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The diffraction of light by high frequency sound waves: Part V

General considerations—oblique incidence and amplitude changes

C V RAMAN
and
N S NAGENDRA NATH
(Department of Physics, Indian Institute of Science, Bangalore)

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1. Introduction

The essential idea that the phenomenon of the diffraction of light by high frequency sound waves depends on the corrugated form of the wave-front of the transmitted light has been pointed out by us in part I of this series of papers[1]. Therein, we considered that the corrugated wave-front of light could be simply obtained by considering the phase changes accompanying the traversing beam which was assumed to undergo no amplitude changes at its various points. This course was adopted by us to bring out the essential features of the theory without unnecessary complications. By a close study of the problem, one can however easily see that the consideration of the phase changes is far more important than the amplitude changes if we desire to understand the essential features of the phenomenon. Indeed, this fact holds if we consider the case when the sound wave field is small and the wavelength of sound is large. This has been experimentally confirmed quite recently by Bär[2].

In part IV of this series of papers, we proposed the method of obtaining the wave function of light by considering the partial differential equation governing the propagation of light in a medium filled with sound waves. *Such a procedure would naturally take account of both the amplitude changes and the phase changes accompanying the beam.* These changes should be however periodic in character. On these considerations we found that, in the case of a progressive sound wave, the nth order diffraction component will be inclined at an angle $\sin^{-1}(-n\lambda/\lambda^*)$ to the direction of propagation of the incident light and will have the frequency $\nu - n\nu^*$ where ν and λ denote the frequency and the wavelength of the incident light while ν^* and λ^* correspond to those of the sound wave. We also showed that when the disturbance in the medium is simple harmonic, the relative intensity of the nth order is given by $|\phi_n|^2$ where ϕ_n is the solution of the equation

$$\mu^2\frac{d^2\phi_n}{d\xi^2} - 2i\mu_0\mu\frac{d\phi_n}{d\xi} - \frac{n^2\lambda^2}{\lambda^{*2}}\phi_n = -\mu_0\mu i(\phi_{n-1} - \phi_{n+1}) \tag{1}$$

where $\xi = 2\pi\mu z/\lambda, \mu_0$ is the refractive index of the undisturbed medium, μ is the amplitude of the variation of the refractive index and the z-axis is along the direction of propagation of the incident light. As μ is of the order 10^{-5} and μ is of the order of unity, we could consider the equation for ϕ_n as given by

$$\boxed{2\frac{d\phi_n}{d\xi} - (\phi_{n-1} - \phi_{n+1}) = \frac{in^2\lambda^2}{\mu_0\mu\lambda^{*2}}\phi_n.} \tag{2}$$

In the case of a stationary sound wave, we obtained the result that, in any even order, radiations with frequencies $\nu \pm 2r\nu^*$ would be present, while in any odd order, radiations with frequencies $\nu \pm \overline{2r+1}\nu^*$ would be present. These results interpret the experimental results of Bär[2] regarding the coherence phenomena among the diffracted orders. If the disturbance in the medium is simple harmonic, we obtained the result that the amplitudes of the various components of the nth order are given by the Fourier analysis of $g_n(\xi, t)$ which satisfies the equation

$$\boxed{2\frac{\partial g_n}{\partial \xi} - \sin\varepsilon(g_{n-1} - g_{n+1}) = \frac{in^2\lambda^2}{\mu_0\mu\lambda^{*2}}g_n} \tag{3}$$

where $\varepsilon = 2\pi\nu^* t$ and the term containing the second derivative of g_n is omitted as its coefficient is very small. *If one however ignores the spectral character of each order*, then the relative intensity of the nth order is given by

$$\int_0^{2\pi} |g_n(\xi, \varepsilon)|^2 \, d\varepsilon. \tag{4}$$

These results pertain to the case of the incident light falling normally on the sound waves. One of the purposes of this paper is to extend the above considerations to the case of the oblique incidence of light to the sound waves. We have found that, in the case of oblique incidence, the intensity of the nth order need not be equal to that of the $-n$th order, thus explaining the results of Debye and Sears[3], Lucas and Biqard[4], Bär and Parthasarathy.[5] We have also investigated the amplitude changes accompanying the traversing wave-front explaining the results of Hiedemann,[7] Bär[2] and Lucas[6].

2. The diffraction of light when it is incident obliquely to the sound waves

We choose the axes of reference such that the x-axis points to the direction of propagation of the sound waves and the Z-axis lies in the plane contained by the directions of propagation of the sound and the incident light waves. With the same considerations as in part IV, the wave function of the light traversing the medium is given by

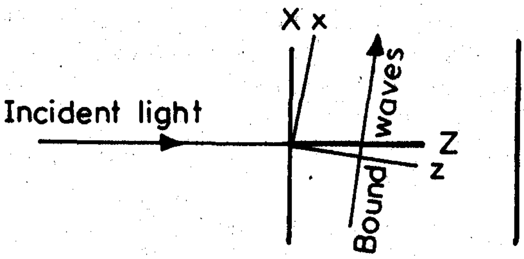

Figure 1

$$\psi = \exp(2\pi i\nu t)\,\Phi(x,z,t), \tag{5}$$

where Φ satisfies the equation

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} = -\frac{4\pi^2}{\lambda^2}\{\mu(x,t)\}^2\Phi. \tag{6}$$

Let $\cos\phi$ and $\sin\phi$ be the z- and x-direction cosines of the direction of propagation of the incident light. The transmitted wave travelling in the medium will suffer periodic fluctuations in its phase and amplitude with the period λ^* $\sec\phi$ along the line in the incident plane of light and the xz plane. Thus,

$$\Phi(x,z,t) = \Phi(x + p\lambda^*, z - p\lambda^*\tan\phi, t). \tag{7}$$

So, the wave travelling in the medium is given by

$$\exp(2\pi i\nu t)\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} f_{rs}(x\sin\phi + z\cos\phi)\exp(2\pi ir)(x\cos\phi - z\sin\phi)/$$
$$\lambda^*\sec\phi\exp(2\pi is\nu^* t). \tag{8}$$

We choose a new axis of reference defined by

$$\begin{aligned} X &= x\cos\phi - z\sin\phi \\ Z &= x\sin\phi + z\cos\phi. \end{aligned} \tag{9}$$

The new Z-axis is along the direction of propagation of the incident light. In the new system of reference, the wave function has to be written as

$$\exp(2\pi i\nu t)\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty} f_{rs}(Z)\exp(2\pi ir)X\cos\phi/\lambda^*\exp(2\pi is\nu^* t). \tag{10}$$

Then

$$\Phi = \sum_{-\infty}^{\infty} \sum_{-\infty}^{\infty} f_{rs}(Z) \exp(2\pi i r) X \cos\phi/\lambda^* \exp(2\pi i s v^* t). \tag{11}$$

In the case of a progressive sound wave

$$\Phi(X + \rho\lambda^* \sec\phi, Z, t) = \Phi(X, Z, t - \rho/v^*). \tag{12}$$

This condition restricts the number of terms in the above expansion (11) so that

$$\Phi = \sum_{-\infty}^{\infty} f_r(Z) \exp(2\pi i r) X \cos\phi/\lambda^* \exp(-2\pi i r v^* t). \tag{13}$$

Thus

$$\psi = \sum_{-\infty}^{\infty} f_r(Z) \exp(2\pi i r) X \cos\phi/\lambda^* \exp(2\pi i)(v - rv^*)t. \tag{14}$$

If one considers the diffraction effects of ψ given by (14), it will be fairly obvious that the nth order will be inclined at an angle $\sin^{-1}(-n\lambda\cos\phi/\lambda^*)$ to the Z-axis and will have the frequency $v - nv^*$ with the relative intensity expression $|f_n(Z)|^2$.

3. The case when the disturbance in the medium is simple harmonic

If we suppose that the vibration in the refractive index of the medium is simple harmonic along the x-axis, it can be represented as

$$\begin{aligned}\mu(x,t) - \mu_0 &= \mu \sin 2\pi(v^* t - x/\lambda^*)\\ &= -\frac{\mu}{2i}\{\exp(i(bx - \varepsilon)) - \exp(-i(bx - \varepsilon))\}\\ &= -\frac{\mu}{2i}\{\exp(i(b\overline{X\cos\phi + Z\sin\phi} - \varepsilon))\\ &\quad - \exp(-i(b\overline{X\cos\phi + Z\sin\phi} - \varepsilon))\}\end{aligned} \tag{15}$$

where $\varepsilon = 2\pi v^* t$ and $b = 2\pi/\lambda^*$.

We know from (6) that Φ satisfies the equation

$$\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2} = -\frac{4\pi^2}{\lambda^2}\{\mu(x,t)\}^2\Phi$$

or

$$\frac{\partial^2\Phi}{\partial X^2} + \frac{\partial^2\Phi}{\partial Z^2} = -\frac{4\pi^2}{\lambda^2}\{\mu(X,Z,t)\}^2\Phi. \tag{16}$$

Substituting the Fourier series (13) and the expression (15) for $\mu(X,Z,t)$ in the equation (16) and neglecting the second order term with the coefficient μ^2, we get by comparing the coefficients

$$\frac{d^2 f_r}{dZ^2} - \frac{4\pi^2 r^2 \cos^2\phi}{\lambda^{*2}} f_r - A f_r$$

$$= \frac{B}{2i}\{f_{r-1}\exp(ibZ)\sin\phi - f_{r+1}\exp(-ibZ)\sin\phi\} \tag{17}$$

where $A = -4\pi^2\mu_0^2/\lambda^2$ and $B = 8\pi^2\mu_0\mu/\lambda^2$.
Putting $f_r(Z) = \exp(-iu\mu_0 Z)\Phi_r(Z)$, where $u = 2\pi/\lambda$, we get

$$\frac{d^2\Phi_r}{dZ^2} - 2iu\mu_0\frac{d\Phi_r}{dZ} - \frac{4\pi^2 r^2\cos^2\phi}{\lambda^{*2}}\Phi_r$$

$$= -\frac{Bi}{2}\{\Phi_{r-1}\exp(ibZ)\sin\phi - \Phi_{r+1}\exp(-ibZ)\sin\Phi\}. \tag{18}$$

Putting $Z = (2\pi\mu)^{-1}\lambda\xi$ we obtain

$$\mu^2\frac{d^2\Phi_r}{d\xi^2} - 2i\mu_0\mu\frac{d\Phi_r}{d\xi} - \frac{r^2\lambda^2\cos^2\phi}{\lambda^{*2}}\Phi_r$$

$$= -\mu_0\mu i\{\Phi_{r-1}\exp(ia\xi)\sin\phi - \Phi_{r+1}\exp(-ia\xi)\sin\phi\}. \tag{19}$$

where $a = \lambda/\mu\lambda^*$.

As μ is of the order 10^{-5} and μ_0 is of the order unity we may omit the first term and consider the equation

$$\boxed{2\frac{d\Phi_r}{d\xi} - (\Phi_{r-1}\exp(ia\xi)\sin\phi - \Phi_{r+1}\exp(-ia\xi)\sin\Phi) = \frac{ir^2\lambda^2\cos^2\phi}{\mu_0\mu\lambda^{*2}}\Phi_r.} \tag{20}$$

The relative intensity of the rth order is given by $|\Phi_r(\xi)|^2$. We may now show that, in general, $|\Phi_r(\xi)|^2 \neq |\Phi_{-r}(\xi)|^2$. We will prove the same by assuming the contradictory result. Suppose

$$\Phi_r(\xi) = \exp(i\rho r)\Phi_{-r}(\xi). \tag{21}$$

Then we get

$$2\frac{d\Phi_{-r}}{d\xi} = (\Phi_{-r+1}\exp(i(\rho_{r-1} - \rho r))\exp(ia\xi)\sin\phi - \Phi_{-r-1}$$

$$\times\exp(i(\rho_{r+1} - \rho_r))\exp(-ia\xi)\sin\phi)$$

$$= \left\{-2i\frac{d\rho_r}{d\xi} + \frac{ir^2\lambda^2\cos^2\phi}{\mu_0\mu\lambda^{*2}}\right\}\Phi_{-r}. \tag{22}$$

The actual equation for Φ_{-r} is

$$2\frac{d\Phi_{-r}}{d\xi}-(\Phi_{-r-1}\exp(ia\xi)\sin\phi-\Phi_{-r+1}\exp((-ia\xi)\sin\phi)$$
$$=\frac{ir^2\lambda^2\cos^2\phi}{\mu_0\mu\lambda^{*2}}\Phi_{-r}. \tag{23}$$

Comparing the equations, we obtain the result that they can never be identical unless $\phi=0$ when $\rho_r=r\pi$. Thus in the case of oblique incidence in which $\phi\neq 0$.

$$\Phi_r(\xi)\neq\exp(i\rho r)\Phi_{-r}(\xi) \tag{24}$$
$$\text{i.e.}\quad |\Phi_r(\xi)|^2\neq|\Phi_{-r}(\xi)|^2.$$

This means that the intensity of the rth order is not equal to the intensity of the $-r$th order. Similar results corresponding to the above could be easily derived in the case of the standing sound waves on the same lines.

In case the coefficient of the term on the right-hand side of the equation (20) has no appreciable influence in the wave-function and ϕ is small, it can be shown that Φ_r approximates to the wave-function given in part II of this series of papers. In this case the diffraction pattern will be very nearly symmetrical.

4. Amplitude changes on the emerging wave-front of light

According to the notation of part IV, the wave-function for a *general* periodic supersonic disturbance in the medium is given by

$$\psi=\exp(2\pi i\nu t)\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}f_{r,s}(z)\exp(2\pi irx/\lambda^*)\exp(2\pi is\nu^*t).$$

In the case of the normal incidence of the incident light to the sound waves. Therefore the intensity is given by

$$|\psi|^2=\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}A_{l,m}\exp(2\pi ilx/\lambda^*)\exp(2\pi im\nu^*t)$$

where

$$A_{l,m}=\sum_{-\infty}^{\infty}\sum_{-\infty}^{\infty}f_{r,s}f_{r-l,s-m}{}^{\dagger}.$$

Thus, the intensity (or the amplitude) will be periodic in x and t on the wave-front which forms the basis of the explanation of the amplitude grating found by Bär[2] and Lucas[6]. This forms also the basis of the explanation of the observability

†Denoting the conjugate.

of the sound waves found by the investigators[7] at Köln. In the case of a standing sound wave, *the average intensity with respect to time* will be given by

$$I(x,z) = \sum B_s B_s^\dagger$$

where

$$B_s = \sum_{r=-\infty}^{\infty} f_{r,s} \exp(2\pi i r x/\lambda^*)$$

and r and s are both even integers or odd integers. It follows from the above that $I(x, z)$ is periodic in $\lambda^*/2$.

The intensity (or the amplitude) will be constant on the wave-front when all $A_{l,m}$ vanish except $A_{0,0}$ as will be so in the case governed by the restrictions of part I.

5. Summary

The essential idea that the phenomenon of the diffraction of light by high frequency sound waves depends on the corrugated nature of the transmitted wave-front of light has been developed on general considerations in this paper to apply for the case of the oblique incidence of the incident light to the sound waves. It is found that the intensity distribution will not be symmetrical in general thus explaining the results of Debye and Sears, Lucas and Biquard, Bär and Parthasarathy. The consideration of the amplitude changes of the traversing beam of light explains the results of Hiedemann, Bär and Lucas.

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