Dirac's Large Numbers Hypothesis

It is often interesting to express values of physical quantities in the units, not as arbitrary as pounds and inches, but that of elementary particles. Suppose one were to express the size of the (observable) Universe in the unit of the size of a nucleon. A natural unit for the nucleon size is a fermi (10^{-13} cm). Taking $t = 10$ billion years as the approximate age of the Universe, the length ct comes to about 3×10^{40} fermi. Given the uncertainties one could say that the ratio of the size of the Universe to that of a fermi is of order 10^{40}.

Let us take another example. One could have a dimensionless number from the ratio of electrical and gravitational forces between a proton and an electron. The value of this ratio ($\frac{e^2}{Gm_e m_p}$) is about 0.2×10^{40}. Seen in a slightly different way, if (e^2/mc^3) is chosen to be the unit of time, then the age of the Universe becomes of the same order as the ratio of electric and gravitational forces.

Another interesting dimensionless number arises from the ratio of the mass of the observable Universe to that of an elementary particle like proton. Assuming an age of $t = 10$ billion year and a mass density which will just about close the Universe (called the 'critical density'), one finds this ratio to be $\sim 5 \times 10^{78}$. It is remarkable that this number is very close to the square of the numbers mentioned earlier.

One then begins to wonder if these numbers are so close to each other, inspite of being very large, by sheer coincidence. The approximate coincidence of the two numbers mentioned here is indeed striking. Dirac proposed in 1937 that such constants of nature are interrelated. Specifically, he suggested that the first two numbers mentioned above are always equal. Since the size of the Universe increases with time, the second ratio is then required to increase in the same way. One of the ways that this ratio can change is to allow the gravitational constant G to vary with time. To be precise, one would require that $G \propto 1/t$.

This variation of G with time would however mean that the Earth-Sun distance varies as $r \propto 1/G$. Since angular momentum remains a constant, the product vr (v being the speed of Earth) is a constant, and therefore $v^2 r^2 = GM_{sun} r =$ constant. Therefore $r \propto 1/G$. Also the brightness of Sun-like stars is proportional to G^7, as argued by Edward Teller in 1948. These two factors would mean that the Earth would have been uninhabitable to life due to scorching heat from sunlight even a few million years ago, in contradiction with fossil records.

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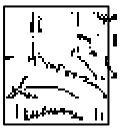

"One must not judge a man's worth from his poorer work; one must always judge him by the best he has done".

–Dirac to Salam about Eddington