1994MNRAS.269..455J

Mon. Not. R. Astron. Soc. **269,** 455–461 (1994)

Magnetic evolution of neutron stars in wide low-mass binary systems

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Accepted 1994 February 23. Received 1994 February 15; in original form 1993 November 30

ABSTRACT

The low magnetic field strengths ($\sim 10^8$ G) of millisecond pulsars could be due either to their progenitor neutron stars being born with such low fields or to a decay of their field strength in the course of evolution. One of the mechanisms envisaged for a decrease in field strength is a slow-down-induced field decay. In this scenario, the interpinning of magnetic fluxoids in the proton superconductor with the angular-momentum-carrying vortex lines in the neutron superfluid in the stellar core results in the expulsion of the magnetic flux out to the crust when neutron vortices are forced to migrate outwards due to the spinning-down of the neutron star. Once deposited in the crust, the field may then decay due to ohmic dissipation. If this is indeed true, then the millisecond pulsars, commonly believed to have been recycled in low-mass X-ray binaries, must have had their spin periods increased to very large values ($\gtrsim 1000$ s) before being spun-up to millisecond periods. To verify the link between the magnetic field evolution and the rotational history of a neutron star, we examine the evolution of a neutron star in a low-mass binary, due to the interaction of its magnetosphere with the stellar wind of the companion. Models for a range of orbital periods, donor masses and mass-loss rates, and certain assumed ohmic decay time-scales in the crust are constructed, and the final surface magnetic field strengths obtained in these cases are presented in this paper. It is concluded that the low magnetic fields of millisecond pulsars may be well accounted for by this mechanism under fairly reasonable circumstances. The models seem to indicate further that an asymptotic value of $\sim 10^8$ G is the lowest possible value for the field obtainable by this mechanism.

Key words: magnetic fields – binaries: general – stars: neutron – pulsars: general.

1 INTRODUCTION

The low magnetic field strengths of binary pulsars, millisecond pulsars and pulsars in globular clusters, all believed to be old neutron stars, strongly support the idea of magnetic field decay in neutron stars. On the other hand, the present field strengths ($\gtrsim 10^8$ G) of these objects seem to be stable (Bhattacharya & Srinivasan 1986; Kulkarni 1986; van den Heuvel, van Paradijs & Taam 1986), ruling out a simple spontaneous ohmic decay as the cause for field reduction. An exponential decay with a time constant of the order of 10^6 yr, for example, would result in negligibly small values for the strengths of magnetic fields in neutron stars older than $\sim 10^9$ yr. Most of these short-period, low-magnetic-field pulsars are believed to be old neutron stars 'recycled' (spun-up during a Roche-lobe overflow mass transfer) in low-mass X-ray binaries (LMXBs) (Radhakrishnan & Srinivasan 1981; Alpar et al. 1982).

According to the standard picture, the evolutionary history of neutron stars in close binaries may be divided into three successive phases (Pringle & Rees 1972; Illarionov & Sunyaev 1975):

(1) the obscured radio pulsar phase, in which the radio emission from a young pulsar is absorbed in the surrounding plasma due to the stellar wind of the companion, while radiation pressure keeps the plasma away from the neutron star magnetosphere;

(2) the slow-down or deceleration phase, in which the magnetosphere of the neutron star acts as a 'propeller', ejecting the infalling accreted wind matter, and

(3) the accretion or spin-up phase, in which the accreted matter falls on to the surface of the neutron star, releasing its gravitational energy. A heavy mass transfer ensues due to Roche-lobe overflow, and the binary starts its new life as a bright X-ray source.

The strength of the dipolar magnetic field of the neutron star is obviously one of the most sensitive factors that decide the phase of the binary evolution. On the other hand, the evolution or the decay of the magnetic field of a neutron star itself seems to be closely linked with its past history in a binary system (Taam & van den Heuvel 1986; Bailes 1989; Srinivasan et al. 1990). However, such a link between the binary evolution and the magnetic field strength has not until recently been explicitly incorporated in any model of binary evolution.

One attempt to account for the decay of the magnetic field of a neutron star in terms of its binary evolution history proposes (Taam & van den Heuvel 1986; Romani 1990) that it is the total amount of the accreted matter on to the star that will set the final strength of its magnetic field.

A second suggestion (Srinivasan et al. 1990) calls upon the spinning-down of the star as the major underlying cause for the decay of its magnetic field. According to this model the flux originally resides in the superconducting core of the star, where no ohmic dissipation can occur. Eventually, however, the interpinning of the fluxoids in the proton superconductor (carrying the magnetic flux) and the vortices in the neutron superfluid (carrying the angular momentum) in the core causes the fluxoids to be carried and deposited in the crust as the neutron star spins down and the vortices decrease in number by migrating radially outwards. The magnetic flux deposited in the crust will then decay due to ohmic dissipation there. The surface magnetic field of the star will therefore keep decreasing until it approaches the value of the residual field in the core.

This model predicts that, while the fields of the isolated neutron stars which have only a small rate of slowing down may not decay significantly, those of the first-born neutron stars in binaries would decay substantially because of the spin-down to the very long periods that can occur in these systems (Bhattacharya & Srinivasan 1991; Srinivasan 1991). The maximum spin period to which the neutron star is slowed down during the 'propeller' phase will uniquely determine the final value of the residual magnetic field in the recycled pulsar that will be born at the end of the accretion phase of the binary (Srinivasan et al. 1990). While the above evolutionary scenario appears attractive, the literature so far contains only qualitative descriptions of such an evolution.

Somewhat more detailed treatment of a related model has recently been performed by Ding, Cheng & Chau (1993). They consider only the case of a spin-down due to dipole radiation, and arrive at low surface field strengths of $\sim 10^8$ G for old isolated neutron stars, as a consequence of the assumption that neutron stars are born with very short spin periods $\sim 10^{-3}$ s. However, it is not at all clear that neutron stars are indeed born with such short periods. For the Crab pulsar the estimated initial period is ~ 20 ms (Trimble & Rees 1970). Absence of a large number of bright centre-filled supernova remnants indicates that most pulsars are born with spin periods greatly exceeding ~ 20 ms (Srinivasan, Bhattacharya & Dwarakanath 1984). Statistical studies of pulsar population have also found evidence for the majority of pulsars being born with periods exceeding ~ 100 ms (Vivekanand & Narayan 1981; Chevalier & Emmering 1986; Narayan 1987; Emmering & Chevalier 1989; Narayan & Ostriker 1990). Therefore the net flux expulsion in an isolated neutron star is expected to be much less than that computed by Ding et al. (1993). In any case, observations seem to suggest that the magnetic field strengths of isolated neutron stars do not decay significantly even in time-scales comparable to the age of the galaxy (Bailes 1989; Bhattacharya & van den Heuvel 1991; Srinivasan 1991; van den Heuvel 1991; Bhattacharya et al. 1992).

In this paper, we explore this problem quantitatively, and ask under what conditions such an evolution would best reproduce the observed properties of young as well as old neutron stars. We consider a simple model for the evolution of neutron stars in wide low-mass binaries, taking into account the spin-down-induced flux expulsion and several possible values for the ohmic decay time-scale in the crust. As is borne out by our calculations, the flux expulsion occurs entirely during the detached phase of the binaries. In wide low-mass binaries (with orbital periods of more than a day) the duration of the detached phase is nearly equal to the main-sequence lifetime of the donor. In this paper, we confine ourselves to only such systems, so as to avoid computing the duration of the detached phase resulting from the highly uncertain degree of magnetic braking in binaries with shorter periods. In Section 2, we describe in detail our adopted model, and our results are discussed in Section 3.

2 MODEL DESCRIPTION

We consider models for the orbital and spin evolution of a newborn neutron star in a binary (orbital period P_{orb}) with a low-mass main-sequence star of mass M_2 which loses mass in the form of a uniform and homogeneous stellar wind with a rate of $\dot{M}$. In the study of X-ray binaries, accretion from the stellar wind of the companion is usually invoked only in the case of high-mass X-ray binaries (HMXBs) (van den Heuvel 1981; Henrichs 1983). This is so because, in the case of low-mass binaries, wind accretion will not be able to produce a perceptible X-ray emission. This feeble stellar wind in low-mass binaries is, nevertheless, the key factor responsible for the spinning-down of the neutron star. The long main-sequence lifetime of the low-mass donor allows for the long (up to a few times 10^9 yr) spin-down (dipole and propeller) phases. In view of the lack of observational data as well as theoretical predictions of mass-loss rates of dwarf main-sequence stars (see, e.g., Dupree 1986; de Jager, Nieuwenhuijzen & van der Hucht 1988; Chiosi, Bertelli & Bressan 1992), we use values both larger and smaller than the observed mass-loss rate of the Sun, namely 2×10^{-14} $\mathrm{M}_\odot$ yr^{-1} (Cassinelli 1979).

In the picture of spin-down-induced field expulsion, the evolution of the spin period and the magnetic field of the neutron star in such a binary would be intimately coupled. While the spin-down process would tend to reduce the field strength, the reduced field strength (together with the increased spin period) will, in turn, affect the rate and the direction of the spin variations. We follow this coupled evolution of the surface magnetic field and the spin period of the neutron star for a period of 10^{10} yr, the expected main-sequence lifetime of the low-mass donor.

We assume that the first phase of the evolution, namely the pulsar phase with a dipole spin-down (during which the neutron star spins down due only to the dipolar radiation torque on it), lasts until the ram pressure of the stellar wind overcomes the pressure of the 'pulsar wind' at the accretion

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radius (see, e.g., Illarionov & Sunyaev 1975; Davis & Pringle 1981). During this period the stellar wind will have no dynamical effect except for the losses from the companion star. The pulsar's core magnetic field will undergo an expulsion corresponding to the dipole spin-down rate. In the subsequent phases where the accreted wind matter interacts directly with the magnetosphere, we assume that a steady Keplerian disc is formed by the accretion flow outside the magnetosphere, with the same sense of rotation as that of the neutron star. This is the least efficient configuration for angular momentum extraction from the neutron star, while more efficient geometries, such as a spherically symmetric radial infall may well occur for the low mass-accretion rates appropriate to these objects (Hunt 1971; Illarionov & Sunyaev 1975; Wang 1981). Therefore we also explore a few cases of radial infall on the neutron star, for comparison.

We note, however, that the sign, magnitude and time-dependence of the accretion torque produced by capture of plasma from a stellar wind is a complex problem that has only partially been solved (Ghosh & Lamb 1991), and the outcome depends on the detailed properties of the flow at the capture radius, which determine the feasibility of formation of an accretion disc (Ho & Arons 1987; Taam, Fu & Fryxell 1991). Also, a second difficulty lies in the fact that, even in the case where a disc is formed, uncertainties in the nature of the interaction of the magnetosphere of the neutron star with the wind plasma have led to widely different predictions for the spin-down torque on the neutron star (Holloway, Kundt & Wang 1978; Kundt 1990; Ghosh & Lamb 1991).

Confronted with the need for a simple and specific model for the spin evolution of the neutron star, we have decided to adopt the following description drawn from the models commonly encountered in the literature (see, e.g., Pringle & Rees 1972; Illarionov & Sunyaev 1975). The accretion flow interacts with the magnetosphere at the so-called Alfvén radius, R_A:

$$R_A = 7.03 \times 10^{-8}\ \mathrm{R}_\odot \left(\frac{B_{s_8}^2}{\dot{M}_{\rm acc}}\right)^{2/7}, \tag{1}$$

where B_{s_8} denotes the surface field strength in units of 10^8 G and $\dot{M}_{\rm acc}$ the rate of capture of wind matter in solar masses per year (Davidson & Ostriker 1973). This interaction spins the neutron star up or down depending on the sign of the quantity $V_{\rm dif}(=V_{\rm co}-V_{\rm kep})$, evaluated at the radius of the magnetosphere. Here $V_{\rm co}$ is the speed of corotation with the neutron star at a given distance from it, and $V_{\rm kep}$ is the Keplerian speed at the same distance. The boundary is determined by the condition of balance between the magnetic pressure and the ram pressure of the infalling flow. In the limiting case, when the corotation velocity $V_{\rm co}$ becomes equal to the Keplerian velocity $V_{\rm kep}$, the neutron star will conserve its spin period while accretion on to the star will continue. The rate of extraction of spin angular momentum $\dot{L}$ is assumed to be:

$$\dot{L} = \xi V_{\rm dif} R_A \dot{M}_{\rm acc}, \tag{2}$$

where ξ is an efficiency factor, included to take into account the uncertainties due to the detailed geometry of the interaction and the actual value of the specific angular momentum carried by the accreted wind just before and after the interaction. The amount of spin angular momentum added to or extracted from the neutron star (and loss in mass and angular momentum of the system) is calculated by assuming that:

(i) in the 'propeller' phase all of the accreted matter is expelled by the magnetosphere and leaves the system at the Alfvén radius with no further interaction, carrying a total orbital angular momentum corresponding to its corotation with the neutron star, as is the case for a 'weak' wind (Holloway et al. 1978), and

(ii) in the accretion phase all of the accreted matter enters the magnetosphere, carrying with it only the excess spin angular momentum due to the difference between the Keplerian and corotation angular velocities.

The fraction of the stellar wind that does not enter the accretion flow towards the neutron star is assumed to escape from the binary system with a specific angular momentum equal to that of the secondary star. The coefficients α and β in equation (5) are computed based on these assumptions.

The spin-up of the neutron star has no effect on the further evolution of its magnetic field in our calculations, and the above relation for angular momentum transfer has been used to ensure the occurrence of an equilibrium spin period, without resorting to a detailed and complicated modelling (as in, say, Ghosh, Lamb & Pethick 1977). The residual magnetic field B_c, which corresponds to the amount of magnetic flux still trapped in the stellar core, is assumed to decrease in proportion to an increase in the spin period P_s, instantaneously. The surface field B_s then approaches B_c exponentially with a constant decay time-scale τ. At the start of the evolution, B_c and B_s are set to be equal.

The assumption of instantaneous expulsion of the magnetic flux in response to an increase in the spin period of the star is, of course, viable only when flux can move freely across the core–crust interface. It has, however, been pointed out by Jones (1987) that, once the flux density in the solid reaches a value of $\gtrsim H_{c1}$, the lower critical field of the proton superconductor, further transport of flux across the boundary is hindered, and a layer of high fluxoid density builds up just below the border. From then on, flux is released into the solid on the same time-scale with which flux transport occurs in the solid crust – by ohmic diffusion, Hall transport or plastic flow.

Estimates of the mechanical strength of the crust suggest that plastic flow might become the major means of flux transport well before the field in the solid builds up to H_{c1} (Ruderman 1991b). The critical strain at the bottom of the crust, estimated to be $\lesssim 10^{-3}$ (Smoluchowski 1970; Baym & Pines 1971; Ruderman 1991a), implies that the maximum magnetic stress that can be sustained in this region is several orders of magnitude less than $H_{c1}^2/8\pi$ (Jones 1987). As long as the time-scale for spin-down is longer than that for plastic flow, flux will thus move more or less freely across the core–crust interface. If the plastic flow time-scale is $\lesssim 10^6$ yr, as estimated by Jones (1987), nearly all of our models will operate in this regime and hence the assumed instantaneous response could be justified.

Nevertheless, since uncertainties in the above numbers are large, we have also examined a few models in which the flux flow across the interface is determined by ohmic diffusion of flux in the solid, and the time-scale of flux transfer from superfluid to the solid is set equal to the assumed ohmic time-scale right from the beginning of evolution (this somewhat overestimates the effect of the hindrance to flux expulsion), in order to gauge the impact of this on our results.

It might also happen that the building up of the fluxoid layer just below the crust reacts with the motion of fluxoids from the interior, perhaps reducing the effectiveness of the pinning interaction between neutron vortices and proton fluxoids. However, no quantitative estimate of this is yet available in the literature to enable us to incorporate it into our models. If this effect is severe, it might not be possible in this scenario to achieve a reduction in the surface magnetic field to values comparable to those of millisecond pulsars.

In the case of 'wide' low-mass binaries, which we are primarily interested in here, the orbital separation a of the binary remains almost constant during the wind phases, since the angular momentum losses due to the gravitational radiation and magnetic braking are very inefficient. However, we do take into account the effects of angular momentum loss as well as the loss and exchange of matter in computing the orbital evolution during this phase.

The accretion radius $R_{\rm acc}$ and the rate of mass accretion $\dot{M}_{\rm acc}$ from the wind are computed following the standard procedure for the case of a supersonic flow in the hydrodynamical approximation (Bondi & Hoyle 1944; Alcock & Illarionov 1980; Ghosh & Lamb 1991):

$$R_{\rm acc} = 3.83 \times 10^5\,{\rm R}_\odot \left(\frac{M_{\rm n}}{V^2 + C_{\rm s}^2}\right), \tag{3}$$

$$\dot{M}_{\rm acc} = \left(\frac{R_{\rm acc}}{2a}\right)^2 \left(\frac{V}{V_{\rm w}}\right) \dot{M}_2, \tag{4}$$

where $V = (V_{\rm w}^2 + V_{\rm o}^2)^{1/2}$, $V_{\rm o}$ is the orbital velocity of the neutron star, $V_{\rm w}$ is the velocity of the wind, and $C_{\rm s}$ is the sound velocity in the wind matter far from the star, all in units of km s^{-1}, and $M_{\rm n}$ is the mass of the neutron star in solar units. The accretion rate $\dot{M}_{\rm acc}$ is affected by the variation of a (the orbital separation), and the instantaneous values of $\dot{M}_{\rm acc}$ and $B_{\rm s}$ determine the Alfvén radius $R_{\rm A}$. The coupled differential equations for $M_{\rm n}$, a, $P_{\rm s}$, $B_{\rm c}$ and $B_{\rm s}$ which are listed below (for the last two evolutionary phases) are integrated numerically, using a fourth-order Runge–Kutta scheme, for various sets of parameters.

$$\frac{{\rm d}a}{{\rm d}t} = 2a \left\{ \left.\frac{\dot{J}}{J}\right|_{\rm losses} - \frac{\dot{M}_2}{M_2}\left[1 + (\alpha - 1)\frac{M_2}{M_{\rm n}} - \frac{1}{2}\,\alpha\frac{M_2}{M} - \alpha\beta\frac{M_{\rm n}}{M}\right]\right\} \tag{5}$$

$$\frac{{\rm d}M_{\rm n}}{{\rm d}t} = \begin{cases} \dot{M}_{\rm acc} & \text{accretion phase} \\ 0.0 & \text{propeller phase} \end{cases} \tag{6}$$

$$\frac{{\rm d}P_{\rm s}}{{\rm d}t} = 3.18 \times 10^{-3}\,{\rm s\,yr^{-1}} \left(\frac{\dot{M}_{\rm acc}}{{\rm M_\odot\,yr^{-1}}}\right)\left(\frac{R_{\rm A}}{\rm km}\right)\left(\frac{P_{\rm s}}{\rm s}\right)^2 \times \left(\frac{V_{\rm dif}}{\rm km\,s^{-1}}\right)\xi \tag{7}$$

$$\frac{{\rm d}B_{\rm c}}{{\rm d}t} = \begin{cases} -(\dot{P}_{\rm s}B_{\rm c})/P_{\rm s} & \text{if } \dot{P}_{\rm s} > 0.0 \\ 0.0 & \text{otherwise} \end{cases} \tag{8}$$

$$\frac{{\rm d}B_{\rm s}}{{\rm d}t} = -\frac{(B_{\rm s} - B_{\rm c})}{\tau({\rm yr})}, \tag{9}$$

where J is the orbital angular momentum, $\dot{J}$ is the rate of change in J except for that due to escaping matter from the system, M is the total mass of the binary, α is the ratio of the mass-loss rate from the system to that from the secondary, and β is the ratio of the effective specific angular momentum of the escaping matter to that of the secondary star. α and β are computed according to the assumptions detailed above.

We follow the evolution of the system over a period of 10^{10} yr, which is of the same order as the main-sequence lifetimes of solar-type stars (Schallar et al. 1992).

3 RESULTS AND DISCUSSION

Fig. 1(a) displays the typical evolutionary behaviour observed for the spin period, and the residual and surface magnetic fields of the neutron star in our models. The values of the spin period following its maximum are somewhat less reliable, for the reasons mentioned above. The spin-up phase considered here is only due to the accretion from the stellar wind of the companion, and will be succeeded by a much more enhanced accretion phase when Roche-lobe overflow ensues during the post-main-sequence evolution of the donor. The spin evolutions of neutron stars in binaries with different orbital periods are compared in Fig. 1(b), where the case of a 10^{10}-yr pure dipole phase (that of a solitary pulsar), which gives rise to the flat portion seen in some of the curves in Fig. 2, has also been included. The time that elapses before the transition of the binary into the 'propeller' phase ranges from 10^8 to 3×10^9 yr (larger $P_{\rm orb}$ and/or lower $\dot{M}_2$ results in larger values for the elapsed time) in different models (Fig. 1b). During this period the neutron star is assumed to spin-down at a small rate due only to a dipole radiation torque, similar to an isolated neutron star: $\dot{P}_{\rm s} = 3.15 \times 10^{-16}$ s yr^{-1} $B_{{\rm s}_8}^2/P_{\rm s}$ (s). The propeller phase, on the other hand, lasts for ~0.3–3 times the duration of the dipole phase, depending on the efficiency factor ξ.

The longest spin periods obtained are of the order of a few times 10^4 s, which correspond to final field strengths of ~10^8 G, while spin periods of ~1000 s result in a value of ~10^9 G for the final strength of the surface field. These long spin periods seem reasonable in view of the long periods observed in some X-ray pulsars (the largest known value being ~835 s; see, e.g., Nagase 1989). Fig. 2 shows the final surface field strengths, $B_{\rm s}$, obtained for models with orbital periods of between 1 and 300 d, for the given values of the parameters. These have been selected from a larger sample of computed models which cover the following range of parameters and initial conditions:

$\xi = 0.02, 0.1, 0.2, 0.6, 1.0$

M_2: 0.8, 1.0 (M$_\odot$)

log τ: 7.0, 8.0, 9.0 (yr)

log $\dot{M}_2$: -16, -15, -14, -13 (M$_\odot$ yr^{-1})

initial $P_{\rm s}$: 0.1, 0.4, 1.0 (s)

$V_{\rm w}$: 400, 600, 800 (km s^{-1})

initial $B_{\rm s}$: 10^{12}, 3×10^{12}, 10^{13} (G)

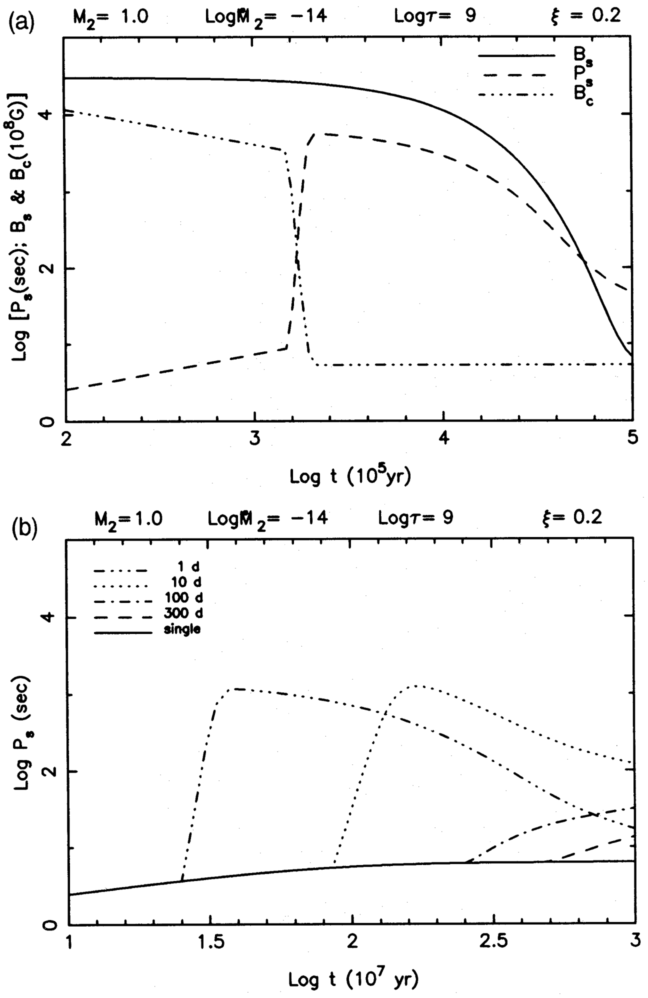

Figure 1. (a) Evolution of the spin period P_s, and the magnetic field strength in the core B_c and at the surface B_s of a neutron star in a low-mass binary, according to the spin-down-induced flux expulsion scenario. M_2 is the companion mass in solar masses, $\dot{M}_2$ is the wind mass-loss rate of the companion star in $M_\odot$ yr^{-1}, τ is the ohmic decay time-scale in the crust in yr, and ξ is an efficiency factor that determines the degree of angular momentum transfer in the magnetosphere-wind interaction. The initial orbital period of the binary has been assumed to be 3 d. (b) Spin evolution of neutron stars in binaries with different initial orbital periods (1, 10, 100 and 300 d), and that of a solitary pulsar. Initial values of $P_s = 0.4$ s, and $B_c = B_s = 10^{12}$ G have been used.

The two sets of initial values used for the spin periods and magnetic field strengths in the models presented in Figs 2(a) and (b) correspond approximately to the extreme values observed in young radio pulsars. We assume that the initial values of spin period and field strength of a neutron star born in a binary are independent of its orbital period, since no mechanism is known so far that would result in a correlation between these quantities.

The final magnetic fields that result from the models presented in Fig. 2 are in the appropriate range for the millisecond and other recycled pulsars. Final field strengths as low as ~10^8 G are, however, obtained only if the ohmic time-scale τ lies in the range $10^{8.5}$–10^9 yr. If τ is lower than this, then the spin-down stops too soon, and not enough flux

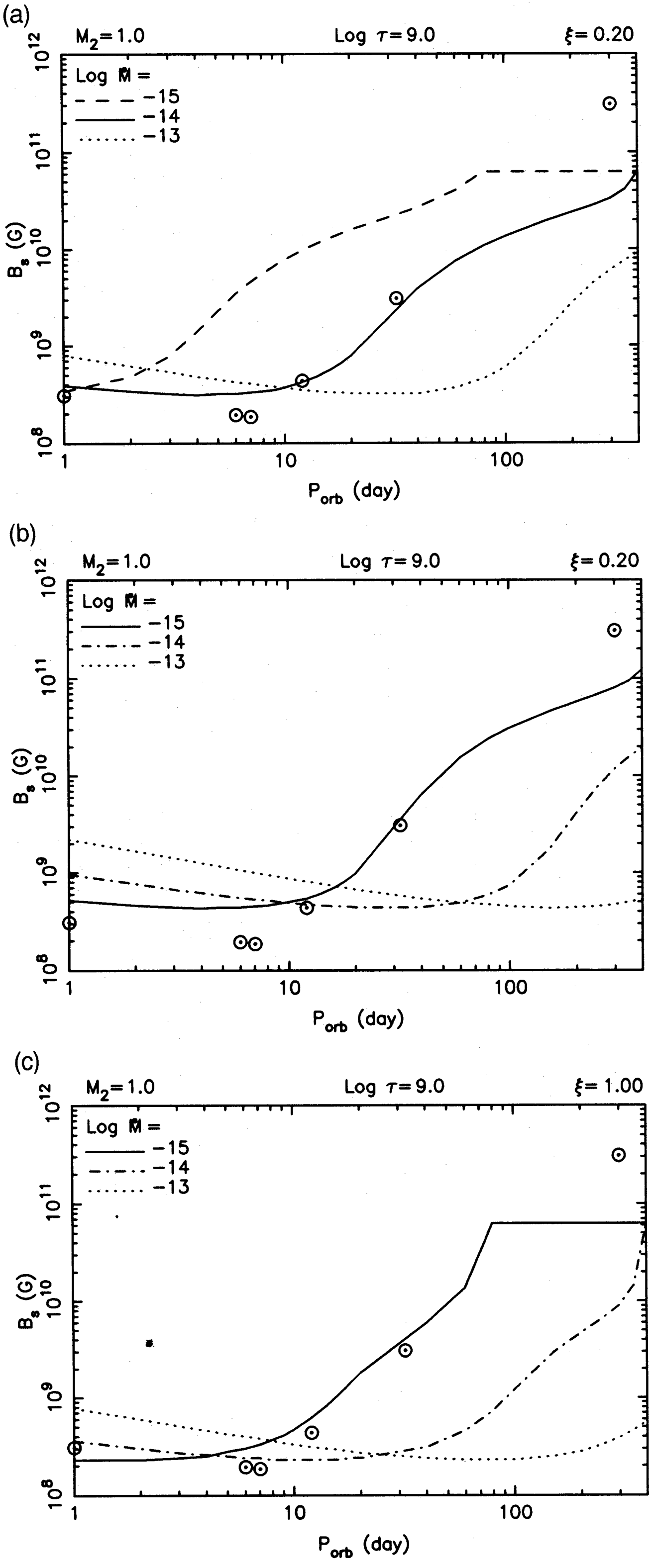

Figure 2. (a) Final values of the surface magnetic field strengths of neutron stars evolved in binaries with various orbital periods P_{orb} and mass-loss rates $\dot{M}$. Initial spin periods and magnetic fields of the neutron stars are assumed to be the same as for Fig. 1(b). Encircled dots represent observed binary radio pulsars that are descendants of wide low-mass binaries, for which the initial orbital periods can be estimated. See the caption of Fig. 1(a) for explanation of legends. (b) As (a), but for initial $P_s = 1.0$ s, and $B_c = B_s = 3 \times 10^{12}$ G. (c) As (a), but for a different efficiency factor, $\xi = 1.0$.

is expelled. For $\tau \gtrsim 10^9$ yr, on the other hand, the surface field does not have time to decay to $\sim 10^8$ G in a Hubble time. It should be noted that values of $\tau \sim 10^9$ yr are shorter than that computed for pure matter at the crust bottom (Baym, Pethick & Pines 1969; Sang & Chanmugam 1987). However, it is by no means clear that the crustal matter of a neutron star is entirely pure, and also it has been shown that even a small amount of impurities and dislocations can drastically reduce the ohmic time-scale (Yakovlev & Urpin 1980; Urpin & Van Riper 1993). Furthermore, the expelled field may eventually undergo a turbulent cascade which would also bring the effective ohmic time-scale down to $\sim 10^8$–10^9 yr (Goldreich & Reisenegger 1992).

Variations in the efficiency factor ξ affect the final results in the same way as the variations in the mass-loss rate, and the best correspondence with observations appears to result from a $\xi \dot{M}_2 \sim 10^{-15}$ $\mathrm{M}_\odot$ yr^{-1}. The variations in final results due to the changes in ξ are, however, not very large; compare Figs 2(a) and (c). This is also indicated by the similar results that we obtain for the case of a radial infall (instead of a Keplerian disc). Therefore, in spite of the fairly crude physical model that has been used, and all the uncertainties in the nature of the captured flow and the details of the magnetospheric interaction with the accreted plasma, the results presented here can be safely used as a good indicator of what is expected from the spin-down-induced magnetic field decay mechanism.

The circles in Fig. 2 represent six observed low-mass binary radio pulsars, which are listed in Table 1 along with some of their properties. These are believed to be the only known descendants of 'wide' LMXBs for which the initial orbital periods can be estimated with reasonable confidence. While the orbital period of such an X-ray binary increases during its final evolutionary phase of Roche-lobe overflow, it remains almost constant throughout its earlier stellar wind mass-transfer phase. The main-sequence lifetime of the donor is therefore spent within a detached binary.

As can be seen from Table 1 and Fig. 2, the surface magnetic field of PSR 0820+02 (3×10^{11} G) is much larger than what is obtained in any of our calculated models, even though the evolution of the model binaries with orbital periods of ~ 300 d is entirely governed by the spin-down due to a dipole torque. Since a value of $\tau = 10^9$ yr is favoured by our results, the large field strength of PSR 0820+02 could be explained if the age of the neutron star is not more than $\sim 4 \times 10^9$ yr, which may happen if either the donor star had a mass somewhat higher than 1 $\mathrm{M}_\odot$ or the neutron star was born in an accretion-induced collapse (AIC) of a white dwarf.

Table 1. The observed low-mass binary pulsars.

PSR	P_s (msec)	P_{orb} (day)	M_2 ($M_\odot$)	$\log B_s$ (G)	initial* P_{orb} (day)
0820+02	864	1232	0.2–0.4	11.48	300
1620−26	11.08	191	0.35	9.48	30
1953+29	6.13	117	0.2–0.4	8.63	12
1855+09	5.36	12.3	0.2–0.4	8.48	1
J1713+0747	4.57	67.83	0.3–0.5	8.28	6
J2019+2425	3.93	76.51	0.3–0.5	8.26	7

References: Bhattacharya & van den Heuvel (1991); Foster, Wolszczan & Camilo (1993); Nice, Taylor & Fruchter (1993).
*As estimated from evolutionary models.

Fig. 2 shows that the adopted model is successful in reproducing the observed magnetic fields of most of the pulsars that are believed to have been recycled in wide low-mass binaries, as well as the apparent relation between the final field strength and the initial orbital period, a possibility that has not been shown to exist in any other scenario.

The model of field decay by mass accretion, which claims to be successful in reproducing the general distribution of the recycled pulsars in the B_s–P_s diagram (Romani 1990), seems to have difficulty in at least one example: the neutron star in 4U 1626−67 has a strong magnetic field of $\sim 10^{12}$ G, although it is expected to have accreted a large amount of mass and hence should have a weak field on that account (Verbunt, Wijers & Burm 1990). However, in the picture presented here the high magnetic field of this neutron star at present seems to be a natural consequence of its evolutionary history. As described by Verbunt et al. (1990), there could have been two evolutionary paths leading to this system. In the first case, the neutron star is formed via AIC of a white dwarf accreting from a ~ 0.02-$\mathrm{M}_\odot$ degenerate donor, in which case obviously the phase of stellar wind interaction is absent and thus the amount of field expulsion is low. In the second case, where the AIC happens due to accretion from a $\gtrsim 0.1$-$\mathrm{M}_\odot$ main-sequence star, our computations show that the neutron star would retain a high magnetic field unless the wind from the donor exceeds a rate of 10^{-14} $\mathrm{M}_\odot$ yr^{-1}, which is unlikely for a star with such a low mass.

As mentioned above, we have also examined cases where the release of the expelled flux from the boundary layer just below the crust to the crustal solid is governed by flux diffusion in the inner crust, which happens over the assumed ohmic time-scale τ. In these cases the delayed release of the expelled flux causes the surface field to remain stronger for a greater length of time, and hence the neutron star can attain a longer spin period. This results in the eventual expulsion of somewhat larger amounts of flux from the interior. Given enough time, the final surface field strengths thus fall below those obtained in our other models. This effect is most pronounced in binaries with longer periods. Therefore, to reproduce the observed trend of surface magnetic field versus orbital period, one needs, in this scenario, smaller ohmic time-scales in the crust. Our calculations show that with $\tau \sim 10^8$ yr one can obtain a correspondence with observations of nearly the same degree as in Fig. 2.

4 CONCLUSIONS

We have explored the effect of spin-down-induced expulsion of magnetic field, and its subsequent decay in a neutron star crust, on the evolution of wide low-mass X-ray binaries. Our results can be summarized as follows.

(1) Magnetic fields of order 10^8–10^9 G, observed in millisecond pulsars, can be obtained under a variety of

circumstances, provided that the ohmic diffusion time-scale τ at the bottom of the crust lies in the range $10^{8.5}$–10^9 yr.

(2) The lowest final surface field strengths B_s achieved in our models are of the order of 10^8 G. This results from the fact that the maximum spin periods acquired by the neutron stars in the course of their binary evolution never exceed a value of $\sim 10^4$ s, irrespective of the initial conditions in the range that we have considered.

(3) Reproduction of the observed trend of final field strengths as a function of the initial orbital period is encouraging, especially in view of the wide range of possible initial conditions that may be obtained in wide LMXBs.

(4) Old, solitary neutron stars are expected to have surface dipole field strengths $\sim 10^{11}$ G.

(5) The transport of flux across the core–crust boundary plays a significant role in determining the final surface field strengths of neutron stars. For example, if this transport occurs on a time-scale similar to the ohmic time-scale in the crust, then the values of τ required to explain observations would be an order of magnitude lower than if the above transport is instantaneous. Extension of the work to the case of very tight orbits ($P_{\rm orb} < 0.5$ d), as well as to binaries with high-mass donor stars, would provide further constraints on this aspect. It will be important to investigate whether the same values of the ohmic time-scale could explain the field strengths of the descendants of both high-mass and low-mass binaries. More detailed and quantitative models of flux transport in the inner crust would be welcome, and will probably be necessary to form a coherent picture of the evolution of the magnetic fields of neutron stars in a wide variety of systems.

ACKNOWLEDGMENTS

We thank E. P. J. van den Heuvel and R. E. Taam for discussions, and G. Srinivasan, V. Radhakrishnan and C. S. Shukre for their comments on the manuscript. We are grateful to an anonymous referee for pointing out the possible importance of the accumulation of magnetic flux in the solid crust and its effects on further flux movement.

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