

The maintenance of forced oscillations of a new type

In a paper "On a Class of Forced Oscillations" published in the *Q. J. Pure Appl. Math.* (No. 148, June 1906), Mr Andrew Stephenson discussed mathematically a proposition which may be stated in his own words thus: periodic non-generating force acting on a system in oscillation about a position of stable equilibrium exerts a cumulative action in intensifying or diminishing the amplitude, if its frequency is contained within any one of a number of ranges lying in the vicinity of 2μ , $2\mu/2$, $2\mu/3$... , where μ is the natural frequency of the system.

Further investigations upon this and other allied subjects appear in seven subsequent issues of the *Philos. Mag.* As regards the forced oscillations discussed in the *Quarterly*, the author gives, in the way of experimental verification of his mathematics, the following: the influence of the disturbing motion becomes feebler as r increases, but it may easily be observed experimentally in a number of cases. For this purpose suspend a load by means of a spiral spring, and attach to it a pendulum light compared with the load, but of such density that the air resistance is negligible; the pendulum being of suitably chosen period, it will be found that when the load is carefully adjusted the relative equilibrium of the pendulum in the vertical motion is unstable.

I believe the beauty and interest of the results obtained by Mr Andrew Stephenson have not been generally realised, otherwise it is nearly certain that something more satisfying in the way of experimental demonstration of these oscillations than mere observation of "instability of equilibrium" in certain cases would have been put in the field. I think an experimentalist would hardly be pleased with anything less than the actual permanent maintenance of oscillations of the type mentioned, i.e. something similar to the experiments of Faraday, Melde and Lord Rayleigh for the case of double frequency, which, as Mr Stephenson points out, is only one particular case of his general theorem.

During the course of certain acoustical work which I have been engaged in during the last two years, I observed certain types of stationary vibration which I find are undoubtedly of the kind contemplated in Mr Stephenson's paper. These observations were made with an apparatus from which any new effects were apparently hardly to be expected. The arrangement was the well known one of a string maintained in vibration by a tuning-fork oscillating in a direction parallel to the string. It is generally supposed that the oscillations permanently maintained have a frequency which is half that of the tuning-fork. I found this was *not* always the case. With an electrically maintained tuning-fork the amplitude of oscillation of which could be readily adjusted, the stationary oscillation of the

string had a frequency of $\frac{1}{2}$ of, equal to, $\frac{3}{2}$ times, twice, &c., that of the tuning-fork, each term in the harmonic series appearing separately by itself with a fairly large amplitude, or with one or more of the others conjointly, according to circumstances. The frequency- and phase-relations could be studied by several methods, most of which were very simple applications of the principle of Lissajous's figures.

The possibility of isolating the harmonics, and also certain serious discrepancies between theory and experiment as regards the phase of the oscillations in the case of double frequency, were traced to the existence of variations of tension in free oscillations of sensible amplitude. These variations of tension were experimentally demonstrated by a special form of monochord denominated the "Ectara" (*vide* the *J. Indian Math. Club* for October, pp. 170-5), in which the sounding surface is a membrane perpendicular to the vibrating string, and emits a tone having *twice* its frequency.

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