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First ranked galaxies in groups and clusters

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Summary. The small scatter in the luminosities of the brightest galaxies in clusters has been a topic of much debate. It has been argued that these galaxies are either special objects or the tail-end of a statistical distribution. In 1928, Fisher and Tippett derived the general form a distribution of extreme sample values should take, independent of the parent distribution from which they are drawn. We compare this asymptotic form with the distribution of first ranked cluster members and conclude that these galaxies are not the extreme members of a statistical population. On the other hand, comparison of first ranked members of 'loose' groups with the extreme value distributions shows that these galaxies are consistent with their being the tail-end of a statistical distribution.

1 Introduction

The small intrinsic dispersion in the magnitudes of the brightest members of rich clusters has made them indispensable as 'standard candles' in observational cosmology. The dispersion of these first ranked members is only 0.35 mag (Hoessel, Gunn & Thuan 1980; Schneider, Gunn & Hoessel 1983). This small scatter in M_1 was apparent from earlier work (Humason, Mayall & Sandage 1956; Sandage 1972; Sandage & Hardy 1973) and in addition, the small dispersion (38 per cent) found in the values of the luminosities of these galaxies has prompted suggestions that they be endowed with a special status (Peach 1969). Unique creation mechanisms or evolutionary processes, like 'cannibalism', (Ostriker & Tremaine 1975; Hausman & Ostriker 1978; Tremaine 1981) which would give them a special status have been suggested. The small scatter in luminosity would also arise if the luminosity function of galaxies was cut off at the bright end, although there is no strong evidence that this is the case (Peterson 1970).

On the other hand, luminosities of first ranked galaxies may be drawn randomly from a universal luminosity function (Scott 1957). Peebles (1968) and Geller & Peebles (1976) have argued that the distribution of their brightest members is consistent with a statistical model based on a universal luminosity function. Peebles (1969) points out that the continuity of M_1

with the extreme end of the luminosities for the fainter cluster members supports the statistical hypothesis, otherwise one would have to attribute this continuity to chance. Tremaine & Richstone (1977) have discussed tests for a statistical hypothesis that are independent of the assumed luminosity function and of its variation from cluster to cluster. Recently, Geller & Postman (1983) have applied these tests to a sample of first ranked galaxies in groups and conclude that the results are consistent with a statistical model.

Here we present a new test for the first ranked members which allows the statistical hypothesis to be checked. It uses known results in extreme value statistics. In 1928 Fisher & Tippett showed that the distribution of the extreme values (smallest or greatest) of subsamples drawn from a large sample tend to a unique asymptotic form that describes their distribution *independent of the parent sample distribution from which the extremes are drawn.* We have investigated two independent data sets: the list of Hoessel *et al.* (1980) consisting of 116 first ranked galaxies in a complete sample of nearby Abell clusters and the data used by Geller & Postman (1983), which lists first ranked members of groups from the CfA distance-limited catalogue, corrected for Virgo Flow. In each case we have studied the agreement between the observed distribution of brightest galaxies and the predicted Fisher–Tippett extreme value distribution.

We find that the brightest *cluster* galaxies do not fit the Fisher–Tippett form at the 99 per cent confidence level. This shows that first ranked members of *rich clusters* cannot be drawn from the tail-end of a statistical distribution of galaxies, no matter how steep the luminosity function. Their *luminosities* (rather than magnitudes) are consistent with being drawn from a Gaussian distribution with a dispersion of 32 per cent around the mean luminosity, suggesting a unique, common mechanism for the origin of these objects. For the brightest galaxies in *groups*, the Fisher–Tippett form is a good fit. A universal form can also be predicted for the frequency distribution of M_{12}, the difference in magnitudes of the first and second ranked galaxies in groups. The agreement of this prediction with the observed distribution is consistent with the statistical hypothesis. These results show that the brightest *group* galaxies are consistent with their being the extreme members of a statistical model in which the luminosities of all group members are drawn from some luminosity function. We conclude that a large proportion of first ranked cluster galaxies comprise a special class of objects whereas the brightest group galaxies are the statistical extremes of a population. The close agreement of the loose group data with the asymptotic extreme value distributions predicted theoretically suggests a common mechanism of formation for these groups of galaxies.

In Section 2 we describe the relevant extreme value statistics; in Section 3 we analyse the cluster data, and in Section 4 the group data before, in Section 5, summarizing our conclusions.

2 Extreme value statistics

Fisher & Tippett (1928) considered the problem of the extreme values of n samples each of size m drawn from the same underlying population (since their paper gives no details we reconstruct their argument here). By noting that the largest value of the whole population of mn objects, x_{ij}, must be equal to the largest of the n largest values taken from the m subsamples, they are able to show that the probability distribution of the largest value, z_{mn}, taken from the mn observations has the same asymptotic form as the probability distribution of the maximum values in samples of size m whenever such an asymptotic form exists. If we suppose x_{ij} are independent real random variables then

$$z_{mn}=\max(x_{11},\dots,x_{1m};\, x_{21},\dots x_{2m};\dots;\, x_{n1},\dots x_{nm})$$
$$=\max(y_1, y_2,\dots y_n) \quad (1)$$

where $y_i = \max(x_{i1}, \ldots x_{im})$; then the cumulative distribution function of z is given by

$$\begin{aligned} F_{mn}(x) &\equiv P(z_{mn} \leqslant x) \\ &= P[\max(y_1, \ldots y_n) \leqslant x] \\ &= P^n(y_i \leqslant x) \end{aligned} \tag{2}$$

where we have used the statistical independence. Fisher & Tippett now argue that, if a stable limiting distribution exists for y as $n \to \infty$ it will tend to the same asymptote as that of the distribution for z. Therefore $P(y \leqslant x)$ must have the same form as $F_{mn}(x)$. Since a linear transformation does not change the form of a distribution we must have

$$P(y \leqslant x) = F_{mn}(\alpha x + \beta) \tag{3}$$

with α, β positive constants. Hence, the distribution function of the extreme values*, $F(x)$, is given by the solution of the non-linear functional equation

$$[F_{mn}(x)]^n = F_{mn}(\alpha x + \beta) \tag{4}$$

We can, henceforth, drop the subscript mn on F but we note that the positive constants, α and β, will in general depend on n so we can rewrite (4) as

$$[F(x)]^n = F(\alpha_n x + \beta_n). \tag{5}$$

The solutions of (5) fall into two equivalence classes according as (α_n, β_n) are taken as $(1, \beta_n)$ or $(\alpha_n, 0)$. In the former case the solution of (5) yields, for the distribution largest values†

$$F(x) = \exp\{-\exp[-a(x - x_0)]\} \tag{6}$$

where $a > 0$ and x_0 are constants independent of n and $-\infty < x < \infty$. For the distribution of smallest values, we put $x \to -x$ in (6) with different a and x_0. The parameter a is related to the steepness of fall of the parent distribution; x_0 is the mode of $F'(x)$.

The Fisher–Tippett asymptote is the distribution of extreme values of samples taken from underlying distributions that are of *exponential type*. The underlying probability distribution $dG(x)$ is said to be of exponential type if (Gnedenko 1941; Gumbel 1966; David 1981)

$$\operatorname*{Lt}_{x \to \infty} \frac{d}{dx} \left[\frac{1 - G(x)}{G'(x)} \right] = 0. \tag{7}$$

Thus, all statistical moments exist for exponential-type distributions and they are only defined for variates that are unbounded to the right or to the left. In particular, this class includes the exponential, $G'(x) = e^{-x}$, gamma, $G'(x) = [\Gamma(\lambda)]^{-1} x^{\lambda-1} e^{-x}$, and normal distribution, $G'(x) = (2\pi)^{-1/2} \exp(-x^2/2)$, within its domain. We note that (6) reveals that a distribution of largest values taken from a parent distribution of exponential type is positively skewed. The other solutions of (5) provide asymptotic forms for the extreme values of distributions whose variates are bounded [when $\beta_n \equiv 0$ in (5)] and for which only a finite number of moments are non-zero (Gumbel 1966). These are not of physical relevance in the problems studied here. We also note that it is not known *a priori* how large n needs to be for good convergence of the distribution of subsample extremes to the asymptote (6) although this will not be a problem in our applications.

* Analogous results apply for minimum value distributions if we use the fact that $\min(y_1, \ldots y_n) = -\max(-y_1, \ldots, -y_n)$.

† To solve (5) when $\alpha_n \equiv 1$ consider $[F(x)]^{mn}$ which leads to the requirement that $\beta_{nm} = \beta_n + \beta_m$ so $\beta_n = -(\log n)/a$, where a is an arbitrary constant. Then take logarithms of (5) twice to obtain (6).

By analogous reasoning it is possible to generalize (6) to obtain the asymptotic distribution function of the mth extreme, $dF_m(x)$, where m is labelled from the top, as (Gumbel 1966)

$$dF_m(x)=\frac{m^m a_m}{(m-1)!}\exp[-my_m-me^{-y_m}]\,dy_m \tag{8}$$

where

$$y_m \equiv a_m(x_m-x_0^{(m)}) \tag{9}$$

where a_m is the density function (steepness of fall) of the parent distribution at the characteristic value of the mth ranked member; $x_0^{(m)}$ is the characteristic value (mode) of the (mth ranked) distribution. The Fisher–Tippett distribution (6) for the first ranked members arises from (8) and (9) in the case $m=1$ where we have $a_1=a$.

The moments are most easily generated using the moment generating function, $\Phi(t)$, for the distribution of mth extremes, (8), which is

$$\Phi(t)=\exp(it\log m)\frac{\Gamma(m-it)}{\Gamma(m)}. \tag{10}$$

To apply (6–10) to the distribution of brightest galaxies in clusters or groups we note that if M_1 is the magnitude of the brightest member in a cluster or group (large negative $M_1 \leftrightarrow$ bright galaxy) then the asymptotic probability distribution for first ranked magnitudes, M_1, will be

$$dP=a\exp\{a(M_1-M_0)-\exp[a(M_1-M_0)]\}\,dM_1 \tag{11}$$

where M_0, the mode of the distribution, is a measure of the total mass of the cluster or group (the total number of objects in the parent distribution). Peebles (1968) had arrived at a formula of this form for a luminosity function of exponential form

$$\psi(M)=a\exp[a(M-M_0)] \tag{12}$$

from first principles, treating the integrated luminosity function

$$\phi_{ab}=\int_{M_a}^{M_b}\psi(M)\,dM \tag{13}$$

as a Poisson process. Since non-overlapping magnitude intervals can be assumed to be statistically independent the probability of n galaxies in $[M_a, M_b]$ is

$$P_n(M_a, M_b)=\frac{(\phi_{ab})^n\exp(-\phi_{ab})}{n!}. \tag{14}$$

By simply evaluating for the probability of one galaxy in $(-\infty, M_1)$ we obtain (11). However, as noted above, the distributions (11) and (8) arise in far more general circumstances than (12).

The *mean* of the distribution (11) is

$$\langle M_1\rangle=M_0-\frac{0.577}{a} \tag{15}$$

where $0.577=-\Gamma'(1)$ is Euler's constant: the *variance* is

$$\sigma^2(M_1)=\frac{\pi^2}{6a^2} \tag{16}$$

and the *median* $m(M_1)$ is

$$m(M_1)=M_0+\frac{\ln(\ln 2)}{a}=M_0-\frac{0.367}{a}. \tag{17}$$

Also useful is the *skewness*, β, given by the second and third moments, μ_2 and μ_3 as (ζ is the Riemann zeta function)

$$\beta \equiv \frac{\mu_3}{(\mu_2)^{3/2}} = \frac{12\zeta(3)\sqrt{6}}{\pi^3} = -1.140. \tag{18}$$

If β vanishes, a distribution is symmetric. These quantities can all be calculated for the mth extremes using (8) and $y_m \to -y_m$; we find for the second and mth brightest members that

$$\langle M_2 \rangle = M_0^{(2)} - \frac{0.577+\ln 2-1}{a_2} = M_0^{(2)} - \frac{0.270}{a_2} \tag{19}$$

$$\langle M_m \rangle = M_0^{(m)} - \frac{1}{a_m}\left[0.577+\ln m - \sum_{i=1}^{m-1} \frac{1}{i}\right] \tag{19a}$$

$$\sigma^2(M_2) = \frac{1}{a_2^2}\left(\frac{\pi^2}{6} - 1\right) = \frac{0.645}{a_2^2} \tag{20}$$

$$\sigma^2(M_m) = \frac{1}{a_m^2}\left(\frac{\pi^2}{6} - \sum_{i=1}^{m-1} \frac{1}{i^2}\right) \tag{20a}$$

$$\beta(M_2) = -0.780. \tag{21}$$

From (15)–(18) one can see that since the mode M_0 can be determined from the observations by determining the mean value $\langle M_1 \rangle$, the only free parameter is a, which is a measure of the steepness of fall in the *original (parent)* distribution at the extreme end. For example, with an exponential parent distribution, (12), a is just the logarithmic derivative of $\psi(M)$.

Another important collection of general distributions can be derived from (8): the exact asymptotic forms for the differences between the mth and $(m+1)$st extremes (where m is counted from the top). If these distances are written as $d_m \geq 0$ where

$$d_m \equiv M_{m+1} - M_m; \quad 1 \leq m \leq n-1 \tag{22}$$

then, so long as n is large and m is small, the probability distribution of the distance d_m tends to the exponential form, (Gumbel 1966)

$$f(d_m) = m a_{m+1} \exp(-m d_m a_{m+1}). \tag{23}$$

Note that the distribution is independent of M_0, and also that for an exponential-type distribution the a_m values are very nearly the same for different m. For the exponential distribution in particular they are identically equal. Henceforth, we write the expressions dropping the subscript on a. Since (23) is exponential, the mean and standard deviation are equal*

$$\langle d_m \rangle = \sigma(d_m) = \frac{1}{ma}. \tag{24}$$

We shall be particularly interested in the distribution of $M_{12} = M_2 - M_1$, the difference between the first and second ranked members, and we note that M_{12} corresponds to d_1 in

*For the pure exponential distribution as the parent population ($a_m \equiv 1$) the distribution (23) is exact for all n, m and not merely asymptotic.

(22). Hence, if first and second galaxies in groups and clusters are drawn from a statistical population we predict

$$\langle M_{12}\rangle=\sigma(M_{12})=\frac{1}{a}. \tag{25}$$

Since $\langle M_{12}\rangle=\langle M_2\rangle-\langle M_1\rangle$, from equation (25) we have

$$\langle M_2\rangle=\langle M_1\rangle+\frac{1}{a}. \tag{26}$$

In general, from (24) and (26), we have

$$\langle M_m\rangle=\langle M_1\rangle+\frac{1}{a}\sum_{i=1}^{m-1}\frac{1}{i} \tag{27}$$

$$\langle M_m\rangle=M_0-\frac{0.577}{a}+\frac{1}{a}\sum_{i=1}^{m-1}\frac{1}{i}. \tag{28}$$

Comparing with (19a), we see that

$$M_0^{(m)}=M_0+\frac{1}{a}\ln(m). \tag{29}$$

The medians are shifted to the right by $a^{-1}\ln(m)$ for the mth ranked galaxy. From (27), the means are shifted to the right by

$$\frac{1}{a}\sum_{i=1}^{m-1}\frac{1}{i}.$$

The difference is:

$$\langle M_m\rangle-M_0^{(m)}=\frac{1}{a}\left[-0.577+\sum_{i=1}^{m-1} i\right]-\frac{1}{a}\ln(m) \tag{30}$$

$$\rightarrow 0 \text{ as } m\rightarrow\infty.$$

Thus the difference decreases and $\sigma(M_m)$ also, by (20a), becomes smaller. Thus, the mth extremes have a more and more symmetric and peaked distribution as m increases.

Incidentally, this allows us to predict the statistics T_1 and T_2 introduced by Tremaine & Richstone (1977) and studied by Geller & Postman (1983). Since these quantities are simply

$$T_1\equiv\frac{\sigma(M_1)}{\langle M_{12}\rangle} \tag{31}$$

$$T_2\equiv\frac{\sigma(M_{12})}{\langle M_{12}\rangle\sqrt{0.677}} \tag{32}$$

we can use (19)–(29) to determine them exactly for M_1 and M_2 chosen from statistical distributions. We find

$$T_1=1.282 \tag{33}$$

$$T_2=1.215. \tag{34}$$

Remarkably, both T_1 and T_2 are independent of a. Thus, a test of the statistical hypothesis is given by conformity of the observations of T_1 and T_2 with (33) and (34) and not just with $T_1 \geqslant 1$ and $T_2 \geqslant 1$. This seems to be a more stringent condition than that set by Tremaine & Richstone (1977) and used by Geller & Postman (1983) in their study of galaxy groups.

We now proceed in Sections 3 and 4 to determine whether or not the observed distributions of first ranked galaxies in groups and clusters follow the functional forms predicted by extreme value statistics. A good fit to these theoretical predictions would lend strong support to the claim that first ranked galaxies are not special, whereas a failure to observe the extreme value form in their distributions would imply that they are a special population of objects.

3 The brightest members of rich clusters

The data used for the analysis of first ranked cluster galaxies is from table 1 of Hoessel *et al.* (1980, HGT). They give absolute visual intrinsic magnitudes for the brightest galaxies in a complete sample of 116 nearby Abell clusters and their photometric data is self-consistent to 0.04 mag. The mean absolute magnitude for this data set is $\langle M_1 \rangle = -22.68$ mag with a dispersion $\sigma(M_1) = 0.35$ mag.

In Fig 1, we have plotted a histogram showing the magnitude distribution of these 116 galaxies. We compare this distribution of M_1 with the Fisher–Tippett form to test the statistical hypothesis. Rewriting equation (11), combined with equation (15), we get

$$\frac{dP}{dM_1} = \exp\left\{a\left(M_1 - \langle M_1 \rangle - \frac{0.577}{a}\right) - \exp\left[a\left(M_1 - \langle M_1 \rangle - \frac{0.577}{a}\right)\right]\right\}. \tag{35}$$

The only free parameter in this equation is a. The value of $\langle M_1 \rangle$ is determined by the observations. We have plotted equation (35) for three values of 'a' as shown in Fig. 1. The constant a is a measure of the steepness of the luminosity function at the bright end of galaxy

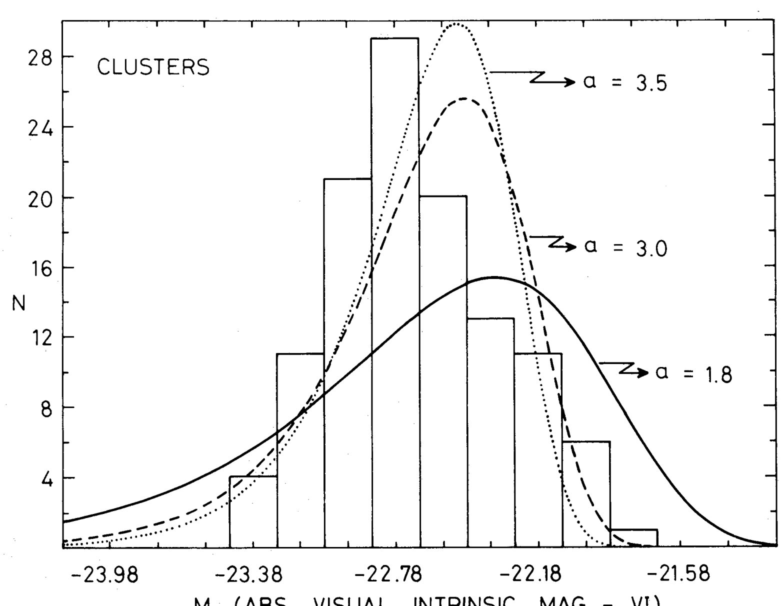

Figure 1. Histogram of frequencies of absolute intrinsic magnitudes, M_1, for 116 first ranked cluster galaxies (Hoesell *et al.* 1980). The data has mean $\langle M_1 \rangle = -22.68$ mag. The curves display the Fisher–Tippett extreme value distributions (equation 35) for three values of $a = 1.8$, 3.0, 3.5. The value $a = 1.8$ is consistent with determinations of the luminosity function at the bright end for rich clusters (see text for discussion).

luminosities. A larger a corresponds to a steeper luminosity function and so a smaller dispersion in the distribution of M_1.

The value of a=1.8 used by Peebles (1968), which is also consistent with later determinations of the luminosity function at the bright end for rich clusters (e.g. see fig. 2 of Schechter 1976), clearly does not describe the distribution of M_1. Peebles (1969) has pointed out that since the luminosity function is not well determined at the bright end a larger value of a would be consistent with observed dispersions in the value of M_1 and the statistical hypothesis could not be ruled out. Sandage (1976) comes to the conclusion that a value of a=5.5 is required to explain another observation: the constancy of M_1 with cluster richness. In Fig. 1 we have shown therefore two additional plots of equation (35) with a=3.0 and a=3.5. A χ^2 test shows that all three curves in Fig. 1 are inconsistent with the data at the 99 per cent confidence level. For larger values of a the fit to the histogram gets even worse.

A Kolmogorov–Smirnov (K–S) test rejects a fit of the data by an extreme value distribution of Fisher–Tippett type at the 99 per cent level for $a \lesssim 2$ and at the 97 per cent level for $a \gtrsim 2$.

If, on the other hand, these galaxies were drawn from some standard mould we would expect their luminosities (rather than their magnitudes) to be distributed as a Gaussian. This would result in the following distribution function for the magnitudes,

$$f(M_1)=\frac{0.92}{\sigma_L\sqrt{2\pi}}\exp(-\mu)\exp\left[-\frac{(10^{-\mu}-\langle L\rangle)^2}{2\sigma_L^2}\right] \tag{36}$$

where

$$\mu\equiv 0.4\,(M_1-\langle M_1\rangle) \tag{37}$$

and $\langle L\rangle$ is the observed mean luminosity in units of $L^*=10^{-0.4\langle M_1\rangle}$ and σ_L is the dispersion in the same units. The data give $\langle L\rangle$=1.046 and σ_L=0.32. In Fig. 2 the distribution (36) is plotted on a smoothed histogram of the data, as in Fig. 1. The K–S test confirms the goodness of fit and would reject it at only the 5 per cent level.

We have also tested the data of Schneider *et al.* (1983, SGH) who give values of M_1, M_2 and M_3 for 83 intermediate distance Abell clusters to see if it fits expected extreme value distributions. Applying the K–S test to the distribution of M_1 shows that an extreme value

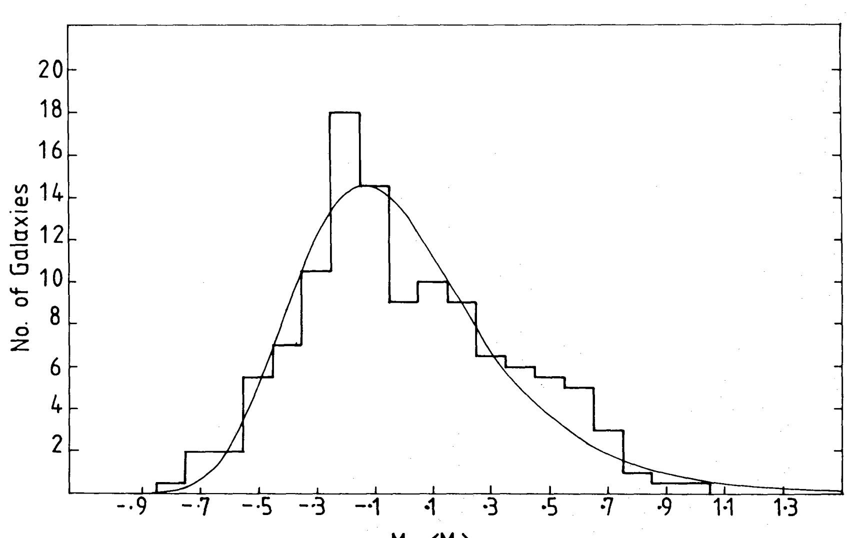

Figure 2. Smoothed histogram of the data displayed in Fig. 1. The curve illustrates the expected magnitude distribution if the luminosities are Gaussian with the observed mean and dispersion.

distribution with $a=3.95$ (best fit) is marginally consistent with the data, but a two-sample K–S test shows that the SGH values for M_1 are also consistent with the HGT sample to a high level of confidence. A K–S test rejects the hypothesis that the second ranked galaxies in the SGH sample (M_2) follow an extreme value distribution at the 95 per cent level around $a=2.75\pm0.3$ and at more than 99 per cent for all other values of a. A two-sample test shows that the distribution of M_2 is inconsistent with the HGT sample at more than 90 per cent confidence.

A χ^2 test rules out both M_1 and M_2 SGH distributions as following an extreme value distribution, at least at the 95 per cent level for all a.

4 Brightest members of groups

The data for 'loose' groups was supplied by Geller & Postman (private communication), and gives the distribution of M_1 for 24 groups. The redshifts for the galaxies in these groups were obtained from the CfA catalogue, and are corrected for our Galaxy's infall into Virgo. The groups are a statistical subset from the catalogue of Geller & Huchra (1983) for which unique infall solutions into Virgo could be obtained. A distance-limiting procedure was supplied to avoid biasing M_{12} (see Geller & Postman 1983). After these procedures, most first and second ranked galaxies in these groups have 10 per cent photometry.

The histogram in Fig. 3 shows the distribution of absolute $B(0)$ magnitudes for M_1 in the 24 groups. The histogram is compared to the Fisher–Tippett form given by (35). The observed $\langle M_1\rangle$ is -19.88 (Geller & Postman, private communication) and is used to determine $\langle M_1\rangle$ in (35). A value of $a=1.44$ achieves the best K–S test fit of (35) with the data and is consistent with the luminosity function for group galaxies (see Fig. 1 in Turner & Gott 1976). The K–S test rejects the fit at only 8.6 per cent confidence (all values of a outside the range $0.7<a<2.6$ are rejected with 90 per cent confidence). This test is preferred by us for the best fit to this small data set because it does not depend on the binning of the data. A χ^2 test shows that the fit to (35) is consistent with the data being drawn from the parent distribution plotted by the curve.

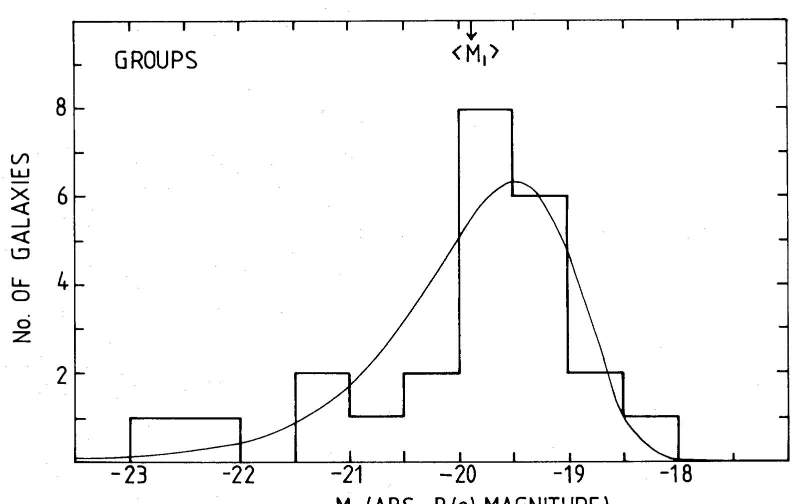

Figure 3. Histogram of frequencies of absolute $B(0)$ magnitudes (corrected for Virgo infall) for 24 first ranked galaxies in groups. This data is a subset of the CfA group catalogue and are groups for which no M_{12} bias is known and has mean $\langle M_1\rangle=-19.89$. Also shown is the best-fit Fisher–Tippett extreme value distribution with $a=1.44$.

Table 1. Comparison of theoretical extreme value parameters with data.

	$\langle M_1\rangle$	$\sigma(M_1)$	$\beta(M_1)$	$\langle M_{12}\rangle$	$\sigma(M_{12})$	T_1 (eqn 26)	T_2 (eqn 27)
Extreme value theory (eqn 35)	–	$\pi/a\sqrt{6}$	μ_3/σ^3	$1/a$	$1/a$	$\pi/\sqrt{6}$	$1/\sqrt{0.677}$
for a=1.44	–	0.89	−1.14	0.69	0.69	1.282	1.215
Samples from Geller–Postman group data							
Total (24)★	−19.885	1.02	−1.15	0.99	1.12	1.03	1.38
Excluding possible discrepant group (23)★	−19.767	0.86	−1.00	0.80	0.69	1.08	1.05
Groups with ⩾ 5 members (15)★	−20.01	0.82	−1.38	0.75	0.68	1.21	1.10
Groups with ⩽5 members (13)★	−19.51	0.76	−0.64	1.06	0.71	0.72	0.81

★number of groups in sample

Table 1 shows the comparison of theoretical values for the dispersion $\sigma(M_1)$, skewness $\beta(M_1)$, $\langle M_{12}\rangle$, $\sigma(M_{12})$, T_1 and T_2 [equations (16), (18), (25), (31) and (32) respectively], with the observations. The first row in Table 1 gives the theoretical values for these quantities, the second row the corresponding numerical values when a=1.44, (β, T_1 and T_2 are independent of a). The third row of numbers gives the observed values for the full data set of the 24 first ranked galaxies. The fourth row gives the same values when one of the groups has been removed from the total sample. This group has an unusually large value for M_{12} (5.29 mag). The chance of M_{12} lying between 5.0 and 5.5 mag in a sample of 24 galaxies with the observed extreme value distribution is only 8×10^{-3}. Therefore, we feel justified in removing this possibility discrepant group from the sample. The values of the statistical parameters, once this group has been removed, fit the theory quite well. The best-fit extreme-value curve has a=1.50.

This data for M_1 is from groups of different sizes. Equation (35) is a valid first approximation to the distribution of M_1. The exact distribution would involve a convolution of equation (35) with the group multiplicity function. This involves introducing two more parameters (the 'slope' and characteristic M^* for the multiplicity function used) and is not a worthwhile exercise in view of our small data set. The freedom allowed is too great and a better 'fit' than obtained with equation (35) would not give us any further insight.

It is more instructive to check for the quantitative trends that we would expect if the first ranked galaxies were indeed statistical extremes derived from a parent sample of varying size. To check this in its simplest form we have divided the data of the 23 galaxies into two subsets. Sample G1 of groups with ⩾5 members and sample G2 of groups with ⩽5 members. The groups with 5 members are included in both samples as the division into these two subsamples is quite arbitrary and chosen so as to get more or less equal numbers of data points in each sample. When histograms of G1 and G2 are plotted they separate distinctly into two distributions. The shift in $\langle M_1\rangle$ has the right direction and the other statistical parameters shown in rows 5 and 6 of Table 1 for G1 and G2 respectively can be compared with the theory. Furthermore, a K–S test shows that each subsample is consistent with an extreme value distribution, with best fits obtained at $a=1.78$ for G1 and $a=1.61$ for G2. Levels of consistency of these fits are as good as that for the total sample. The trend in the value of a, for the samples G1 and G2 individually and the composite sample (for which $a=1.50$) is also in the right direction since we would expect the curve to get stretched and flattened due to the superposition of several extreme value distributions displaced slightly from each other. Thus the best-fit values obtained for a are all lower limits and a value of $a=1.8$ consistent also with the luminosity function of galaxies seems consistent also with the distribution of M_1.

Let us also quantify the shift in $\langle M_1 \rangle$ when the size of the parent population from which M_1 is drawn varies. Theoretically, for a larger parent sample with the same underlying distribution (luminosity function in this case), equation (35) maintains its form but shifts towards larger negative values of M_1 proportional to $\ln N$, where N is the number of members in the parent sample. Thus if ΔM is the shift in $\langle M_1 \rangle$ and the two distributions of M_1 come from parent samples of size N_1 and N_2 then

$$\Delta M = \frac{1}{a} \ln \frac{N_1}{N_2}. \quad (38)$$

Thus the ratio of the sizes of the parent populations is given by $\exp(a\Delta M)$. Taking as representative the two largest groups of 248 members (Virgo) and 170 members (the third largest group has 23 members) and the five smallest groups of three members each, we find the following: $\langle M_1 \rangle_{\text{largest}} = -21.64$, $\langle M_1 \rangle_{\text{smallest}} = -19.31$; therefore $\Delta M = 2.33$ mag; for $a=1.8$ this corresponds to a ratio of 66 in the sizes of the two extreme parent populations. The actual ratio in the number of members in the two samples is $\langle 248+170 \rangle /3 = 209/3 = 69$.

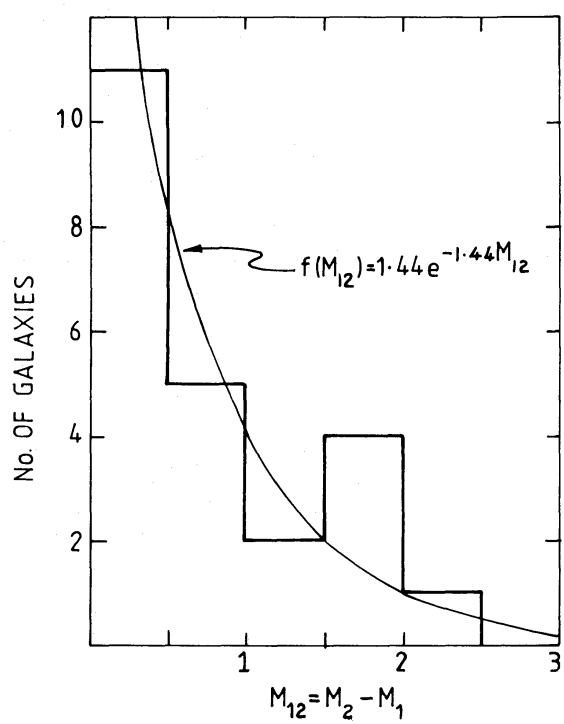

Figure 4. Histogram of frequency of $M_{12}=M_2-M_1$ for the 24 groups in the data set of Fig. 3 (excluding one point at $M_{12}=5.29$ which lies outside the frame and involves the brightest galaxy, $M_1=-22.68$ in Fig. 3. The continuous curve is the Fisher–Tippett asymptote for $a=1.44$ corresponding to the M_1 extreme curve of Fig. 3.

We also compared the observed distribution of M_{12} for groups with the exponential form predicted by the extreme value statistic, (23), which is

$$f(M_{12})=1.44\exp(-1.44\,M_{12}). \tag{39}$$

This is plotted on the data in Fig. 4.

We conclude that the brightest members of groups are consistent with having been drawn from the extreme end of a statistical distribution of galaxies. The fact that first ranked group members are consistent with a statistical hypothesis makes them useful as distance estimators as suggested by Schechter & Press (1976).

5 Conclusions

The distribution of extremes of a large number of subsamples drawn from a single population has, in general, a universal asymptotic probability distribution which is independent of the underlying population. Using this result we have investigated whether the first ranked members of galaxy clusters and groups possess these predicted universal distributions. The distributions are uniquely specified by the mean value of the brightest members and are positively skewed.

The distribution of magnitudes of first ranked galaxies in clusters rules out the hypothesis that they are the extremes of a statistical distribution at 95–99 per cent confidence (depending on the slope of the luminosity function). Even if the slope of the luminosity function increases at the bright end, as suggested by Peebles (1969), the distribution of M_1 is inconsistent with a statistical hypothesis, as is the distribution of M_2. The magnitudes of first ranked cluster galaxies (Hoessel *et al.* 1980) are consistent with having been drawn from a Gaussian distribution of luminosities with a dispersion equal to 0.32 of the mean luminosity. The first ranked members of groups are found to be consistent with the hypothesis that they are extreme members of a statistical population of galaxies. Our results are derived using $B(0)$ magnitudes; [for a discussion of possible colour-dependent biases see Geller & Postman (1983)]. We also find that the distribution of M_{12} in groups follow the predicted universal extreme value distribution.

These results support the idea that first ranked cluster members are an independent population of objects or have been influenced by evolutionary effects like mergers and weigh strongly against the claim that they are statistical extremes. In contrast, they support the hypothesis that the first and second ranked galaxies of the loose groups we studied are simply statistical extremes and have not been strongly influenced by evolution or cannibalism.

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References

David, H. A., 1981. *Order Statistics,* 2nd edn, John Wiley, New York.
Fisher, R. A. & Tippett, L. H. C., 1928. *Proc. Camb. Phil. Soc.,* **24,** 180.
Geller, M. J. & Huchra, J. P., 1983. *Astrophys. J. Suppl.,* **52,** 61.
Geller, M. J. & Peebles, P. J. E., 1976. *Astrophys. J.,* **206,** 939.

Geller, M. J. & Postman, M., 1983. *Astrophys. J.*, **274**, 31.
Gnedenko, B. V., 1941. *Ann. Math.*, **44**, 423.
Gumbel, E. J., 1966. *Statistics of Extremes,* Columbia University Press, New York.
Hausman, M. A. & Ostriker, J. P., 1978. *Astrophys. J.*, **224**, 320.
Hoessel, J. G., Gunn, J. E. & Thuan, T. X., 1980. *Astrophys. J.*, **241**, 486.
Humason, M. L., Mayall, N. U. & Sandage, A., 1956. *Astr. J.*, **61**, 97.
Ostriker, J. P. & Tremaine, S. D., 1975. *Astrophys. J. Lett.*, **202**, L113.
Peach, J. V., 1969. *Nature,* **223**, 1140.
Peebles, P. J. E., 1968. *Astrophys. J.*, **153**, 13.
Peebles, P. J. E., 1969. *Nature,* **224**, 1093.
Peterson, B. A., 1970. *Astrophys. J.*, **159**, 333.
Sandage, A., 1972. *Astrophys. J.*, **178**, 1.
Sandage, A. & Hardy, E., 1973. *Astrophys. J.*, **183**, 743.
Sandage, A., 1976. *Astrophys. J.*, **205**, 6.
Schechter, P., 1976. *Astrophys. J.*, **203**, 297.
Schechter, P. & Press, W. H., 1976. *Astrophys. J.*, **203**, 557.
Schneider, D. P., Gunn, J. E. & Hoessel, J. G., 1983. *Astrophys. J.*, **264**, 337.
Scott, E. L., 1957. *Astr. J.*, **62**, 248.
Tremaine, S. D., 1981. *The Structure and Evolution of Normal Galaxies,* p. 67, eds Fall, S. M. & Lyndell-Bell, D., Cambridge University Press, Cambridge.
Tremaine, S. D. & Richstone, D. O., 1977. *Astrophys. J.*, **212**, 311.
Turner, E. L. & Gott, J. R., 1976. *Astrophys. J.*, **209**, 1.