COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "Modified photon equation of motion as a test for the principle of equivalence"

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In a recent paper, a modification of the geodesic equation was proposed for spinning photons containing a spin-curvature coupling term. The difference in arrival times of opposite circular polarizations starting simultaneously from a source was computed, obtaining a result linear in the coupling parameter. It is pointed out here that this linear term violates causality and, more generally, Fermat's principle, implying calculational errors. Even if these are corrected, there is a violation of covariance in the way the photon spin was introduced. Rectifying this makes the effect computed vanish entirely.

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In a recent paper [1], Bhawal, Mani, and Vishveshwara studied a modified equation of motion (EOM) for spinning photons in curved spacetime which contained a possible equivalence-principle-violating spin-curvature coupling. Their aim was to use the possible difference in arrival times of opposite circular polarizations from a pulsar to constrain f, the coupling constant. In this Comment, an error of principle in the final result for the arrival time as well as one in the way the equation of motion was set up are pointed out. There also appears to be a more general difficulty with this type of formulation.

Photons obeying the proposed EOM still travel along null curves which reduce to null geodesics when f becomes zero. The final result for the arrival times, computed to lowest order in f in Eq. (11) of Ref. [1], has the structure

$$t_{\pm} = t_0 \pm f \times (\text{something}) \ . \tag{1}$$

Here $t_{\pm}$ are the arrival times of two opposite circular polarizations at the observer (which started at the same time from the source), t_0 is the arrival time for a null geodesic, and the quantity in parentheses multiplying f is a nonvanishing combination of source and receiver coordinates. This immediately violates causality since one of the signals has come out of the light cone (Fig. 1) for any sign of f. This is not possible for null curves such as those obeying the EOM of Ref. [1]. We therefore conclude that some calculational error is responsible for the nonvanishing term linear in f in Eq. (1).

The argument from causality, while compelling, is not completely general. One can have a situation, also illustrated in Fig. 1, when more than one light ray reaches an observer from the given event E. The later images to arrive are in the interior of the causal future set of E, and one might conclude that first-order terms in f in the arrival time formula are allowed for these images. However, this is not so, as is seen by applying Fermat's principle. Consider the arrival time of light traveling along a null geodesic from a source to a given timelike curve (the observer word line). Compare this with the arrival times for nearby null curves which are not geodesics but also reach the observer. According to Fermat's principle, the arrival time is an extremum with respect to such variations. Applied to the present problem this shows that the arrival time can have no linear term in f, since that would violate the extremal property. For static spacetimes, such as that in Ref. [1], the discussion of Fermat's

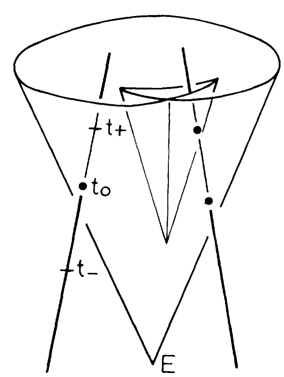

FIG. 1. Spacetime diagram showing a family of light rays (null geodesics) emitted from an event E. On the left, an observer world line intersecting the family once at t_0 is shown. The causality violation due to $t_{\pm}$ being on either side of t_0 is apparent. On the right, an observer who receives more than one ray from the event E is shown.

principle in the text by Weyl [2] suffices. A recent discussion of Fermat's principle for general spacetimes has been given by Kovner [3].

Without pinpointing the algebraic error which led to the linear term in Eq. (1), it may be noted that the minimum impact parameter r_0 of Ref. [1] should have a term linear in f which does not appear to have been allowed for. In general, one expects the leading term in the arrival time *difference* to be of order f^3, since an f^2 term would have the same effect on both circular polarizations.

We now examine the extra term added in Ref. [1] to the usual geodesic equation, viz.,

$$fR^a_{bcd}u^b u^c S^d \,, \tag{2}$$

where u^a is the photon four-velocity dx^a/dp, p being a parameter along its path and R the Riemann tensor. The photon spin vector S^d in Ref. [1] was chosen parallel to the spatial part of the photon four-velocity u^a with a zero time component. This was no doubt motivated by the physical idea that the spin is along the direction of propagation. However, this choice violates covariance since the parallelism will not be true in other coordinate systems. If one wants the space parts of u and S to be parallel in all systems, the only possibility is $S^a=\alpha u^a$. When this is used in Eq. (2), the term $fRuuu$ vanishes by the (anti)symmetry properties of the curvature tensor R.

The spin four-vector W^a is defined in terms of the total angular momentum J_{bc} and momentum P_d by $W^a=\epsilon^{abcd}J_{bc}P_d$. J_{bc} itself is not suitable for inclusion in spin-curvature coupling since it is origin dependent and hence cannot be regarded as an intrinsic property of the photon. More precisely, a change of origin by Δx^a adds $\Delta x^a P^b-\Delta x^b P^a$ to J^{ab}, which can be interpreted as additional orbital angular momentum. The extra piece vanishes when forming W, which is origin independent. But there is no frame-independent way to separate an (origin-independent) spin piece from J for a massless particle which has no rest frame.

It is to be hoped that future work in this area will take account of the constraints set by causality, Fermat's principle, and covariance.

[1] B. Bhawal, H. S. Mani, and C. V. Vishveshwara, Phys. Rev. D **44**, 1323 (1991).
[2] H. Weyl, *Space, Time, Matter* (Dover, New York, 1952), p. 244. [Original edition 1920.]
[3] I. Kovner, Astrophys. J. **351**, 114 (1990).