Quantum gravity effects in the CGHS model of collapse to a black hole

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We show that only a sector of the classical solution space of the CGHS model describes the formation of black holes through the collapse of matter. This sector has either right or left moving matter. We describe the sector which has left moving matter in canonical language. In the nonperturbative quantum theory all operators are expressed in terms of the matter field operator which is represented on a Fock space. We discuss the existence of large quantum fluctuations of the metric operator when the matter field is approximately classical. We end with some comments which may pertain to Hawking radiation in the context of the model. [S0556-2821(98)03706-0]

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I. INTRODUCTION

We regard the Callan-Giddings-Harvey-Strominger (CGHS) model [1] as a 2D classical field theory in its own right rather than as string inspired. We view the dilaton field as just another classical field rather than as string coupling. The theory shares two important features with 4D general relativity—it is a nonlinear diffeomorphism invariant field theory in which the solutions describe spacetime metrics and *some* of these solutions correspond to black hole formation through matter collapse. Since the model is classically exactly solvable and, modulo certain very important qualifications, has been (non-perturbatively) canonically quantized via the Dirac procedure [2], we use it as a toy model for these features of 4D quantum general relativity.

In this work we prove some results regarding the properties of classical solutions to the model using a general relativist's point of view. Next, we discuss some quantum properties of the spacetime geometry along the lines of [3]. We end with a few speculative remarks concerning Hawking radiation in the model. The discussion and analysis of the quantum mechanics of the model are based on recent work [2].

The outline of the paper is as follows. In order to interpret the quantum theory of [2] it is essential to understand the classical solution space [4]. In Sec. II, we show that ''most'' classical solutions *do not* correspond to matter collapsing to form a black hole. More specifically, we show that if both left and right moving matter is present, the spacetime does not represent black hole formation through matter collapse.[1] However, if only ''one-sided'' matter is present, it is possible to obtain solutions describing the collapse to a black hole.

In fact, without the restriction to one-sided collapse, it is difficult to characterize the broad properties of the spacetime in terms of the properties of the matter field distribution (see, however, [4]). We have very little control over the solution space and do not understand exactly what facets of 4D general relativistic physics, if any, are modeled by the solutions. In order to retain the solutions corresponding to the collapsing black hole spacetimes as well as to have a better control on the space of solutions, we restrict our attention to the one-sided collapse sector in the remainder of the paper.

Solutions to the CGHS model are most simply described in Kruskal-like null cone coordinates $X^{\pm}$ [2]. The one-sided collapse solutions which we analyze describe a single black hole spacetime and correspond to the restriction $X^{\pm}>0$. These solutions can be analytically extended through the entire $X^{\pm}$ plane (indeed, the quantum theory of [2] seems to require consideration of such extensions). We show, through Penrose diagrams, how the physical spacetime is embedded in, and analytically extended to, the full $X^{\pm}$ plane. This completes our analysis of the classical solution space.

In Sec. III we turn to the Hamiltonian description of the model. Since we are interested in the one-sided collapse situation, we restrict the description in [2] suitably, by setting the left mass of the spacetime and the right moving matter modes to zero. We adapt the quantization of [2] to the one-sided collapse case. To make contact with the semiclassical treatment of Hawking radiation in the literature (see, for example, [5]), left *and* right moving modes are needed. Since one set of modes is frozen in our analysis, we do not discuss Hawking radiation related issues except for some comments in Sec. V. Instead, we focus on other issues in quantum gravity. We calculate quantum fluctuations of the metric operator when the matter fields are approximately classical (metric operator fluctuations were discussed earlier in [6] and, in the context of spacetimes with an internal boundary, in [7]). We show that large quantum gravity effects as in the case of cylindrical waves [3] are manifested even far away from the singularity (although not at spatial infinity).

The discussion of this section pertains to the quantum version of the analytic extension to the entire $X^{\pm}$ plane, of the one-sided collapse solutions. In contrast, in Sec. IV, we deal with (the canonical classical and quantum theory of) only the physical spacetime region $X^{\pm}>0$. To do this we appropriately modify the analysis of asymptotics in [2]. The most direct route to the quantum theory is to first gauge fix (in the manner of Miković [6]) and then quantize the resulting description. We obtain a Fock space representation based on the time choice $\ln(X^{+}/X^{-})$ in contrast to the Fock space of Sec. III which was based on the time choice $(X^{+}+X^{-})/2$.

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[1] A similar result is asserted in section 8 of [4].

We repeat the analysis of Sec. III regarding large quantum gravity effects. In the process we find that the operator corresponding to the spacetime metric at large values of X^+ cannot be represented on the Fock space of the quantum theory. The implication is that the most natural representation (which we have chosen) for the quantum theory may not be the correct one. We leave this as an open problem. Section V contains concluding remarks including some comments on Hawking radiation in the context of the model. We do not attempt to review the vast amount of pertinent literature but instead refer the reader to review articles such as [5].

Notation. Besides standard conventions, we will use the following notation (from [2]) throughout this paper: In the double null coordinates $X^\alpha=(X^+,X^-)$, many quantities depend only on X^+ or X^-, but not on both variables. We will emphasize this by using only X^+ or X^- as an argument of that function or functional. For example, while $f(X)$ means that f is a function of both X^+ and X^-, $f_{,+}(X^+)$ and $f_{,-}(X^-)$ mean that the derivatives $f_{,+}$ and $f_{,-}$ depend only on X^+ and X^-, respectively. Moreover, $f_{,\pm}(X^\pm)$ will serve as a shorthand notation to denote the function dependence of both $f_{,+}$ and $f_{,-}$ simultaneously. $\mathcal{I}_L^-$, $\mathcal{I}_R^-$, $\mathcal{I}_L^+$, and $\mathcal{I}_R^+$ denote past left, past right, future left, and future right null infinity, respectively.

II. ANALYSIS OF THE CLASSICAL SOLUTION SPACE

A. The action and the solution to the field equations

We briefly recall the action and the solution to the field equations for the CGHS model in the notation of [2] (for details, see [2]). In units in which the velocity of light, c, and the gravitational constant, G, are unity, the action is

$$S[y,\gamma_{\alpha\beta},f]=\frac{1}{2}\int d^2X|\gamma|^{1/2}(yR[\gamma]+4\kappa^2-\gamma^{\alpha\beta}f_{,\alpha}f_{,\beta}). \tag{1}$$

Here y is the dilaton field, $\gamma_{\alpha\beta}$ is the spacetime metric [signature $(-+)$], and f is a conformally coupled scalar field. $R[\gamma]$ denotes the scalar curvature of $\gamma_{\alpha\beta}$, and κ is a positive definite constant having the dimensions of inverse length.

To interpret the theory, we will treat $\gamma_{\alpha\beta}$ as an auxiliary metric and

$$\overline{\gamma}_{\alpha\beta}:=y^{-1}\gamma_{\alpha\beta} \tag{2}$$

as the physical ''black hole'' metric. Since y is a conformal factor, it is restricted to be positive. However, note that since the field equations and Eq. (1) are well defined for $y\leq 0$, solutions with positive y admit analytic extensions to $y\leq 0$. The solution to the field equations arising from (1) is as follows (for details, see [2]). $\gamma_{\alpha\beta}$ is flat. The remaining fields are most elegantly described in terms of the double null coordinates $X^\pm=Z\pm T$, where (Z,T) are the Minkowskian coordinates associated with the flat auxiliary metric. Then the spacetime line element associated with the metric $\gamma_{\alpha\beta}$ is

$$ds^2=dX^+dX^-, \tag{3}$$

the matter field is the sum of left and right movers:

$$f(X)=f_+(X^+)+f_-(X^-), \tag{4}$$

and in the conformal gauge [1]

$$y(X)=\kappa^2X^+X^-+y_+(X^+)+y_-(X^-). \tag{5}$$

Here

$$y_\pm(X^\pm)=-\int^{X^\pm}d\bar{X}^\pm\int^{\bar{X}^\pm}d\bar{\bar{X}}^\pm(f_{,\pm}(\bar{\bar{X}}^\pm))^2. \tag{6}$$

Finally, the line element corresponding to the physical metric $\overline{\gamma}_{\alpha\beta}$ is

$$d\overline{s^2}=\frac{dX^+dX^-}{y}. \tag{7}$$

Its scalar curvature is

$$\overline{R}=4\left(\frac{y_{,+-}}{y}-\frac{y_{,+}y_{,-}}{y^2}\right). \tag{8}$$

For smooth matter fields, it is easy to see that curvature singularities can occur only when $y=0$ or $y=\infty$ (the converse may not be true).

B. Unphysical nature of solutions with ''both-sided matter''

We now analyze the physical spacetime structure corresponding to the solutions described above. We are interested in those solutions which describe matter collapse to a black hole. So for spacetimes of physical interest we require the following.

(i) A notion of (left past and future, right past and future) null infinities exists such that any light ray originating within the physical spacetime, traversing a region of no curvature singularities and reaching null infinity should exhaust infinite affine parameter to do so. Further, null infinity is the locus of all such points. Each of left past, right past, left future, and right future null infinity is a null surface diffeomorphic to the real line and forms part of the boundary of the spacetime.

(ii) Only future singularities should exist. Note that since y is a conformal factor (7), it is required to be positive. Any region *within* the physical spacetime where $y\leq 0$ is defined to be singular.

For simplicity we restrict the spacetime topology to be R^2. We also assume that the matter fields be of compact support at past null infinity. Since the null infinities are null boundaries of the spacetime, they are labeled by lines of constant $X^\pm$ (the constant could be finite or infinite). Thus, the physical spacetime is a subset of the entire Minkowskian plane framed by boundaries made up of lines of constant $X^\pm$ and a future singularity. With this picture in mind let us further analyze the consequences of (i) and (ii).

Further consideration of (i) results in the following lemma.

Lemma. If a section of null infinity is labeled by $X^+=0$ or $X^-=0$ then (i) implies that $y=0$.

Proof. Let a section of (right future or left past) null infinity be labeled by $X^+=0$. Approach $X^+=0$, through a nonsingular region along an $X^-=$ constant $=a^-$ line from $X^+=a^+$ (a^+ is finite). Let the normal to this line be $k_a=\alpha(dX^-)_a$. For this line to be a geodesic $\partial\alpha/\partial X^+=0$.

Choose $\alpha=1$. Let the affine parameter along this geodesic be λ. (i) implies

$$|\lambda(X^-=a^-,X^+=0)-\lambda(a^-,a^+)|=\left|\int_{a^+}^{0}\frac{dX^+}{\alpha y}\right|=\infty. \quad (9)$$

From (i), if $X^+=0$ is to label null infinity, $y\to 0$ as $X^+\to 0$ in such a way as to make the integral diverge. Hence $y(a^-,0)=0$.

We now show that (i) or (ii) is violated if both left and right moving matter is present. For this, we examine Eqs. (5) and (6) and choose the lower limits of integration in Eq. (6) as follows. Let the least value of X^- be X_0^- on left future null infinity and that of X^+ be X_0^+ on right past null infinity. Then we specify y as

$$y(X)=\kappa^2X^+X^-+y_+(X^+)+y_-(X^-)+a_+X^++a_-X^-+b, \quad (10)$$

where $a_\pm ,b$ are constants and

$$y_\pm(X^\pm)=-\int_{X_0^\pm}^{X^\pm}d\bar{X}^\pm\int_{X_0^\pm}^{\bar{X}^\pm}d\bar{\bar{X}}^\pm(f_{,\pm}(\bar{\bar{X}}^\pm))^2. \quad (11)$$

The auxiliary flat metric determines $X^\pm$ only up to Poincare transformations. In this section, if X_0^+ (X_0^-) happens to be finite, we use the translational freedom in X^+ (X^-) to set $X_0^+=0$ ($X_0^-=0$).

Our strategy will be to demand **(i)** or **(ii)** and use the lemma for exhaustive choices of ranges of $X^\pm$. Thus we *assume* that the physical spacetime satisfies (i), (ii) and that the boundaries of these ranges label the infinities of the spacetime. Singularities will occur inside the ranges when $y<0$.

(A) $-\infty<X^\pm<\infty$. Past timelike infinity is labeled by $(X^-,X^+)=(\infty,-\infty)$. As we approach this point, the first term on the right-hand side of Eq. (10) becomes arbitrarily negative and since it dominates the behavior of y, it drives y to negative values. The region $y<0$ must ''intersect'' past left and past right infinity. Since y cannot be negative, there must be a past singularity in the spacetime. Thus **(ii)** rules out this range for $X^\pm$.

(B) $-\infty<X^-<\infty,\ 0<X^+<\infty$. Left past null infinity is labeled by $X^+=0$. From the lemma, $y(X^-,0)=0$. From Eq. (10) this gives, on left past null infinity,

$$y^-(X^-)+a_-X^-+b=0. \quad (12)$$

Differentiating this equation with respect X^- yields $f_{,-}=0$. Thus **(i)** implies that there cannot be right moving matter for this choice of range.

All other choices of range can be handled by using the arguments in (A),(B). The conclusion is that either there is a past singularity in the spacetime so that the corresponding range is ruled out or $f_{,-}$ or $f_{,+}$ vanish. Thus we have proved the following statement:

In the conformal gauge, if (i) and (ii) hold, then either left moving or right moving matter must vanish. Note that we have *not* proved the converse of this statement.

C. One-sided collapse to a black hole

Having established that classical solutions of physical interest contain only ''one-sided'' matter, we turn to the analysis of Eq. (10) with $f^-=0$ (a similar analysis can be done for $f^+=0$).

We first identify the region of the $X^\pm$ plane corresponding to the physical spacetime. Let us fix the translation freedom in $X^\pm$ by setting $a_\pm=0$ in (10).[2] Using arguments similar to those in the lemma, (A) and (B), it can be shown that the only possible labelings of past null infinity which do not contradict **(i)**, **(ii)** and $f_{,+}\neq 0$ are past left null infinity at $X^+=0$ and past right null infinity at $X^-=\infty$.

Thus the solution of interest for the rest of the paper is

$$y(X)=\kappa^2X^+X^-+y_+(X^+), \quad (13)$$

with

$$y_+(X^+)=-\int_0^{X^+}d\bar{X}^+\int_0^{\bar{X}^+}d\bar{\bar{X}}^+(f_{,+}(\bar{\bar{X}}^+))^2, \quad (14)$$

and $X^+>0$. Let the support of f^+ be $\alpha<X^+<\beta$. Note that within the physical spacetime $X^-\geqslant 0$, otherwise $y(x)$ can become negative. Note that

$$y(x)=\kappa^2X^+X^- \quad \text{for} \quad X^+<\alpha, \quad X^-\geqslant 0, \quad (15)$$

and the spacetime is flat. For this region the null line $X^-=0$ is part of $\mathcal{I}_L^+$. Similarly, $\mathcal{I}_R^-$ is found to be $X^-=\infty$. Consideration of $X^+>\beta$ fixes $\mathcal{I}_R^+$ to be at $X^+=\infty$.

Next we examine the locus of the singularity:

$$y(x)=0\Rightarrow\kappa^2X^+X^-=\int_0^{X^+}d\bar{X}^+\int_0^{\bar{X}^+}d\bar{\bar{X}}^+(f_{,+}(\bar{\bar{X}}^+))^2 \quad (16)$$

$$\Rightarrow X^-=\frac{1}{\kappa^2X^+}\int_0^{X^+}d\bar{X}^+\int_0^{\bar{X}^+}d\bar{\bar{X}}^+(f_{,+}(\bar{\bar{X}}^+))^2. \quad (17)$$

The singularity intersects $\mathcal{I}_L^+$ at ($X^-=0$, $X^+=\alpha$). It can be checked that the normal $n_a=\partial_a y$ to the curve corresponding to the singularity has a norm in the auxilary metric given by

$$n^an_a=\kappa^2\left(\int_0^{X^+}d\bar{X}^+\int_0^{\bar{X}^+}d\bar{\bar{X}}^+(f_{,+}(\bar{\bar{X}}^+))^2 - X^+\int_0^{X^+}d\bar{X}^+(f_{,+}(\bar{X}^+))^2\right) \quad (18)$$

$$=-\kappa^2\int_0^{X^+}d\bar{X}^+\bar{X}^+(f_{,+}(\bar{X}^+))^2. \quad (19)$$

This is clearly negative for $X^+>\alpha$. Thus the singularity is spacelike. From Eq. (17) the singularity intersects future right null infinity at

[2]This is a different choice from that used in Sec. II B.

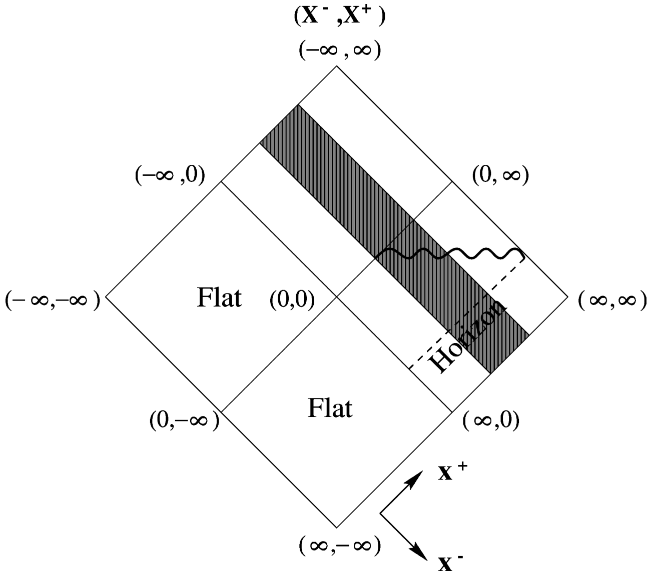

FIG. 1. The black hole spacetime is embedded in its analytic continuation to the entire Minkowskian plane. The curly line denotes the singularity in the black hole spacetime and the shaded region the (left moving) matter.

$$X^- = \frac{1}{\kappa^2}\int_0^\infty dX^+ (f_{,+}(X^+))^2. \tag{20}$$

Equation (20) gives the position of the horizon for the black hole formed by the collapse of the left moving matter.

That there is a single spacelike curve solving Eq. (17) can be seen from the following argument.[3] Consider y for fixed X^- as a function of X^+. Let $X^+ = X^+_{sing} > \alpha$ solve Eq. (17). It can be checked that for $X^+ > X^+_{sing}$, $y_{,+} < 0$. Thus, for a given X^-, $y = 0$ occurs at a single value of X^+.

This completes the discussion of the physical spacetime. As mentioned before, this solution admits an analytic extension to the whole $X^\pm$ plane. We now analyze this extension.

The full Minkowskian plane is divided into the following.

(1) $X^\pm > 0$. The physical spacetime lies within this range. It has an analytic extension ''above'' the singularity in which $y < 0$ and the metric acquires the signature $+-$ instead of $-+$.

(2) $X^- > 0$, $X^+ < 0$. Equation (13) gives $y = \kappa^2 X^+ X^-$. This describes a (complete) flat spacetime with $y < 0$ (there is a ''signature flip'' for the analytically continued metric).

(3) $X^\pm < 0$: $y = \kappa^2 X^+ X^-$ describes a complete flat spacetime with $y > 0$.

(4) $X^- < 0$, $X^+ > 0$. Both terms on the right-hand side of Eq. (13) survive and both are negative. So there is a signature flip in this region with $y < 0$.

The structure in the full Minkowskian plane is shown schematically in Fig. 1.

III. CANONICAL DESCRIPTION ON THE ENTIRE MINKOWSKIAN PLANE

We describe the one-sided collapse situation in classical canonical language. This is achieved by switching off degrees of freedom associated with the right moving matter fields in a consistent manner, in the description of [2]. Hence, we shall use the results and the notation, and adapt the procedures, of [2]. Rather than repeat the content of that paper here, we refer the readers to [2]. Henceforth we shall assume familiarity with that work. We shall also use the results from [8] regarding the canonical transformation to the Heisenberg picture. Although that work dealt with a spacetime topology $S^1 \times R$, the transformation to the Heisenberg picture as well as other basic ideas such as the relation of canonical data with the spacetime solution of the Klein Gordon equation go through in the R^2 case which is of relevance here.

It would be straightforward, in what follows, to use the gauge fixing procedure of [6]. Unfortunately, the gauge fixing conditions (67) *in conjunction with* the asymptotic conditions of [2] result in a foliation inappropriate for the entire Minkowskian plane. More precisely, the foliation consists of boosted planes all passing through $X^+ = X^- = 0$ and does not cover the timelike wedges $X^+X^- < 0$.[4] Such a foliation *does* cover the region $X^\pm > 0$ and this is why we use it in Sec. IV.

In what follows, x is a coordinate on the constant t spatial slice and the $1+1$ Hamiltonian decomposition is in the context of a foliation of spacetime by such slices. We use notation such that for a given field $g(x,t)$, $\partial g/\partial x$ is denoted by g' and $\partial g/\partial t$ is denoted by $\dot{g}$.

A. Classical theory

In [2] the CGHS model is mapped to a parametrized free-field theory on a flat 2D spacetime. The transformation from Eq. (1) (after parametrization at infinities) is made to a description in terms of embedding variables and the (canonical form of the) action in these variables is

$$S[X^\pm, \Pi_\pm, f, \pi_f, \overline{N}, N^1; p, m_R] = \int dt \int_{-\infty}^{\infty} dx \ \ (\Pi_+ \dot{X}^+ + \Pi_- \dot{X}^- + \pi_f \dot{f} - \overline{NH} - N^1 H_1) + \int dt \ \ p\dot{m}_R. \tag{21}$$

Here $X^\pm$ are the embedding variables (they correspond to the light cone coordinates we have been using to describe the solution in earlier sections), $\Pi_\pm$ are their conjuagte momenta, f is the scalar field and π_f its conjugate momentum, $\overline{N}$ and N^1 are the rescaled lapse and the shift, and $\overline{H}$ and H_1 are the rescaled super-Hamiltonian and supermomentum constraints which take the form of constraints for a parametrized massless scalar field on a 2-dimensional Minkowski spacetime. It is convenient to deal with the Virasoro combinations

[3] I thank J. Samuel for suggesting this argument.

[4] This does not necessarily rule out the existence of a *different* set of asymptotic conditions, which together with the same gauge fixing conditions (67), gives a foliation which covers the entire Minkowskian plane.

$$H^{\pm}:=\frac{1}{2}(\overline{H}\pm H_1)=\pm\Pi_{\pm}X^{\pm\prime}+\frac{1}{4}(\pi_f\pm f')^2\approx 0. \quad (22)$$

m_R is the right mass of the spacetime and p, its conjugate momentum, has the interpretation of the difference between the parametrization time and the proper time at right spatial infinity with the left parametrization time chosen to agree with the left proper time. It is useful to recall from [2] that

$$y(x)=\kappa^2X^+(x)X^-(x)-\int_{\infty}^{x}d\overline{x}\;X^{-\prime}(\overline{x})\int_{\infty}^{\overline{x}}d\overline{\overline{x}}\;\Pi_-(\overline{\overline{x}})+\int_{-\infty}^{x}d\overline{x}\;X^{+\prime}(\overline{x})\int_{-\infty}^{\overline{x}}d\overline{\overline{x}}\;\Pi_+(\overline{\overline{x}})+\int_{-\infty}^{\infty}dx\;\;X^+(x)\Pi_+(x)+\frac{m_R}{\kappa}. \quad (23)$$

Note that the right mass is related to the left mass by

$$\frac{m_L}{\kappa}=\frac{m_R}{\kappa}+\int_{-\infty}^{\infty}dx\;\;X^+(x)\Pi_+(x)-\int_{-\infty}^{\infty}dx\;\;X^-(x)\Pi_-(x). \quad (24)$$

That the right mass appears in Eq. (21) rather than the left mass m_L is a matter of choice. In [2] if the authors had chosen to synchronize the right parametrization clock with the right proper time, the last term in Eq. (21) would be $\int dt\overline{p}\dot{m}_L$, where $\overline{p}$ denotes the difference between the left parametrization and the left proper time. So an action equivalent to Eq. (21) is

$$S[X^{\pm},\Pi_{\pm},f,\pi_f,\overline{N},N^1;\overline{p},m_L)=\int dt\int_{-\infty}^{\infty}dx\;\;(\Pi_+\dot{X}^++\Pi_-\dot{X}^-+\pi_f\dot{f}-\overline{NH}-N^1H_1)+\int dt\overline{p}\dot{m}_L. \quad (25)$$

Note that $m_L,\overline{p}$ are constants of motion. To make contact with the solution in Sec. II C, we first freeze the left mass to zero and simultaneously put $\overline{p}=0$. The reduced action, without this pair,

$$S[X^{\pm},\Pi_{\pm},f,\pi_f,\overline{N},N^1;\overline{p}=0,m_L=0)=\int dt\int_{-\infty}^{\infty}dx\;\;(\Pi_+\dot{X}^++\Pi_-\dot{X}^-+\pi_f\dot{f}-\overline{NH}-N^1H_1) \quad (26)$$

reproduces the correct equations of motion. Alternatively, if one is not familiar with the procedures for parametrization at infinities, it can be checked from Eq. (21) that m_R,p are constants of motion. We can, therefore, consistently freeze this degree of freedom by setting m_R,p equal to *constants* of motion and then simply use the reduced action (26). Since

$$\frac{m}{\kappa}:=-\int_{-\infty}^{\infty}dx\;\;X^+(x)\Pi_+(x)+\int_{-\infty}^{\infty}dx\;\;X^-(x)\Pi_-(x) \quad (27)$$

commutes with the constraints, it is a constant of a motion and we can consistently set $m_R=m$ and $p=0$.

Next, in order to have a description of the one-sided collapse situation, we must set $f^-=0$. This is done in the canonical treatment as follows. Through a Hamilton-Jacobi type of transformation we pass from the description in Eq. (26) to the Heisenberg picture [2,8]. The new variables are the Fourier modes (they can be interpreted as determining the matter field and momentum on an initial slice) $a_{\pm}(k)$ $(k>0)$, their complex conjugates $a^*_{\pm}(k)$, the embedding variables $X^{\pm}(x)$ (these are unchanged), and the new embedding momenta $\overline{\Pi}_{\pm}$. The Fourier modes and the new embedding momenta are given by

$$a_{\pm}(k)=\frac{i}{2\sqrt{\pi k}}\int_{-\infty}^{\infty}(\pi_f\pm f')e^{ikX^+(x)}, \quad (28)$$

$$\overline{\Pi}_{\pm}=\frac{H^{\pm}}{X^{\pm\prime}}. \quad (29)$$

Thus, the vanishing of the constraints is equivalent to the vanishing of the new embedding momenta. The only nontrivial Poisson brackets for the new variables are

$$\{a_{\pm}(k),a^*_{\pm}(l)\}=-i\,\delta(k,l),\quad \{X^{\pm}(x),\overline{\Pi}_{\pm}(y)\}=\delta(x,y). \quad (30)$$

To summarize, the scalar field and momenta are replaced by their Fourier modes, which can be thought of as coordinatizing their values on an initial slice given by $X^+(x)-X^-(x)=0$. The embedding coordinates are unchanged and the new embedding momenta are essentially the old constraints.

Setting $f^-=0$ is equivalent in the canonical language to demanding $\pi_f-f'=0$ [8]. From Eq. (28), this is achieved by setting $a_-(k)=a^*_-(k)=0$ and this can be done consistently, since the $+$ and $-$ modes are not dynamically coupled. So the final variables for the theory are $a_+(k)$, $a^*_+(k)$, $X^{\pm}(x)$, and $\overline{\Pi}_{\pm}(x)$. The latter vanish on the constraint surface. The connection to the variables $X^{\pm},\Pi_{\pm},f,\pi_f$ [2] is through Eqs. (28) and (29). The $X^{\pm},\Pi_{\pm}$ variables are related to the geometric variables of interest (the dilaton and its canonically conjugate momentum, the induced metric on the spatial slice and its conjugate momentum) in [2]. Since we are dealing with $-\infty<X^{\pm}<\infty$ [2], this analysis (and the next two sections on quantum theory) pertains to the analytically extended one-sided collapse solution.

In this paper we examine only the dilaton field (23). As mentioned in [2], solving the constraints $H^{\pm}$ expresses $\Pi_{\pm}$ in terms of $X^{\pm}$ and the scalar field and its momentum. Substituting this in Eq. (23) and using $\pi_f+f'=2X^{+\prime}f_{,+}$ from [8], one is led back to the spacetime solution

$$y(X)=\kappa^2X^+X^--\int_{-\infty}^{X^+}d\overline{X}^+\int_{-\infty}^{\overline{X}^+}d\overline{\overline{X}}^+(f_{,+}(\overline{\overline{X}}^+))^2. \quad (31)$$

For the next section it is useful to examine the large X^+ behavior of Eq. (31). For X^+ large enough that it is outside the support of the matter

$$y=\kappa^2X^+X^- - X^+H+\frac{m_R}{\kappa} \tag{32}$$

with

$$H=\int_{-\infty}^{\infty}d\overline{X^+}(f_{,+}(\overline{X^+}))^2 \tag{33}$$

and m_R given by the right-hand side of Eq. (27).

B. Quantum theory

The passage to quantum theory is straightforward. From [2], the operators $\hat{X}^{\pm}$ are represented by multiplication, $\hat{\Pi}_{\pm} = -i\hbar\,\delta/\delta X^{\pm}$ and $\hat{a}_+(k),\hat{a}^{\dagger}_+(k)$ by representation on a Fock space. Note that $\sqrt{k}\hat{\mathbf{a}}(-k)$, $(k>0)$ in [2] corresponds here to $\hat{a}_+(k)$ and that the commutator

$$[\hat{a}_+(k),\hat{a}^{\dagger}_+(l)]=\hbar\,\delta(k,l). \tag{34}$$

The imposition of the quantum version of the classical Heisenberg picture constraints leads us to the quantum Heisenberg picture, wherein states lie in the standard embedding-independent Fock space. Note that the Fock space here is spanned by the restriction of the Fock basis of [2] to negative momenta because we have frozen the right moving modes.

In the next section, we show the existence of large quantum gravity effects at large X^+. This involves a calculation of fluctuations of operators, $\hat{Q}$, of the form

$$\hat{Q}=\int_{-\infty}^{\infty}dX^+Q(X^+):(\hat{f}_{,+}(X^+))^2: , \tag{35}$$

where $Q(X^+)$ is a c-number function, :: refers to normal ordering, and

$$\hat{f}(X^+)=\frac{1}{2\sqrt{\pi}}\int_0^{\infty}\frac{dk}{\sqrt{k}}[\hat{a}_+(k)e^{-ikX^+}+\hat{a}^{\dagger}_+(k)e^{ikX^+}]. \tag{36}$$

Consider the coherent state

$$|\psi_c\rangle=e^{-(1/2\hbar)\int_0^{\infty}|c_+(k)|^2dk}\exp\left(\int_0^{\infty}\frac{dk}{\hbar}c_+(k)\hat{a}^{\dagger}_+(k)\right)|0\rangle \tag{37}$$

where $|0\rangle$ is the Fock vacuum and $c_+(k)$ are the (c number) modes of the classical field $f_c(X^+)$,

$$f_c(X^+)=\frac{1}{2\sqrt{\pi}}\int_0^{\infty}\frac{dk}{\sqrt{k}}[c_+(k)e^{-ikX^+}+c^*_+(k)e^{ikX^+}]. \tag{38}$$

The mean value $\overline{Q}$ of the operator $\hat{Q}$ in this coherent state is given by

$$\langle\psi_c|\hat{Q}|\psi_c\rangle=\int_{-\infty}^{\infty}dX^+Q(X^+)(f_{c,+}(X^+))^2 \tag{39}$$

as expected.

The (square) of the fluctuation in $\hat{Q}$ is given by

$$(\Delta Q)^2=\langle\psi_c|\hat{Q}^2|\psi_c\rangle-\overline{Q}^2=\frac{\hbar^2}{8\pi}\int_0^{\infty}dkk^3|Q(k)|^2 + \frac{\hbar}{4}\int_0^{\infty}dkk|Q_f(k)|^2, \tag{40}$$

where $Q(k)$ is the Fourier transform of $Q(X^+)$ and $Q_f(k)$ is the Fourier transform of the function $Q_f(X^+)$: $=Q(X^+)f_{c,+}(X^+)$. The Fourier transform of the function $g(X^+)$ is

$$g(k)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}dX^+e^{ikX^+}g(X^+). \tag{41}$$

Note that by virtue of its being independent of $f_c(X^+)$ the $\hbar^2$ term in Eq. (40) is the vacuum fluctuation of $\hat{Q}$.

C. Large quantum gravity effects

We examine the fluctuations of the dilaton field, y, which plays the role of a conformal factor for the physical metric and hence encodes all the nontrivial metrical behavior. The expression for y simplifies at large X^+ and we shall calculate the fluctuations of $\hat{y}$ in this limit. $y(X)$ is turned into the operator $\hat{y}(X)$ by substituting the appropriate embedding dependent Heisenberg field operators (36) in Eq. (31). Similarly H and m_R are turned into operators $\hat{H}$ and $\hat{m}_R$. Note that $y(x)$ is not a Dirac observable. However, it can be turned into one using the evolving constants of motion interpretation (see [9] and references therein).

Straightforward calculations result in the following expression for the ratio of the fluctuation in $\hat{y}$ to its mean value, at large X^+:

$$\left(\frac{\Delta y}{\overline{y}}\right)^2=\frac{(\Delta H)^2+(\Delta m_R/\kappa X^+)^2-(1/X^+\kappa)([\hat{H},\hat{\overline{m}}_R]_+-2\overline{Hm_R})}{(\kappa^2X^-)^2(1-\overline{H}/\kappa^2X^-+\overline{m}_R/\kappa^3X^+X^-)^2}. \tag{42}$$

Here

$$[\hat{H},\overline{\hat{m}_R}]_+ = \langle \psi_c | \hat{H}\hat{m}_R + \hat{m}_R \hat{H} | \psi_c \rangle. \tag{43}$$

To make contact with the classical solution of Sec. II, we choose the coherent state to be such that $f_c(X^+)$ is of compact support. Further,

$$f_c(X^+) = 0 \quad \text{for} \quad X^+ \leqslant 0. \tag{44}$$

$\hat{H}$ corresponds to $Q(X^+) =: H(X^+) = 1$ in Eq. (35) and $\hat{m}_R/\kappa$ to $Q(X^+) =: m_R(X^+)/\kappa = X^+$. From Eq. (39) and the fact that $f_c(X^+)$ is of compact support, it is easy to see that $\overline{H}, \overline{m_R}$ are finite.

Using Eq. (40) it can be seen (in obvious notation) that

$$(\Delta H)^2 = \frac{\hbar}{4}\int_0^\infty dk k |H_f(k)|^2 = \hbar \int_0^\infty dk k^2 |c_+(k)|^2. \tag{45}$$

Since f_c is of compact support, its Fourier modes decrease rapidly at infinity and have a sufficiently good infrared behavior that the integral above is both ultraviolet as well as infrared finite. This shows that the fluctuation in $\hat{H}$ is finite.

The fluctuation in $\hat{m}_R$ is

$$(\Delta m_R)^2 = \frac{\hbar^2}{8\pi}\int_0^\infty dk k^3 |m_R(k)|^2 + \frac{\hbar}{4}\int_0^\infty dk k |m_{Rf}(k)|^2. \tag{46}$$

Again, the fact that f_c is of compact support renders the second term on the right hand side of Eq. (46) UV and IR finite. We now argue that the first term corresponding to the vacuum fluctuation,

$$(\Delta_0 m_R)^2 := \frac{\hbar^2}{8\pi}\int_0^\infty dk k^3 |m_R(k)|^2 \tag{47}$$

is finite. Since $m_R(X^+) = \kappa X^+$, its Fourier transform $m_R(k)$ is ill defined. We calculate, instead, the vacuum fluctuation of the regulated operator $\hat{m}_R^{(D)}$ defined by setting

$$m_R^{(D)}(X^+) = \kappa X^+ e^{-(X^+)^2/D^2}. \tag{48}$$

We shall take the $D \to \infty$ limit at the end of the calculation to obtain the vacuum fluctuation of $\hat{m}_R = \hat{m}_R^{(\infty)}$.

Now $m_R^{(D)}(X^+)$ is a function of sufficiently rapid decrease at infinity that $(\Delta_0 m_R^{(D)})^2$ exists. It is evaluated to be

$$(\Delta_0 m_R^{(D)})^2 = \frac{\hbar^2 \kappa^2}{6}, \tag{49}$$

which is finite and independent of D. Thus the $D \to \infty$ limit can be taken and we have

$$(\Delta_0 m_R)^2 = \frac{\hbar^2 \kappa^2}{6}. \tag{50}$$

Finally, a straightforward calculation shows Eq. (43) also to be finite. We evaluate Eq. (42) for two cases:

Case I. Near right spatial infinity:

Here $X^{\pm} \to \infty$. So

$$\left(\frac{\Delta y}{\overline{y}}\right)^2 \to 0 \tag{51}$$

as $O(1/(X^-)^2)$. Thus, unlike the cylindrical wave case [3], there are no large quantum fluctuations of the metric near spatial infinity. This is because the leading order behavior of the metric is dictated by $\kappa^2 X^+ X^-$, which is a state-independent c-number function, unlike in the cylindrical wave case.

Case II. $X^- = \overline{H}/\kappa^2 - \overline{m}_R/\kappa^3 X^+ + d$ and X^+ large:

Here d is a real parameter. It can be checked that d measures the distance in X^- from the singularity which occurs at $d=0$, see Eq. (17). It is easy to see that

$$\left(\frac{\Delta y}{\overline{y}}\right)^2 = \frac{(\Delta H)^2}{\kappa^4 d^2} + O\left(\frac{1}{(X^+)^2}\right). \tag{52}$$

This expression makes *no* assumptions on the size of d. Using Eq. (45) in Eq. (52) and reinstating explicitly the factors[5] of G (and keeping $c=1$), we find that up to leading order in X^+

$$\left(\frac{\Delta y}{\overline{y}}\right)^2 = \frac{\hbar G^2}{\kappa^4 d^2}\int_0^\infty dl l^2 |c_+(l)|^2 \tag{53}$$

$$= \frac{\hbar G}{\kappa^2 d^2}\int_0^\infty d\frac{l}{\kappa}\left(\frac{l}{\kappa}\right)^2 (G\kappa |c_+(l)|^2). \tag{54}$$

Note that in $c=1$ units, $[G] = M^{-1}L^{-1}$, $[\kappa] = L^{-1}$, $[c_+(l)] = M^{1/2}L$, and $[\hbar] = ML$. Thus $\hbar G$ is the dimensionless Planck number, and κd, $l\kappa$, and $G\kappa|c_+(l)|^2$ are all dimensionless. From Eq. (54) there are large fluctuations in $\hat{y}$ when

$$\hbar G \gg \frac{\kappa^2 d^2}{\int_0^\infty d(l/\kappa)(l/\kappa)^2 (G\kappa |c_+(l)|^2)}. \tag{55}$$

This does not require d to be small. Large fluctuations can occur even if $\kappa d \gg 1$, provided the integral in Eq. (55) is large enough. Two cases when this is possible is when there are a large enough number of low-frequency scalar field excitations or if there is a high-frequency ''blip'' in the scalar field. This is very similar to what happens in [3]. Note that in a classical solution with mass m_R, the classical scalar curvature at a distance d from the singularity as a function of X^+ is at a large enough X^+,

$$R = \frac{m_R}{\kappa X^+ d} \tag{56}$$

and vanishes at $X^+ = \infty$.

The horizon is located (approximately) at $X_H^- := \overline{H}/\kappa^2$ (20). Therefore, if $d<0$, the region under consideration lies within X_H^-; if $d>0$ and X^+ is large enough, the region lies

[5] Units are discussed in [2].

outside X_H^-. Thus, for states satisfying Eq. (55), large quantum fluctuations in the metric occur both within and outside the mean location of the horizon. But from Eq. (20) this location itself fluctuates by $\Delta H/\kappa^2$. Thus, if the Planck number is much less than 1, the above calculation does not show the existence of large quantum fluctuations *outside* the *fluctuating* horizon.[6]

IV. CANONICAL DESCRIPTION ON THE $X^{\pm}>0$ SECTOR OF THE MINKOWSKIAN PLANE

Section III is applicable to the analytic extension of the one-sided collapse situation to the full Minkowskian plane. In this section we attempt to deal *only* with the physical spacetime and not with its analytic extension. We modify the analysis of [2], pertinent to the entire Minkowskian plane $-\infty<X^{\pm}<\infty$ in order to treat the case when $X^{\pm}>0$. This involves a modification of the asymptotics at left spatial infinity. Now, x is restricted to be positive and $x=0$ labels left spatial infinity. As mentioned in Sec. III, the simplest route to quantum theory is through gauge fixing [6] the description in terms of the original geometric variables rather than by transforming to embedding variables.

A. Classical theory

As in the previous section we assume familiarity with [2]. The canonical form of the action in the original geometric variables is

$$S[y,\pi_y,\sigma,p_\sigma,f,\pi_f,\overline{N},N^1]=\int dt\int_{-\infty}^{\infty}dx\ \ (\pi_y\dot{y}+p_\sigma\dot{\sigma}+\pi_f\dot{f}-\overline{NH}-N^1H_1) \quad (57)$$

with

$$\overline{H}=-\pi_y\sigma p_\sigma+y''-\sigma^{-1}\sigma'y'-2\kappa^2\sigma^2+\frac{1}{2}(\pi_f^2+f'^2) \quad (58)$$

and

$$H_1=\pi_y y'-\sigma p_\sigma'+\pi_f f'. \quad (59)$$

Here π_y is the momentum conjugate to the dilaton, σ is the spatial metric (induced from the auxiliary spacetime metric), and p_σ is its conjugate momentum.

The asymptotic conditions at right spatial infinity (which corresponds to $x=\infty$) are unchanged from [2]. The left spatial infinity is labeled by $x=0$. We require, as $x\to 0$,

$$y=\kappa^2x^2+O(x^3)\qquad \sigma=1+O(x^2) \quad (60)$$

$$\pi_y=O(x)\qquad p_\sigma=O(x^2) \quad (61)$$

$$\overline{N}=\alpha_L x+O(x^3)\qquad N^1=O(x^3), \quad (62)$$

where α_L is a real parameter.

Equation (57) is augmented with surface terms to render it functionally differentiable. The result is

$$S[y,\pi_y,\sigma,p_\sigma,f,\pi_f,\overline{N},N^1]=\int dt\int_0^{\infty}dx\ \ (\pi_y\dot{y}+p_\sigma\dot{\sigma}+\pi_f\dot{f}-\overline{NH}-N^1H_1)+\int dt\ \left(-\alpha_R\frac{m_R}{\kappa}\right). \quad (63)$$

Here α_R is related to the asymptotic behavior of the lapse at right spatial infinity [2]. It can be checked that with f,π_f of compact support, all the asymptotic conditions are preserved under evolution. Note that Eq. (60) automatically ensures that $m_L=0$.

To make contact with the one-sided collapse solution the right moving modes must be set to zero. We do this as follows. Note that up to total time derivatives

$$2\int_0^{\infty}\pi_f\dot{f}=\int_0^{\infty}dx\left(\int_0^x\pi_-(\overline{x})d\overline{x}\right)\dot{\pi}_-(x)-\int_0^{\infty}dx\left(\int_0^x\pi_+(\overline{x})d\overline{x}\right)\dot{\pi}_+(x), \quad (64)$$

where

$$\pi_\pm:=\pi_f\pm f'. \quad (65)$$

Thus, we can replace f,π_f by $\pi_\pm$, with the new Poisson brackets being

$$\{\pi_\pm(x),\pi_\pm(y)\}=\pm 2\frac{d\delta(x,y)}{dx},\quad \{\pi_+(x),\pi_-(y)\}=0. \quad (66)$$

Since π_+ and π_- do not couple dynamically, we can consistently freeze $\pi_-=0$. This corresponds to setting $f^-=0$.

Now, with a view towards quantization we introduce the gauge fixing conditions [6]

$$\pi_y=0,\qquad \sigma=1. \quad (67)$$

Using Eq. (67) in the constraints, the general solution for y and p_σ, in terms of π_+, consistent with the asymptotic conditions at left and right spatial infinity is

$$y=\kappa^2x^2-\int_0^x d\overline{x}\int_0^{\overline{x}}d\overline{\overline{x}}\frac{\pi_+^2(\overline{\overline{x}})}{4} \quad (68)$$

$$p_\sigma=\int_0^x d\overline{x}\frac{\pi_+^2(\overline{x})}{4}. \quad (69)$$

Requiring the preservation of Eq. (67) under evolution along with consistency with the asymptotic conditions fixes

$$\overline{N}=\alpha x,\qquad N^1=0, \quad (70)$$

where α is a real parameter. From [2] and Eq. (68),

[6] I thank Sukanta Bose for comments regarding this point.

$$\frac{m_R}{\kappa}=\int_0^\infty dx x\frac{\pi_+^2(x)}{4}. \tag{71}$$

Substituting this in Eq. (63) and using Eq. (64), we get

$$S[\pi_+(x)]=-\int dt\left[\int_0^\infty dx\left(\int_0^x \frac{\pi_+(\bar{x})}{2}d\bar{x}\right)\dot{\pi}_+(x) -\int_0^\infty dx x\frac{\pi_+^2(x)}{4}\right]. \tag{72}$$

In the above equation put

$$\kappa r:=\ln(\kappa x), \quad \bar{\pi}_+=e^{\kappa r}\pi_+ \tag{73}$$

to get

$$S[\bar{\pi}_+(x)]=-\int dt\left[\int_{-\infty}^\infty dr\left(\int_{-\infty}^r \frac{\bar{\pi}_+(\bar{r})}{2}d\bar{r}\right)\dot{\bar{\pi}}_+(r) -\int_{-\infty}^\infty dr\frac{\bar{\pi}_+^2(r)}{4}\right]. \tag{74}$$

Note that the last term ($=m_R/\kappa$) simplifies. The equations of motion are

$$\dot{\bar{\pi}}_+(r,t)=\{\bar{\pi}_+(r,t),m_R/\kappa\}=\frac{\partial\bar{\pi}_+(r,t)}{\partial r}. \tag{75}$$

The appropriate mode expansion which solves this is

$$\bar{\pi}_+(r,t)=\frac{1}{\sqrt{\pi}}\int_0^\infty dk\sqrt{k}[-i\bar{a}_+(k)e^{-ikr^+}+i\bar{a}_+^*(k)e^{ikr^+}], \tag{76}$$

where $r^+:=r+t$. From Eqs. (66) and (76), the only nontrivial Poisson brackets between the mode coefficients are

$$\{\bar{a}_+(k),\bar{a}_+^*(l)\}=(-i)\delta(k,l). \tag{77}$$

From [2] one can understand the slicing of the spacetime corresponding to the gauge fixing conditions (67) we have used. In particular, on a solution, one can see that (r,t) is related to $X^\pm$ by

$$\kappa X^\pm=e^{\kappa(r\pm t)} \tag{78}$$

and that the physical metric in these coordinates is manifestly asymptotically flat at spatial infinities.

The large X^+ behavior of y is again given by Eq. (32). Now y, H, and m_R/κ, which are evaluated in new coordinates, take the form

$$y=\kappa^2e^{2\kappa r}-e^{\kappa r^+}H+\frac{m_R}{\kappa}, \tag{79}$$

$$H=\int_0^\infty dX^+\frac{(\pi_+(X^+))^2}{4}=\int_{-\infty}^\infty dr^+e^{-\kappa r^+}\frac{(\bar{\pi}_+(r^+))^2}{4}, \tag{80}$$

and

$$\frac{m_R}{\kappa}=\int_0^\infty dX^+X^+(\pi_+(X^+))^2=\int_{-\infty}^\infty dr^+\frac{\bar{\pi}_+^2(r^+)}{4}. \tag{81}$$

In the $X^\pm$ coordinates H Eq. (33) took the form of a conventional Hamiltonian for the free-field theory on the entire $X^\pm$ plane, but m_R/κ was more complicated. In the (r,t) coordinates, m_R/κ takes the form of a conventional Hamiltonian for the free-field theory on the entire (r,t) plane (this is just the $X^\pm>0$ part of the entire $X^\pm$ plane), but H is complicated.

B. Quantum theory

The mode operators $\hat{\bar{a}}_+(k),\hat{\bar{a}}_+^\dagger(k)$ are represented in a standard way on the Fock space with vacuum $|\bar{0}\rangle$. They have the standard commutation relations

$$[\hat{\bar{a}}_+(k),\hat{\bar{a}}_+^\dagger(l)]=\hbar\delta(k,l). \tag{82}$$

Following the pattern of Secs. III B and III C we attempt to calculate the fluctuations of $\hat{y}$ in the large X^+ region, in the coherent state:

$$|\psi_c\rangle =\exp\left(-\int_0^\infty\frac{dk}{2\hbar}|\bar{c}_+(k)|^2dk\right)\exp\left(\int_0^\infty\frac{dk}{\hbar}\bar{c}_+(k)\hat{\bar{a}}_+^\dagger(k)\right)|\bar{0}\rangle \tag{83}$$

corresponding to the classical field

$$\bar{\pi}_{+c}(r^+)=\frac{1}{\sqrt{\pi}}\int_0^\infty dk\sqrt{k}[-i\bar{c}_+(k)e^{-ikr^+}+i\bar{c}_+^*(k)e^{ikr^+}] \tag{84}$$

which is of compact support in r^+.

Formally, Eq. (42) again expresses the fluctuations in $\hat{y}$. However, now the crucial operator is $\hat{H}$. It is obtained from the corresponding classical expression (80) in an obvious way. Similar calculations to those in Sec. III B give

$$(\Delta H)^2=\frac{\hbar^2}{8\pi}\int_0^\infty dkk^3|H(k)|^2+\frac{\hbar}{4}\int_0^\infty dkk|H_{\bar{\pi}}(k)|^2, \tag{85}$$

where $H(k)$ is the Fourier transform of

$$H(r^+):=e^{-\kappa r^+} \tag{86}$$

and $H_{\bar{\pi}}(k)$ is the Fourier transform of the function

$$H_{\bar{\pi}}(r^+):=H(r^+)\frac{\bar{\pi}_{+c}(r^+)}{4}. \tag{87}$$

The Fourier transform of the function $g(r^+)$ is

$$g(k)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^\infty dr^+e^{ikr^+}g(r^+). \tag{88}$$

Since $e^{-\kappa r^{+}}$ is not a function of rapid decrease in the r^{+} variable, the first integral in Eq. (85) is ill defined. It can be regulated by introducing the regulator $e^{-(r^{+})^{2}/D^{2}}$. The relevant regulated integral diverges in the limit $D\rightarrow\infty$ as $e^{2D^{2}}/D^{2}$. Thus the vacuum fluctuations of $\hat{H}$ diverge. Therefore, in this representation we cannot proceed further with the analysis and $\hat{y}$, as it stands, cannot be given meaning as an operator on the Fock space.

V. DISCUSSION

One way of describing the spacetime geometries which arise in the CGHS model is as follows. Consider the Minkowskian plane with a flat auxiliary metric (3), on which a scalar field propagates in accordance with the flat space wave equation. The spacetime metric is conformal to the auxiliary flat metric. The conformal factor y is determined by the matter distribution through Eq. (5) and is required to be positive. The field equations continue to make sense for $y \leqslant 0$. If one removes the restriction of positivity of y, then the following picture emerges. The Minkowskian plane is divided into spacetimes, each of which has $y>0$ or $y<0$. The former have the signature $-+$ and the latter $+-$. As far as we know, typically, $y=0$ labels singularities or boundaries at infinity for these spacetimes and some of these singularities may be past singularities.

This is the classical picture which corresponds to the quantum theory in [2]. Among all these classical solutions there are solutions which describe black holes formed from matter collapse. It may be that the entire solution space and the associated quantum theory [2,6,10] is required in order to understand issues that arise from black hole formation. In particular it may be that an understanding of Hawking radiation from a nonperturbative quantum theoretic viewpoint requires a treatment as in [6].

However, in this paper we have adopted the viewpoint that only solutions which describe the physically interesting situation of black hole formation through matter collapse are to be taken as the basis for passage to quantum theory. We have shown that these solutions have only left or right moving matter. We concentrated on the solution with left moving matter which described a collapsing black hole spacetime in the $X^{\pm}>0$ part of the plane. This solution admitted an analytic extension to the full Minkowskian plane and we showed the existence of large quantum gravity effects away from the singularity in a quantum theory based on this set of analytically extended solutions. Large quantum fluctuations of the metric occur even when the classical curvature is small. However, since the position of the horizon also fluctuates, our calculation does not prove the existence of large fluctuations outside the (fluctuating) horizon. Next, we dealt with the classical and quantum theory based on only the $X^{\pm}>0$ region. Note that even within this region there is an analytic extension of the solution above the singularity. In the quantum theory a quantity of interest, H (80), could not be represented as an operator on the Fock space of the theory. This is unfortunate because the classical theory captures physically relevant collapse situations (modulo the extension through the singularity). Note that m_R/κ takes the form of usual Hamiltonian for the free-field theory and is also a quantity of interest. We do not know if such a representation of quantum scalar field theory exists so that both H and m_R can be promoted to operators.

For the quantum theory based on the entire $X^{\pm}$ plane there were no such difficulties. In a sense, the quantum theories on the entire plane and the $X^{\pm}>0$ region are unitarily inequivalent. The former uses a positive-negative frequency split based on the time choice $T=(X^{+}+X^{-})/2$ and the latter on a time choice $t=(1/\kappa)\ln T$. This is very reminiscent of what happens in the Unruh effect in 2D [11], with the exception that there, both sets of modes are present. The role of acceleration in the Unruh effect is taken by κ.

The following comments regarding Hawking radiation are speculative. It seems significant that the Hawking temperature to leading order in the mass from semiclassical calculations [5] is independent of mass and is precisely the Unruh temperature for observers accelerating with κ. This line of thought has been pursued in [12] in the semiclassical context.

It seems that both right and left moving matters are required to calculate the Hawking effect. Therefore, let us switch the right moving modes on and go back to the quantization of [2]. The quantum theory is a standard unitary quantum field theory on a Fock space. But, as emphasized before, it corresponds to an analytic extension of the usual CGHS model. We beleive that it is the analytic extension which plays a key role in obtaining a unitary theory. A possibility is that the correlations in the quantum field which appear to have been lost by passage into the singularity reappear in the analytic extension beyond the singularity in the new ''universe'' which lies in the other side of the singularity.

Note that instead of freezing the degrees of freedom corresponding to the right moving modes, as is done in this work, one can continue to use the results of [2], but evaluate quantities pertaining to one-sided collapse by restricting the right moving part of the quantum states to the (right moving) Fock vacuum. Then vacuum fluctuations of the right moving modes would contribute to various quantities but we believe that the large quantum gravity effects away from the singularity (see Sec. II C) will persist. Maybe one can also examine Hawking effect issues since the right moving modes are not switched off.

Finally, from the point of view of 4D quantum general relativity, we feel that the CGHS model could be improved to a more realistic model of black holes if somehow an internal reflecting boundary in the spacetime existed [13,7]. The lack of such a boundary and the fact that the matter is conformally coupled so that it does not ''see'' the singularities, are, we believe, the key unphysical features present in the model but absent in the (effectively 2D) spherical collapse of a scalar field in 4D general relativity. The latter is of course a physically realistic situation, but unfortunately technically very complicated. It would be interesting to try to apply the techniques of [2] to the model with a boundary described in [13,7] and to try to compute the metric quantum fluctuations and to compare the results with those in [7]. Since the boundary in [7] is itself dynamically determined, it is not clear to us to what extent the model is solvable. It would be good to have a technically solvable model which was closer to 4D collapse situations.

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