Decoherence due to the Geometric Phase in a "Welcher-Weg" Experiment

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We describe a new version of the well-known gedanken experiments to determine which of the two paths a particle took in a two-beam interference experiment. In this version the presence of a geometric phase can be deduced from the requirement that the interference pattern must disappear as a result of the welcher-weg (which path) information.

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Ever since the advent of quantum mechanics in the early part of this century, one of the problems that has intrigued physicists is duality, i.e., wave and particle behavior exhibited by both matter as well as light. Young's two-slit interference experiment has, from the very beginning, provided the standard framework for discussion of the issues arising out of wave-particle duality. The famous Bohr-Einstein debate [1] surrounding the recoiling slit experiment proposed by Einstein [2] made it clear that it is impossible to determine which of the two slits the particles went through without destroying the wave aspect, namely, the interference pattern. More recent work on the two-slit interference involving photon interference [3], neutron interference [4,5], and atom interference [6–8] has brought the issue into the realm of real experiments and has focused on the question of partial determination of the path of the particle leaving the interference pattern largely unaffected. It is generally agreed that a complete determination of the path of the particles must result in destruction of the interference pattern. However, there is an unsettled question of the mechanism of disappearance of the interference pattern. For example, it is claimed [7,8] that in the experiment of Scully and Walther [7] the interference does not disappear due to a randomization of the phase of the interfering waves as envisaged in the original arguments of Bohr and Einstein.

Another important development in physics in recent years has been the discovery of the geometric phase in the context of adiabatic evolution of quantum systems by Berry [9], that of its more general counterpart in nonadiabatic evolution by Aharonov and Anandan [10], and the discovery of an anticipation of the geometric phase by Pancharatnam [11] in his work on polarization transformations of light. The relevance of Pancharatnam's work to phase changes in general evolutions in quantum mechanics including noncyclic and nonunitary evolutions has been pointed out by Samuel and the present author [12]. The phase predicted by Pancharatnam has been the subject of several experimental studies [13–16] and is by now a well-accepted idea. More recently, it was found [17] that phase changes occurring in very simple experimental situations in optics, e.g., linearly polarized light passing through a rotating birefringent plate, present, at first sight, paradoxes that disappear when the geometric phase as defined by Aharonov and Anandan [10] is accounted for.

In this Letter we present an interesting link between the above two fields in the form of a new version of Einstein's gedanken experiment. We start with the hypothesis that destruction of the interference as a result of the welcher-weg information must be due to a randomization of the phase of the interfering waves and we look for the source of the random phase. In the proposed experiment we find this to be a geometric phase. Encouraged by this we analyze the proposal of Ref. [7] and find that in that scheme too there must be a random phase introduced as a result of the welcher-weg information and that it can be looked upon as a geometric phase.

Consider an interference experiment in which a right-hand circularly polarized beam of light is split by a beam splitter into two beams each of which pass through a half-wave plate (HWP) and then recombine as shown in Fig. 1, the beams being routed through ideal metal-mirror reflections. In passing through a HWP each pho-

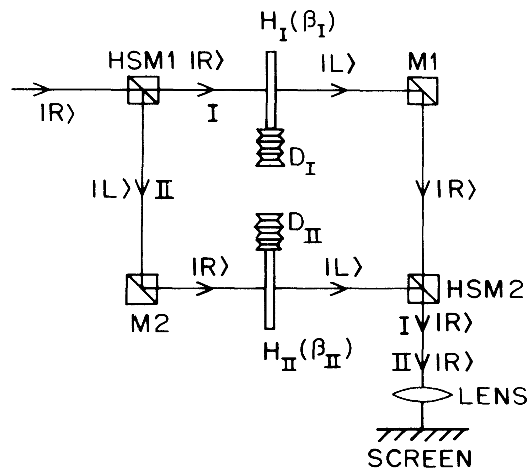

FIG. 1. Schematic diagram of the proposed gedanken experiment. $|R\rangle$ and $|L\rangle$ represent right-hand and left-hand circularly polarized states of a monochromatic light beam. $H_I(\beta_I)$ and $H_{II}(\beta_{II})$ are half-wave plates with their principal axes oriented at angles β_I and β_{II} with the x direction which lies normal to the plane of the interferometer. D_I and D_{II} are detectors capable of detecting a change in the angular momentum of H_I, H_{II}.

ton reverses its helicity, i.e., its spin angular momentum reverses sign and in the process imparts an angular momentum equal to $2\hbar$ to the HWP. Such angular momentum transfers have been experimentally observed by Beth [18]. Let us assume that to each HWP is attached a device D_I or D_{II} that can detect an angular impulse imparted to it by a passing photon. If this angular momentum change could be unambiguously determined one would have the required welcher-weg information. The interference pattern must then disappear. What causes the random phase that does this? The answer is essentially the same as that in the Bohr-Einstein debate. For an unambiguous determination of an angular momentum change equal to $2\hbar$, one must first prepare the HWPs in states in which their angular momenta are defined to an accuracy much better than $2\hbar$, i.e., more or less in an eigenstate of L_z, where L_z is the component of the angular momentum of the HWP in the direction of the beam axis which is taken to be along $\hat{\mathbf{z}}$. The uncertainty relation between L_z and the angular orientation ϕ of the HWP about the beam axis, $\Delta L_z \Delta\phi > \hbar/2$ then requires that $\Delta\phi \gg \frac{1}{4}$, i.e., be completely uncertain [19,20].

How does an uncertain ϕ result in a random phase of the interfering beams? It is a straightforward deduction from the results of several experiments already reported in literature [14,21] that in an arrangement shown in Fig. 1, each of the beams has a phase proportional to the angle of orientation of the HWP and is the Pancharatnam phase. We have also explicitly verified experimentally that in an arrangement equivalent to that shown in Fig. 3, a rotation of any of the two HWPs by an angle β results in a change in the phase difference between the two beams by $\pm 2\beta$. Figure 2 shows the tracks followed by the polarization state of the two beams on the Poincaré sphere which is the projective Hilbert space for any two-state system, for example, light polarization or a spin-$\frac{1}{2}$ particle. Let the x axis be chosen to be the direction normal to the plane of the interferometer. Then, in passing through the HWPs the state of the beams I and II follow the geodesic arcs $|R\rangle A_I|L\rangle$ and $|R\rangle A_{II}|L\rangle$, respectively, where $\alpha_i = 2\beta_i - 90°$, $i = \text{I,II}$. The angles β_I and β_{II} are the angles made by the fast axes of the HWPs with the x axis. Since M1 and HSM2, assumed to be ideal metal mirrors, can each be replaced with a HWP with its fast axis along $\hat{\mathbf{x}}$ as far as the polarization transformations are concerned [22], both beams return to the state $|R\rangle$ along the geodesic $|L\rangle M|R\rangle$. Each beam picks up a phase equal to half the solid angle subtended by the area enclosed by the closed circuit traced out by its polarization state on the Poincaré sphere. Any change in the relative orientation of the two HWPs therefore results in a phase change between the two beams equal to half the solid angle subtended by the change in area enclosed by the arcs $|R\rangle A_I|L\rangle$ and $|R\rangle A_{II}|L\rangle$.

We also note that the mirror reflection bringing the beams back to the state $|R\rangle$ is not essential. To get a geometric phase difference it is enough that the two beams traverse different arcs in going from $|R\rangle$ to $|L\rangle$. In principle, therefore, an experiment similar to Einstein's gedanken experiment shown in Fig. 3 would do as well, where any change in the relative orientation between the two HWPs would result in a proportional fringe shift. A completely uncertain orientation resulting from preparation of the HWPs in angular momentum eigenstates, necessary for detection of an angular momentum change of $2\hbar$ due to passage of the photons would therefore result in a completely random phase difference between the two waves and therefore in the disappearance of the in-

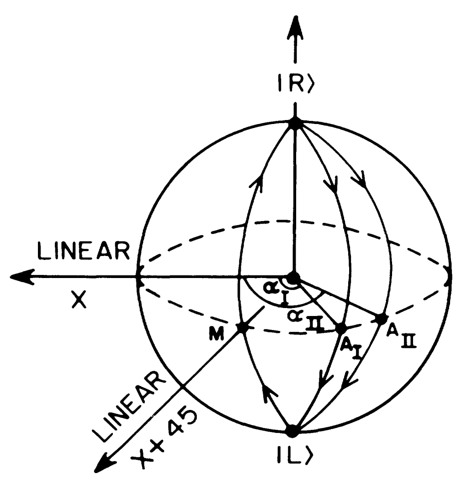

FIG. 2. The polarization states of beams I and II trace the geodesic arcs $|R\rangle A_I|L\rangle$ and $|R\rangle A_{II}|L\rangle$ on the Poincaré sphere while passing through the half-wave plates, and the arc $|L\rangle M|R\rangle$ while reflecting off the mirror M or HSM2, thus picking up geometric phases equal to α_I or α_{II}, where $\alpha_i = 2\beta_i - 90°$.

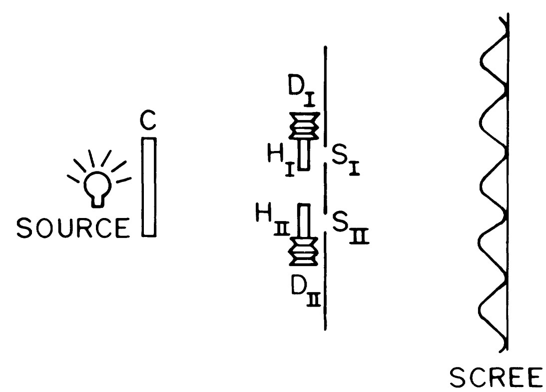

FIG. 3. An experiment equivalent to that of Fig. 1 shows the similarity in principle of the proposed experiment with Einstein's gedanken experiment. Photons from a monochromatic source pass through a circular polarizer C and then have a choice of one of two classical paths to the screen, i.e., via H_I-S_I or H_{II}-S_{II}, where H_I,H_{II} are HWPs and S_I,S_{II} are slits. D_I,D_{II} are angular momentum detectors that can detect any angular momentum transfer to a HWP by a passing circularly polarized photon.

terference pattern.

We next show by a two-step argument that there is no difference in principle between the gedanken experiments of Einstein [2] and the proposed experiments of Scully and Walther [7] in which the welcher-weg information is sought to be obtained by detection of a photon emitted by a passing atom in a maser cavity; in both cases the disappearance of the interference is due to a randomization of the phase as a result of the welcher-weg information. (1) Einstein's experiment and the one proposed in this paper are exactly similar except for the replacement of the conjugate variables x and p in Einstein's proposal with the variables ϕ and L_z in the present one. (2) The experiment of Scully and Walther is exactly similar to our proposed experiment except for the replacement of the conjugate variables L_z,ϕ in the latter with the pair N,θ in the former, where N stands for the number of energy quanta and θ for the phase of the oscillating cavity mode. The quantum mechanics of the variables L_z and ϕ for a rigid rotator is similar to the quantum mechanics of the variables N and θ for a harmonic oscillator which is an electromagnetic cavity mode [20]. The precise correspondence is as follows: (i) The angular momentum of the rotator corresponds to the energy of the oscillator; (ii) the orientation ϕ of the rotator corresponds to the phase θ of the oscillating cavity mode; and (iii) the number of angular momentum quanta in the rotator corresponds to the number of energy quanta in the oscillator. The N-θ uncertainty relation $\Delta N \Delta\theta > \frac{1}{2}$ must therefore play the same part in the Scully-Walther experiment as the L_z-ϕ uncertainty relation $\Delta L_z \Delta\phi > \hbar/2$ in the present one. It follows, therefore, that associated with a phase difference $\theta_1 - \theta_2$ between the two oscillating cavities there must exist a phase difference ψ between the wave functions of the two interfering atom beams. In fact this is clearly brought out in the analysis of Ref. [6]. When the cavities are prepared in number states, i.e., in eigenstates of N, both θ_1 and θ_2 are completely random [23], and hence $\theta_1 - \theta_2$ and consequently ψ are completely random. The correlation between $\theta_1 - \theta_2$ and ψ has also been observed experimentally in the "quantum beat" experiments of Badurek, Rauch, and Tuppinger [4] where a small frequency difference between the radio frequency coils in the two beams (a slowly varying phase difference) resulted in the quantum beating, i.e., a slowly time-varying phase difference between the interfering neutron beams. This has been interpreted by Wagh and Rakhecha [24] as a time-varying geometric phase.

To sum up, there is no difference in principle between the three experiments discussed in this paper; the difference is only in the pair of conjugate variables of the measuring apparatus used to obtain the welcher-weg information. In fact in any other scheme, if a variable A is used to obtain the path information, the conjugate variable B which is related to A by an uncertainty relation must be correlated with the phase of the interfering waves, and this principle can be used to identify the phase if it is not already obvious.

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