of the size of the bubble, namely, a

*C V Raman and B Banerji, *Philos. Mag.*, 1921, **41**, 860.

†I R Rao, *Indian J. Phys.*, 1927, **2**, 167. Some photographs illustrating the effect are reproduced with this paper.

Figure 70. Mixed plates in the Foucalt test.

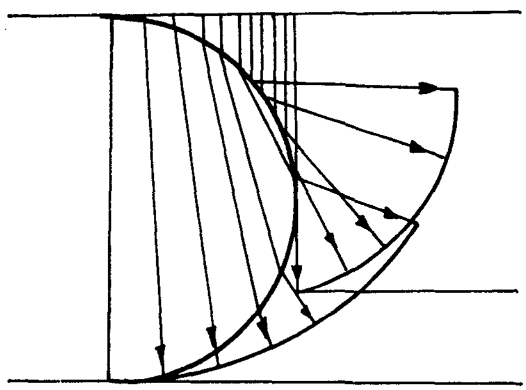


Figure 71. Form of wave-front in mixed plates.

semi-circular arc whose diameter is equal to the thickness of the film. The manner in which the wave-front of the light is modified in its passage through the film is indicated in figure 71. The wave incident on the curved liquid-air boundary is in part twice refracted and in part totally reflected at the interface. The twice-refracted part forms a curved continuation of the wave-front which has passed through the air, while the totally reflected part forms a cusp-like appendage to the wave-front which has passed through the liquid. The appearance of the diffracted radiation towards the interior is thus strongly favoured, while towards the exterior it is greatly weakened. The explanation of the asymmetry of diffraction indicated by figure 71 is completely confirmed by viewing the edge under sufficiently high powers of the microscope. The emergence of the refracted and reflected rays from distinct points on the meniscus and their approach to each other with increasing obliquity can actually be observed when the films are fairly thick. The disappearance of the exterior diffraction at larger angles is found to occur more rapidly with films of greater thickness.

It is thus evident that the interior and exterior diffractions by the laminar boundaries appear superposed in the halo. The two are of the same intensity at small angles of diffraction, but at larger angles the interior diffraction is much more intense and principally determines the observed phenomena. In either case, the edge radiations are derived simultaneously from the two parts of the wave-front which have passed through the film. If we ignore the light transmitted or reflected by the liquid by the liquid meniscus, the edge radiations from the wave-fronts which emerge from liquid and air respectively would interfere under a path difference

$$(\mu - 1)d - \tfrac{1}{2}d\cdot\sin\theta - \tfrac{1}{2}\lambda$$

for interior diffraction, and under a path difference

$$(\mu - 1)d + \tfrac{1}{2}d\sin\theta + \tfrac{1}{2}\lambda$$

for exterior diffraction, θ being the angle of diffraction, μ the refractive index of the liquid and d is the diameter of the meniscus, which is also the thickness of the film. When θ is sufficiently small, $\frac{1}{2}d\sin\theta$ may be neglected and the expressions show that the colour of the diffracted light would be complementary to the interference colour of the light transmitted through the film. For larger angles of diffraction, the path difference increases for exterior diffraction and diminishes for interior diffraction. But at such angles, the effects due to the meniscus become of great importance in interior diffraction. Here the case may be treated as practically one of interference between the rays which are totally reflected and those twice refracted at the meniscus. Their path difference is easily shown to be

$$d(1 - \mu\cdot\sin i)(\mu\cos i - \sqrt{1 - \mu^2\sin^2 i}) - \delta,$$

where i is the angle of incidence at the meniscus of the light which is twice refracted, and δ is the correction for the change of phase in total reflection. The angle of diffraction θ of the light emerging from the film is given by the formula

$$\sin\theta = \mu\sin 2(r - i), \quad \text{where } \sin i = \mu\sin r.$$

For small values of i and θ, it is readily shown that the path difference given by this formula is sensibly the same as in the one given above, namely,

$$\alpha(\mu - 1) - \tfrac{1}{2}d\sin\theta - \tfrac{1}{2}\lambda.$$

For larger values of i and θ, the path difference falls off more rapidly, finally vanishing when i is equal to the critical angle for the liquid and $\sin\theta = \mu \sin 2i$. The corresponding direction of emergence of the light from the film would be outside the observable limit of the diffraction halo.

The diffraction halo as observed thus consists of two sets of rings, the intensities of which in any direction are superposed. In one of them, the path difference of the interfering rays diminishes with increasing angle of diffraction and finally vanishes in the direction of the achromatic ring. In the other set of rings which has a relatively small angular extension, the path difference becomes larger with the increasing angle of diffraction. The superposition of the two sets of rings whose angular positions are not the same thus leads to fluctuations in their visibility at small angles. From the formulae, the angular positions of the rings due to interior diffraction can be calculated and compared with observation and a satisfactory agreement is found.* The formula also enables a calculation to be made of the diameters of the dark and bright rings localised on a film of non-uniform thickness at any given angle of observation, and the particular angle at which a blurring of the rings would occur for a given thickness of the film. In every case the theory is confirmed by the actual measurements. Since the phase change occurring in total reflection is different for light polarised in and at right angles to the plane of incidence, there should be a corresponding small difference in the positions of the rings in the two cases. Even this fine point in the theory is confirmed by observation.† It is noticed that a plate which is too thick to show colours when viewed normally shows them if seen obliquely. Further, a film which shows colours when viewed normally appears achromatic when observed obliquely. These facts receive a satisfactory explanation on the theory.

It may be ramarked that the edge of each bubble in the film gives the complete diffraction halo, the diameter of the rings being, however, independent of the size of the bubble. The intensity of the halo in any particular direction depends on the aggregate length of the laminar edges running in the perpendicular direction. Hence, if the bubbles show a bias towards elongation in any particular direction, the halo appears intensified in the transverse direction, the rings, however, remaining circular.

An easy extension of the theory enables the oval haloes observed with obliquely held plates and the corresponding phenomena with non-uniform plates to be explained. As already mentioned, dry films exhibit phenomena of a quite different nature. For these and other details, reference may be made to the original papers.‡

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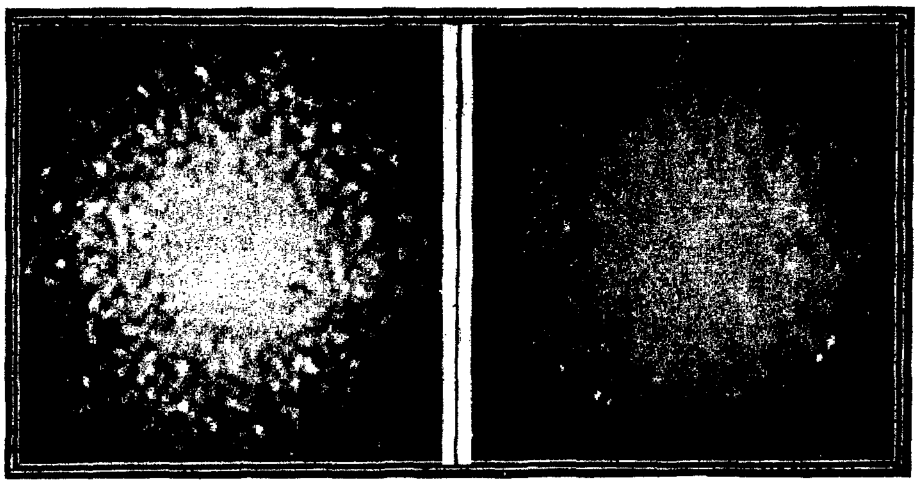
*C V Raman and K. Seshagiri Rao, *Philos. Mag.*, 1921, **42**, 679.

†It follows that if the incident light be plane-polarised in an arbitrary azimuth, the light diffracted at the boundary would, in general, be elliptically polarised.

‡See also K Seshagiri Rao, *Proc. Indian Assoc. Cultiv. Sci.*, 1923, **8**, 243. In this paper, the intensity distribution in the diffraction halo of mixed plates and the phenomena presented by dry films are discussed.

Intensity fluctuations in coronae: We shall now proceed to a closer examination of the nature of the diffraction pattern produced by a randomly distributed cloud of particles. As remarked earlier, such a pattern is *statistically* a summation of the effects of the individual particles but differs from them vastly in detail. Figure 66(a) on page 502 exhibits the central region of the corona observed around a *monochromatic* source of light of small angular extension, when viewed through a glass plate lightly dusted with lycopodium. The central disk of the corona is over-exposed in the photograph and shows no detail, but the granular structure of the pattern is seen very clearly in the first ring surrounding it. *Each of the bright spots in the field is a focussed image of the original source of light, formed by the joint action of the diffracting particles and the lens of the photographic camera.* This is verified by varying the size or shape of the source of light and noting its effect on the appearance of the pattern. It is then noticed that all the bright spots in the field alter in the same way and have the same form as the source. This is illustrated in figure 72 which shows the central disc of the corona photographed with a smaller exposure and on a larger scale than in figure 66(a), so as to clearly bring out the structure of the pattern. A small circular aperture and another in the form of a somewhat larger equilateral triangle were used as sources in photographing the two patterns reproduced. The circular and triangular shapes of the individual spots appearing in figures 72(a) and (b) can easily be recognised. The triangles in figure 72(b) appear inverted on the plate with respect to the source, as they should be in the images formed by a converging lens.*

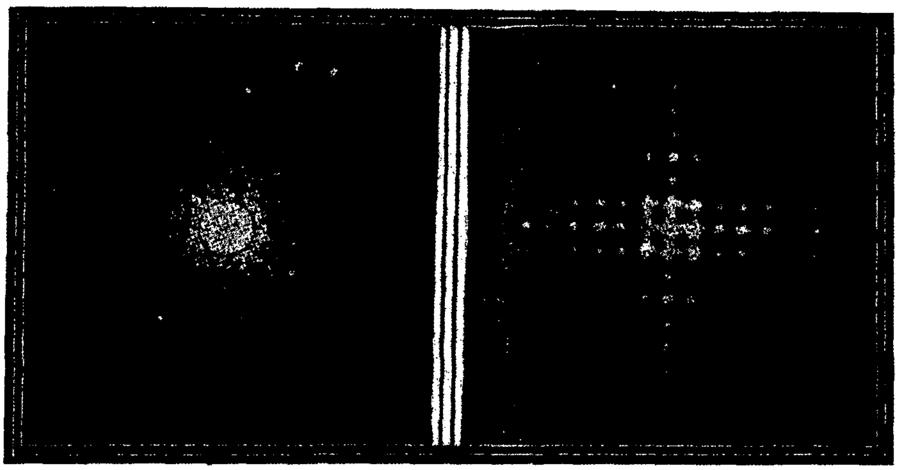


(a) (b)

Figure 72. Central disc of corona in monochromatic light with (a) a circular pin-hole and (b) a triangular aperture as source.

*G N Ramachandran, *Proc. Indian Acad. Sci.*, 1943, A18, 190.

It is familiar knowledge that a regularly spaced arrangement of apertures or obstacles can function as a diffraction grating and in combination with a lens give focussed spectra which in effect are monochromatic images of the source of light employed. The patterns reproduced in figure 72 show that a perfectly random arrangement of diffracting apertures or obstacles can also give well-defined images; the superiority of the regular grating is that it gives fewer and correspondingly more intense images in easily calculable positions instead of a great many feeble and irregularly spaced ones. The results are readily understood, since the optical effect in the focal plane of the lens can always be regarded as due to a plane wave of appropriate amplitude covering the entire area of the lens and travelling in such a direction that it comes to a focus at the point under consideration. The definition of the image of the source appearing at such point would be determined in every case by the configuration of the boundary of the lens and not by the disposition of the individual apertures or obstacles over its area. That the images formed by a random distribution of diffracting particles are not inferior in definition to those given by a regular diffraction grating is



(a) (b)

Figure 73. Comparison of corona with diffraction spectra given by a grating: (a) corona and (b) diffraction spectra.

illustrated in figures 73(a) and (b). These reproduce respectively the central part of the corona observed through a glass plate dusted with lycopodium and the diffraction spectra given by a fine sieve of metallic wires. A fine pin-hole illuminated by the 5461 Å radiation of a mercury lamp was the source and the optical conditions were also otherwise completely identical in the two cases.

The relation between the structure of the corona and the distribution of the diffracting particles on the plate can be illustrated in various ways. If, for example, the plate is moved in front of the eye, keeping the latter fixed on the source, the ring-system does not undergo any change, but the fine structure of the corona appears to move relative to the pattern of rings in *the same direction* as the motion of the plate. *Vice versa*, if one moves the eye, keeping the plate fixed, all the while looking at the source, the structure of the corona appears to move in *the opposite direction*. If the plate is rotated, the structure rotates in the same direction. The prettiest effects are those observed when a very small aperture is held immediately before the eye so as to limit the effective area of the lycopodium-dusted plate held in front of it. As the plate is moved relative to the aperature, different areas of the former become operative, and the spots in the corona appear and vanish at random positions in the field, thus simulating the effects seen in a spinthariscope.

The theoretical law of distribution of intensities resulting from random interferences which was derived earlier, viz., that $dW = p(f)\,df = \exp(-f)\,df$ has been tested* making use of the photograph reproduced in figure 73(a) for counting the spots and classifying them according to their observed intensities. The average intensity in a corona falls away from the centre in the proportion $J_1^2(x)/x^2$, where $x = 2\pi a \sin\phi/\lambda$, a being the radius of the particles, ϕ the angle of diffraction, and λ the wavelength of the light. To take account of this factor, the parts of the corona where the spots could be clearly seen were divided into five annular regions as marked in the photograph; the spots in each annulus were counted and classified in a scale of intensities established with the aid of the diffraction pattern of the grating photographed under strictly comparable conditions [figure 73(b)]. Using this tabulated list, the average intensity for each region was computed, and thus the values of $p(f)$ and f were determined for that

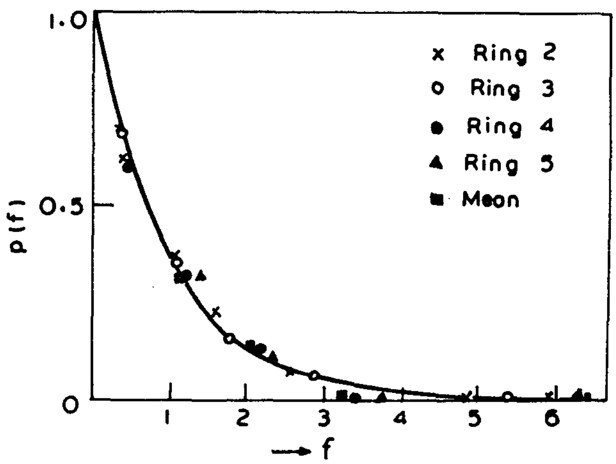


Figure 74. Verification of the statistical law of intensity fluctuation.

*G N Ramachandran, *Proc. Indian Acad. Sci.*, 1943, **A18**, 190.

region. These were plotted in figure 74 for four regions (the innermost one being too dense to facilitate counting), the continuous curve in the figure being the one calculated from theory. A final average for the four regions was obtained by dividing the intensities of the spots in each by the mean value of $J_1^2(x)/x^2$ for it, and the values of $p(f)$ thus obtained appear represented by black dots in figure 74. It will be seen from this figure that the experimental values fit the exponential formula indicated by the theory remarkably well.

The phenomenon of the radiant spectrum: Since, as we have seen, the bright spots in a diffraction corona as seen with monochromatic light are real images of the source employed, it follows that when white light is used instead, each bright spot would be drawn out into a spectrum, the different radiations appearing at distances from the centre of the pattern proportional to their respective wavelengths. This explains why in such circumstances, coronae exhibit numerous long coloured streamers or spectra located at random but directed radially outwards from the centre of the pattern. The streamers are most clearly seen in the outer parts of the corona, traversing its marginal rings and extending to the farthest visible limits of its extension. The streamers are distinguishable also in the central disc of the corona [see figure 66(b) on page 502], but their radial distribution and their colours are least conspicuous near the centre of the pattern. It is interesting to observe the coronae through a filter which transmits only two well-separated regions in the spectrum, e.g., the red and green regions. The entire pattern then appears filled with red and green spots; every green spot is accompanied by a red one, the two being along the same radius and the red spot at a distance from the centre greater than that of the green spot in the proportion of the two wavelengths. It is also of interest to view the diffraction corona through a dispersing prism held in front of the eye. The source of light then itself appears drawn out into a spectrum, and the radiant spectra are drawn out or shortened and also tilted one way or another, according to the direction in which they run. As the result of these changes, the "achromatic centre" of the diffraction pattern from which the coloured streaks appear to diverge is shifted away from its original position to a point lying well beyond the violet end of the spectrum into which the light-source is itself seen dispersed.

A familiar example of "radiant spectra" are those noticed when a small intense source of light is viewed directly by the normal eye against a dark background. Long coloured streamers of light are seen to diverge from the source in all directions, and faint coloured haloes also appear encircling the source near the outer limit of the streamers. Curious movements are also noticed within these streamers which may be controlled to some extent by fixing the eye on the source. The fact that the streamers disappear and are replaced by numerous bright points of illumination when a monochromatic source is used instead of white light, clearly indicates that we are here dealing with diffraction effects analogous to those discussed in the foregoing pages.* The diffracting structures are evidently those present in the refractive media of the eye, including especially the cornea, and the crystalline lens, and possibly also the vitreous and aqueous humors. To give rise to such effects, it is not necessary that the diffracting particles should be opaque or spherical or of uniform size. Even small differences of refractive index in regions of appropriate size should be sufficient to give the observed phenomena. The angular dimensions of the brightest region of the diffraction corona are in accord with the supposition that it owes its origin to the known structure of the cornea of the eye, while it appears probable that the outer coloured haloes arise from the fibrous structure of the crystalline lens around its margin.

Holding a dispersing prism in front of the eye and viewing a bright source of light through it, the radiant streamers now appear to diverge from a point well beyond the violet end of the spectrum into which the light-source is itself dispersed. This effect, noticed long ago by Brewster, is clearly analogous to that observed with diffraction coronae and discussed above.† It is, of course, necessary that the prism used should have clean and well-polished surfaces so that it does not itself give rise to disturbing effects of a similar nature.‡

Observation of Brownian movements without a microscope: As illustrated by figures 72(a) and 73(a) appearing on earlier pages, the corona due to a cloud of diffracting particles exhibits strongly marked local variations of intensity. These variations are determined by the distribution of the diffracting particles in space, and if this alters with time, there would necessarily be corresponding changes in the corona. If the movements of the particles are large and rapid, all trace of visible structure would disappear from the field. If, however, the movements are sufficiently small and slow, it should be possible to follow the changes in the corona from instant to instant and thus obtain visual evidence that the diffracting particles are in motion.

As is well known, the individual particles in colloidal suspensions and emulsions execute "Brownian movements", which are most lively when the particles are very small and are suspended in an inviscid fluid. For our present purpose, it is necessary to select a substance in which the particles are of fair size so that the coronal disc is of sufficient intensity and also exhibits a visible structure. Fresh milk is the most easily available material satisfying this requirement. When a little of it is flowed on to a clean glass plate and then allowed to drain away as completely as possible, a thin film remains firmly adherent to the plate. A small aperture illuminated by a mercury arc lamp and viewed through

*C V Raman, *Philos. Mag.*, 1919, **38**, 568.
†C V Raman, *Philos. Mag.*, 1922, **43**, 357.
‡C V Raman, *Nature (London)*, 1922, **109**, 175.

such a film exhibits an extended field of diffuse illumination surrounding it. Fixing the attention over a limited area of the field, it is noticed that this exhibits a structure which is not static but is continually changing. Bright points of illumination continually appear in the field and others disappear. These changes become less rapid and ultimately stop when the film is dry; the structure of the field is then completely static.*

*Unpublished observations by the author and G N Ramachandran.