Surface Tension and the Cosmological Constant

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One of the few predictions from quantum gravity models is Sorkin's observation that the cosmological constant has quantum fluctuations originating in the fundamental discreteness of spacetime at the Planck scale. Here we present a compelling analogy between the cosmological constant of the Universe and the surface tension of fluid membranes. The discreteness of spacetime on the Planck scale translates into the discrete molecular structure of a fluid membrane. We propose an analog quantum gravity experiment which realizes Sorkin's idea in the laboratory. We also notice that the analogy sheds light on the cosmological constant problem, suggesting a mechanism for dynamically generating a vanishingly small cosmological constant. We emphasize the generality of Sorkin's idea and suggest that similar effects occur generically in quantum gravity models.

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One of the outstanding problems of theoretical physics today is quantum gravity. A generic feature of quantum gravity models is an underlying discreteness of spacetime at the Planck scale. Events can be specified only to Planck scale accuracy. Though the smooth manifold model of spacetime works well over a range of length scales, such a picture may not hold at the Planck scale (see remarks by Einstein quoted in Ref. [1]). In studying gravity at the Planck scale, experiments cannot be performed because they are beyond our reach in energy. The nearest one can get to ''experimental quantum gravity'' are laboratory analogs. While condensed matter analogs of classical and semiclassical gravity have been discussed before [2–6], none of these deal with true quantum gravity effects. There have been some attempts at using analog condensed matter systems to address quantum gravity phenomenology [7]. However, these probe violations of local Lorentz invariance which is present in *some* quantum gravity models [8]. A more fundamental property of *all* quantum gravity models is discreteness of spacetime in some form. Here we propose an analog experiment which probes the discreteness of spacetime, a genuine quantum gravity prediction due to Sorkin [9].

Sorkin suggested a quantum gravity explanation for the observed value of the cosmological constant. In this Letter, we develop an analogy between the cosmological constant λ and the surface tension σ of membranes. We first describe the cosmological constant problem in general terms and then show using our analogy that similar problems appear in soft condensed matter physics. The analogy is fairly compelling, and we are able to translate ideas from one context to the other. Let us briefly recount the cosmological constant problem. In general relativity (GR), a spacetime is a pair $(\mathcal{M}, g)$, where $\mathcal{M}$ is a four-dimensional manifold and g a Lorentzian metric. We will refer to $(\mathcal{M}, g)$ as a *history* $\mathcal{H}$. The dynamics of GR is described by the Einstein-Hilbert action $I_2 = c_2 \int d^4x\sqrt{-g}R$, modified by the addition of a cosmological term $I_0 = c_0 \int d^4x\sqrt{-g}$. In standard notation, $c_2 = 1/(16\pi G)$, where G is Newton's constant and c_0 is λ. Usually, higher derivative terms such as $I_4 = c_4 \int d^4x\sqrt{-g}R^2$ are dropped as being negligible. This is entirely in the spirit of effective field theory (or Landau theory in condensed matter physics), where we expect that the low energy physics will be dominated by the lower derivative terms. However, applying this logic consistently, we would expect the cosmological constant term I_0 to dominate over the Einstein-Hilbert term I_2. A crude dimensional analysis would suggest a value for the cosmological constant which is of order 1 in dimensionless units ($G = c = \hbar = 1$). In fact, the observed value of the cosmological constant is practically zero. But not exactly zero. The Planck length $l_{\mathrm{Planck}} = 10^{-33}$ cm serves as a natural unit of length in this problem, and astronomical observations give $\lambda l^4_{\mathrm{Planck}} \approx 10^{-120}$: tiny but nonzero. The dilemma of the cosmological constant thus has two horns [9]: (a) Why is the cosmological constant nearly zero? (b) Why is it not *exactly* zero? It seems hard to come up with a natural explanation for *both* of these facts: One could conceivably construct models (for example, by invoking a symmetry) in which λ exactly vanishes. But why then does it only approximately [10] vanish in the real world?

There is a beautiful idea due to Sorkin [9] that quantum gravity may provide a natural explanation stemming from a fundamental discreteness of spacetime at the Planck scale. Sorkin's proposal for solving the cosmological constant problem was made in the framework of causal sets. In Sorkin's approach of causal set theory, one replaces spacetime by a discrete structure, a collection of points carrying causal relations. The number N of points is associated with the total four-volume $\mathcal{V} = \int d^4x\sqrt{-g} = l^4_{\mathrm{Planck}}N$ of spacetime. More precisely, the region of integration is the causal past of a cosmic observer. The rest of the metrical information of spacetime (the conformal structure) is captured in causal relations between points. Spacetime is regarded as an emergent notion, when the number of points

N gets large. The spacetime four-volume $\mathcal{V}$ also plays a role in unimodular gravity [11,12], which is classically equivalent to GR modified by a cosmological constant. However, unlike in GR, the cosmological constant λ is not a coupling constant but a dynamical variable conjugate to $\mathcal{V}$, which is subject to quantum fluctuations.

Sorkin's proposal addresses part (b) of the cosmological constant dilemma. Let us for the moment suppose that part (a) has been solved: Some mechanism has been found for ensuring that the expectation value $\langle\lambda\rangle$ of the cosmological constant is zero. Sorkin's idea is that there will be fluctuations $\Delta\lambda$ about this mean value which result in a small nonzero cosmological constant $\lambda = \langle\lambda\rangle + \Delta\lambda = \Delta\lambda$. From the uncertainty principle, $\Delta\lambda\Delta\mathcal{V} \approx 1$. We also have $\Delta N \approx \sqrt{N}$. These $\sqrt{N}$ fluctuations are the mechanism for a small and nonzero cosmological constant. Based on this argument, Sorkin predicts [9] the following order of magnitude for the fluctuations in λ: $\Delta\lambda \approx l_{\text{Planck}}^{-4}/\sqrt{N}$. These fluctuations, which have their origin in quantum gravity, are of order $1/\sqrt{N}$, where N is the four-volume of the Universe in Planck units. In this model, the quantum rms fluctuations in the vacuum energy density (λ) are comparable in magnitude to the matter density at *all* epochs [9].

These predictions are consistent [10] with astronomical data (redshift-luminosity distance relations) from type I supernovae: The observations show that the Universe is accelerating at the present epoch, indicative of a positive cosmological constant. Sorkin's argument predicts the correct order of magnitude for λ. Other researchers [13,14] have since taken up this idea with slight variations. In this Letter, we have followed Sorkin's original proposal [9] and treatment.

Let us now turn from GR and the cosmological constant to membranes in soft condensed matter physics. A configuration $\mathcal{C}$ of a membrane is described as a two-dimensional surface Σ embedded in flat three-dimensional space. The surface Σ has [15] extrinsic curvature H and intrinsic curvature K. Note that H has the dimension of inverse length $1/L$, while K has dimension $1/L^2$. To complete this description of a membrane, we need to specify the energy $\mathcal{E}(\mathcal{C})$ of a configuration $\mathcal{C}$. We restrict ourselves to membranes which have two "sides" (orientable) and which are symmetric in their two sides. The latter implies that the energy is invariant under $H \rightarrow -H$. In the spirit of Landau theory, we write down terms with the lowest number of derivatives consistent with the symmetry of the problem [15]: $\mathcal{E}_0 = a_0 \int_\Sigma d^2x\sqrt{\gamma}$, $\mathcal{E}_2 = a_2 \int_\Sigma d^2x\sqrt{\gamma}H^2 + a_2' \int_\Sigma d^2x\sqrt{\gamma}K$. The leading term here is the surface tension a_0, which is conventionally denoted as σ. Higher derivative terms such as $\int_\Sigma d^2x\sqrt{\gamma}H^4$ are negligible in the long wavelength description. The physics of membranes is then contained in the partition function $Z = \Sigma_\mathcal{C} \exp[-\mathcal{E}(\mathcal{C})/(k_B T)]$, where $\mathcal{E} = \mathcal{E}_0 + \mathcal{E}_2 + \ldots$ is an expansion of the energy in inverse powers of length. We will sometimes set Boltzmann's constant k_B to unity and measure temperature in energy units. This mathematical model of a membrane can be physically realized as an interface between fluids.

There is a clear analogy (summarized in Table I) between the GR situation and the soft matter one. The analogy is based on the usual correspondence between quantum physics and statistical mechanics. A history $\mathcal{H}$ in GR is replaced by a configuration $\mathcal{C}$ in statistical physics. A sum over histories in quantum GR $\Sigma_\mathcal{H} \exp[iI(\mathcal{H})/\hbar]$ is replaced by the partition function, a sum over configurations with Boltzmann weight. The action $I(\mathcal{H})$ is replaced by the energy $\mathcal{E}(\mathcal{C})$. The role of Planck's constant is played by the temperature T. The leading term in the action is the cosmological constant term just as the leading term in the energy of a membrane is the surface tension term. In particular, the surface tension has the interpretation of "energy cost per unit area of membrane": One has to supply energy to increase the area of the membrane. This is usually supplied in the form of mechanical work when one works up a lather while shampooing or beating an egg. In GR, the cosmological constant is the "action cost per unit four-volume of spacetime."

The geometric description of a membrane as a smooth two manifold Σ embedded in space is only a mathematical idealization. A real membrane in the laboratory is composed of molecules. The smooth manifold picture of Σ is valid only at length scales large compared to the molecular length scale l_{mol}. This is quite similar to the breakdown of the smooth manifold picture of spacetime at the Planck scale. The role of the Planck length is played here by the mean intermolecular spacing l_{mol}, which is [16] about 0.3 nm. At mesoscopic scales, the membrane appears smooth and, in a statistical sense, locally homogenous and isotropic. Just as a gas respects Euclidean translational and rotational symmetries, a random distribution of spacetime points leads to local Lorentz invariance on mesoscopic scales. For instance, the probability of having a void of area $\mathcal{A}_{\text{void}}$ in a membrane of area $\mathcal{A}$ can be crudely

TABLE I. The analogy.

Membranes	Universe
Configuration $\mathcal{C}$	History $\mathcal{H}$
Area of a configuration $\mathcal{A}$	Four-volume of a history $\mathcal{V}$
Sum over configurations	Sum over histories
Energy $\mathcal{E}(\mathcal{C})$	Classical action $I(\mathcal{H})$
Temperature T	Planck's constant $\hbar$
Thermal fluctuations	Quantum fluctuations
Surface tension σ	Cosmological constant λ
Molecular length l_{mol}	Planck length l_{Planck}
Molecules	Causet elements
Free energy	Quantum action
$E_0 = a_0 \int d^2x\sqrt{\gamma}$	$I_0 = c_0 \int d^4x\sqrt{-g}$
$E_2 = \int d^2x\sqrt{\gamma}H^2$	$I_2 = c_2 \int d^4x\sqrt{-g}R$
Spatial discreteness	Spacetime discreteness
Plumber's nightmare phase	Spacetime foam

estimated as $P_{\rm void} \approx \mathcal{A}/\mathcal{A}_{\rm void} \exp[-\mathcal{A}_{\rm void}/l_{\rm mol}^2]$. This works out to $P_{\rm void} \approx \mathcal{A}/\mathcal{A}_{\rm void} \exp[-10^7]$, for a micron-sized void. This is similar in spirit to estimates [17] in the causet framework for the probability of nuclear-sized voids in the age of the Universe: $P_{\rm void} \sim e^{-10^{80}}$.

Using the analogy, we would expect that the surface tension of a membrane σ is of order 1 in dimensionless units, that is, $\sigma \approx T/l_{\rm mol}^2$. Using the values $k_B T = 1/40$ eV (corresponding to 300 °K) and $l_{\rm mol} = 0.3$ nm, we expect the surface tension of membranes to be around 40 in units of millijoules per square meter. This expectation turns out to be correct. Most interfaces of simple liquids have the expected order of magnitude [16] (e.g., the air-water interface has a σ of 72 mJ/m^2) and do not suffer from the analog of the cosmological constant problem.

However, there is an exception which is of great interest from the present perspective: fluid membranes. These are characterized by a negligibly small surface tension σ, orders of magnitude below that predicted by the dimensional argument. The statistical mechanics of fluid membranes is dominated [18,19] by the curvature terms $\mathcal{E}_2$ rather than by the surface tension term $\mathcal{E}_0$. This is an exact counterpart of the fact that I_2 dominates over I_0 in GR. Fluid membranes thus provide us with an example in which part (a) of the cosmological constant problem is naturally solved. Clearly, there is something to learn from this for cosmology. Let us consider fluid membranes and understand why they have vanishing surface tension [19].

A fluid membrane [19] is composed of amphiphilic molecules, which consist of hydrophilic (water loving) polar head groups and hydrophobic (water hating) hydrocarbon tails. As one increases the volume fraction of amphiphiles, the molecules pack more and more densely in the membrane and the area per molecule α decreases. There is a limit to this packing density, however, and, at a critical value of $\alpha = \alpha_0$, there is a minimum in the free energy per molecule $f(\alpha)$. A further increase in the number of molecules does not decrease α but increases the area of the interface (by rippling, for example) so as to accommodate the increase of molecules. Such a membrane is said to be saturated. At the saturation point $\alpha = \alpha_0$, the free energy per molecule has a minimum

$$\frac{\partial f}{\partial \alpha}\bigg|_{\alpha=\alpha_0} = 0. \tag{1}$$

Consider a saturated membrane with a fixed area $\mathcal{A}$ and $N = \mathcal{A}/\alpha$ molecules. The total free energy is given by [15,19] $F(\mathcal{A}) = Nf(\alpha)$. The expected value of the surface tension of the membrane

$$\langle \sigma \rangle = \frac{\partial F}{\partial \mathcal{A}} = \frac{\partial f}{\partial \alpha}\bigg|_{\alpha_0} = 0 \tag{2}$$

is zero, thus solving part (a).

Interestingly, the second horn of the dilemma can also be addressed in this condensed matter context. A fluid membrane consists of a finite number N of discrete elements or molecules. Therefore, just like the fluctuations in the cosmological constant which appear in discrete quantum gravity models [9], a fluid membrane consisting of a finite number N of molecules has an interfacial tension σ which fluctuates about zero. The mean square statistical fluctuation in the surface tension is

$$(\Delta \sigma)^2 = \langle (\sigma - \langle \sigma \rangle)^2 \rangle = T \frac{\partial^2 F}{\partial \mathcal{A}^2} = \frac{T}{N} \frac{\partial^2 f}{\partial \alpha^2}\bigg|_{\alpha_0}. \tag{3}$$

We can (naturally) expect $T(\partial^2 f/\partial \alpha^2)|_{\alpha_0}$ to be of order 1 in dimensionless units and so

$$\Delta \sigma \sim \frac{1}{\sqrt{N}} \tag{4}$$

in complete analogy to Sorkin's proposal in the cosmological context.

We propose an analog quantum gravity experiment to probe this small nonzero fluctuating cosmological constant. Consider a cylindrical fluid membrane in an ambient buffer solution stretched between two tiny rings, both of radius r in tens of nanometers. One of the rings is attached to a piezoelectric translation stage which can be moved in nanometer steps. The other ring is attached to a micron-sized bead which is confined in an optical trap. Fixing the separation L by using a feedback loop, one can measure the force F on the bead. This force F is related to the surface tension by $F = 2\pi r \sigma$. We expect to see fluctuations [20,21] in the surface tension due to finiteness of N over and above any instrumental noise which may be present. Experiments very similar to the one proposed above have already been done [22], albeit with a completely different motivation. They [22] pull out a tube (80 nm radius) of lipid membrane from a multilamellar vesicle of DDAB, stretch it out over tens of microns, and measure its force extension relation. They do measure a surface tension, but it is not the effect that we discuss here since the number of molecules involved is not small enough. Such an experiment can be viewed as an analog quantum gravity experiment probing a small nonzero fluctuating cosmological constant.

We have developed an analogy between the surface tension of membranes and the cosmological constant. The analogy is based on the standard mapping between quantum field theory and statistical mechanics. We have shown that the cosmological constant problem has its counterpart in the context of membranes and suggested experimental probes for measuring a fluctuating surface tension, thus realizing in analogy Sorkin's proposal of a fluctuating cosmological constant. While the quantum gravity prediction of a fluctuating λ was first made in the context of causets, it appears that the result is more general and follows just from the discreteness of spacetime. Discreteness of spacetime is generically present in quantum gravity models. Indeed, any quantum gravity model which predicts a finite entropy for black holes [23] must have discreteness in some form.

The main new point of this Letter is the *connection* between two disparate fields. As some aspects are better understood in one field and some in the other, we are able to derive insights from *both* fields. For example, part (b) is discussed in cosmology [9] but we have not seen a corresponding discussion in membranes. The reverse is true [19] for part (a). Apart from drawing attention to the generality of Sorkin's prediction, our analogy also suggests a way of solving part (a) of the cosmological constant problem. One could use the analogy to transport this discussion to cosmology. It appears that fluid membranes solve the cosmological constant problem by exchanging molecules with an external reservoir. In GR, too, one could invoke a ''grand canonical ensemble'' to provide discrete elements of spacetime at Planck density [9]. Indeed, the idea of a ''trans-Planckian reservoir'' has been discussed by Brout [24] and Volovik [25]. Carrying over ideas from the membrane context, one could introduce in analogy to $f(\alpha)$ a ''quantum action per element,'' a function of four-density which has a minimum at the Planck four-density. Both part (a) and part (b) seem to emerge naturally from this description. We hope to interest the quantum gravity community in implementing this idea in technical detail. While the details of the implementation may vary from model to model, we expect that the general idea will work provided only that there is some form of discreteness in the model.

The analogy we develop here is a fertile one. There is a membrane counterpart of dimensional reduction by Kaluza-Klein compactification, a cylindrical tubule with length much greater than its width. Small extra dimensions are nanotubules, and large extra dimensions are microtubules [22,26]. The analog of spacetime foam is a membrane with proliferating handles [16], a plumber's nightmare. We hope to interest both the soft matter and the quantum gravity communities in exploring these ideas further.

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[1] R. D. Sorkin, in *Causal Sets: Discrete Gravity*, Lectures on Quantum Gravity, edited by A. Gomberoff and D. Marolf (Springer, New York, 2005).
[2] W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981); Phys. Rev. D **51**, 2827 (1995).
[3] C. Barcelo, S. Liberati, and M. Visser, Living Rev. Relativity **8**, 12 (2005), and references therein.
[4] See, for instance, G. E. Volovik, *The Universe in a Helium Droplet* (Clarendon, Oxford, 2003).
[5] M. J. Bowick, L. Chandar, E. A. Schiff, and A. M. Srivastava, Science **263**, 943 (1994); R. Ray and A. M. Srivastava, Phys. Rev. D **69**, 103525 (2004).
[6] See, for instance, A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, England, 1994); T. Vachaspati and A. Vilenkin, Phys. Rev. D **30**, 2036 (1984).
[7] S. Liberati, M. Visser, and S. Weinfurtner, Phys. Rev. Lett. **96**, 151301 (2006); Classical Quantum Gravity **23**, 3129 (2006).
[8] G. Amelino-Camelia *et al.*, Nature (London) **393**, 763 (1998).
[9] R. D. Sorkin, in *Proceedings of the Ninth Italian Conference on GR and Gravitational Physics, Capri, Italy, 1990*, edited by R. Ciani, R. de Ritis, M. Francaviglia, G. Marmo, C. Rubano, and P. Scudellaro (World Scientific, Singapore, 1991), pp 68–70; R. D. Sorkin, Int. J. Theor. Phys. **36**, 2759 (1997), and references therein; M. Ahmed, S. Dodelson, P. B. Greene, and R. Sorkin, Phys. Rev. D **69**, 103523 (2004).
[10] A. G. Riess *et al.*, Astron. J. **116**, 1009 (1998); S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999).
[11] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
[12] W. G. Unruh, Phys. Rev. D **40**, 1048 (1989); W. G. Unruh and R. M. Wald, Phys. Rev. D **40**, 2598 (1989).
[13] T. Padmanabhan, Classical Quantum Gravity **19**, L167 (2002).
[14] G. E. Volovik, JETP Lett. **80**, 465 (2004).
[15] *Statistical Mechanics of Membranes and Surfaces*, edited by D. Nelson, S. Weinberg, and T. Piran (World Scientific, Singapore, 2004).
[16] M. Daoud and C. E. Williams, *Soft Matter Physics* (Springer-Verlag, Berlin, 1999).
[17] F. Dowker, J. Henson, and R. D. Sorkin, Mod. Phys. Lett. A **19**, 1829 (2004).
[18] L. Peliti and S. Leibler, Phys. Rev. Lett. **54**, 1690 (1985).
[19] Samuel A. Safran, *Statistical Thermodynamics of Surfaces, Interfaces and Membranes* (Addison-Wesley, Reading, MA, 1994).
[20] J. Samuel and S. Sinha, Phys. Rev. E **66**, 050801(R) (2002).
[21] S. Sinha and J. Samuel, Phys. Rev. E **71**, 021104 (2005).
[22] T. Roopa and G. V. Shivashankar, Appl. Phys. Lett. **82**, 1631 (2003), and references therein.
[23] R. D. Sorkin, Braz. J. Phys. **35**, 280 (2005).
[24] R. Brout, gr-qc/0305054.
[25] G. E. Volovik, gr-qc/0604062.
[26] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999); **83**, 4690 (1999).