Appearance of the central singularity in spherical collapse

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We analyze here the structure of nonradial nonspacelike geodesics terminating in the past at a naked singularity formed as the end state of inhomogeneous dust collapse. The spectrum of outgoing nonspacelike geodesics is examined analytically. The local and global visibility of the singularity is also examined by integrating numerically the null geodesics equations. The possible implications of the existence of such families for the appearance of a star in the late stages of gravitational collapse are considered. It is seen that the outgoing nonradial geodesics give an appearance to the naked central singularity like that of an expanding ball whose radius reaches a maximum before the star goes within its apparent horizon. The radiated energy (along the null geodesics), however, is shown to decay very sharply in the neighborhood of the singularity. Thus the total energy escaping via nonradial null geodesics from the naked central singularity vanishes in the scenario considered here.

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I. INTRODUCTION

The continual gravitational collapse of a massive matter cloud results in either a black hole or a naked singularity, depending on the nature of the regular initial data from which the collapse develops. An important development is the emerging realization that at least in the spherically symmetric case both these final states seem to occur generically [1].

The theoretical properties and possible observational signatures of a black hole and a naked singularity would be quite significantly different from each other. An immediate distinction is, that if the collapse ends in a black hole, an event horizon develops well before the occurrence of the singularity, and thus the regions of extreme physical conditions (e.g., blowing up densities and curvatures) are hidden from the outside world. On the other hand, if the collapse develops into a globally naked singularity, then the energy of the regions neighboring the singularity can escape via the available nonspacelike geodesic paths or via other nongeodetic, nonspacelike trajectories to a distant observer.

In such a case, as postulated by some authors [2], a huge amount of energy could possibly be released, in principle, during the final stages of collapse from the regions close to the singularity. As suggested in [2] an enormous amount of energy might be generated either by some kind of quantum gravity mechanism, or by means of an astrophysical process wherein the region simply turns into a fireball creating shocks in the surrounding medium. One way to study the structure of these extreme regions is to examine the complete spectrum of all nonspacelike geodesics through which this energy could escape. Even if a fraction of the energy so generated is able to escape to a distant observer, an observational signature could be generated. It therefore becomes quite important and interesting to look into this possibility in some detail, and to consider possible observable differences in the two scenarios from the perspective of a faraway observer.

We therefore make such an attempt here to study the appearance of the late stages of collapse. While our emphasis in the present study will be on examining the null spectrum, which is important, we also need to keep in mind the possibility that emissions could as well come through other nonspacelike geodesics or nongeodetic paths, which represent high energy particles.

If the collapse ends in a black hole, the neighboring regions of singularity will appear dark, hidden within the event horizon. However, the naked singularity scenario could be different in principle. Hence we analyze the possible appearance of these high curvature regions when the singularity is naked. In this connection, one of the most extensively studied models has been that describing the collapse of inhomogeneous dust, where various features arising as collapse end states are well understood. It is known in this case that both black holes and (locally and globally) naked singularities, with either an asymptotically flat or a cosmological background, can develop depending on the nature of the initial data, which is specified in terms of the density and velocity profiles for the collapsing shells at the onset of the collapse [3–5]. Hence, these provide an ideal situation to study a problem such as the above and to study the scenario developing in the later stages of collapse.

It has been shown that in the case of collapse ending in a naked singularity, outgoing future directed radial null geodesic (RNG) families terminate in the past at the central naked singularity. The observed appearance of a distant object in the electromagnetic spectrum depends on the behavior of the full spectrum of null geodesics, including radial as well as

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nonradial trajectories. The behavior and contribution of nonradial null geodesics (NRNGs) could be significant for the appearance of the singularity. Although the study of outgoing nonradial null geodesics terminating at a naked singularity is of theoretical interest in its own right, it becomes particularly essential in the context of the appearance of naked singular regions when one realizes that past calculations of the appearance of a collapsing star (nonsingular regions) depended on the behavior of nonradial null geodesics in the late stages of collapse [6]. In fact, in the late stages of collapse the main contribution comes from nonradial null geodesics, which form a ring at $R=3M$. This ring then decays exponentially, giving a distinct rapid decline of luminosity within a short time interval. It is also of interest to check if timelike nonradial geodesics (NRGs) also come out of the singularity.

We thus need to understand and analyze the behavior of nonradial geodesics in the vicinity of a naked singularity toward this end, which has not been done so far. Such an analysis not only is of theoretical interest in its own right, but will also clarify several issues such as those above, thus revealing the structure of the naked singularity better. In the following, Sec. II describes basic equations governing the Tolman-Bondi-Lamaître (TBL) models, and in Sec. III we consider nonradial geodesics emerging from the naked singularity forming in such a collapse. Section IV discusses the luminosity aspect and in Sec. V some conclusions are proposed.

II. THE TBL MODEL AND GRAVITATIONAL COLLAPSE

A collapsing dust cloud is described by the TBL metric [7], which is given in comoving coordinates as

$$ds^2=-dt^2+\frac{R'^2}{1+f}dr^2+R^2(d\theta^2+\sin^2\theta\, d\phi^2). \tag{1}$$

The energy-momentum tensor is that of dust:

$$T^{ij}=\epsilon\delta^i_t\delta^j_t, \quad \epsilon=\epsilon(t,r)=\frac{F'}{R^2R'}, \tag{2}$$

where ϵ is the energy density, and the area radius $R=R(t,r)$ is given by

$$\dot{R}^2=f(r)+\frac{F(r)}{R}. \tag{3}$$

Here the overdot and prime denote partial derivatives with respect to the coordinates t and r, respectively, and for the case of collapse we have $\dot{R}<0$. The functions F and f are called the mass and energy functions, respectively, and they are related to the initial mass profile and velocity distribution of shells in the cloud.

At this point we limit ourselves to the marginally bound case because of the simplicity and clarity of the analysis. The results in the general case would be the same, however. For a marginally bound cloud ($f=0$), the integration of Eq. (3) gives

$$t-t_0(r)=-\frac{2R^{3/2}}{3\sqrt{F}}, \tag{4}$$

where $t_0(r)$ is a constant of integration. Using the coordinate freedom for rescaling the radial coordinate r we set

$$R(0,r)=r, \tag{5}$$

which gives

$$t_0(r)=\frac{2r^{3/2}}{3\sqrt{F}}. \tag{6}$$

At the time $t=t_0(r)$, the shell labeled by the coordinate radius r becomes singular where the area radius R for the shell becomes zero. We consider only situations where there are no shell crossings in the spacetime. A sufficient condition for this is that the density be a decreasing function of r, which may be considered actually to be a physically reasonable requirement, because for any realistic density profile the density should be higher at the center, decreasing away from the center. The ranges of coordinates are given by

$$0\leq r_b<\infty, \quad -\infty<t<t_0(r), \tag{7}$$

where $r=r_b$ denotes the boundary of the cloud. The quantity R', which is also needed later in the equation of the geodesics to check the visibility or otherwise of the central singularity, can be written as

$$R'=\frac{F'R}{3F}+\left(1-\frac{rF'}{3F}\right)\sqrt{r/R}. \tag{8}$$

III. NONRADIAL GEODESICS IN TBL MODELS

The regular center in a spherically symmetric spacetime can be the source of only radial geodesics. So when the central singularity is covered the center is observed by a distant observer by intercepting radial null geodesics till the time the event horizon forms, beyond which nothing is seen. An outside observer would see radial photons from the center, which would disappear much before the formation of the singularity. These photons do not stay in the cloud longer. At this point it is important to note that, as in earlier discussions of a collapsing star entering the event horizon, the role of nonradial geodesics cannot be overemphasized. The nonradial null geodesics emitted in late stages of collapse tend to revolve around the star and thus take a longer time to reach the distant observer. Therefore, even after the star has gone through its Schwarzschild radius, it leaves a ring of photons at $R=3M$. So a distant observer observing late stages of collapse sees a bright ring at $R=3M$ which decays exponentially. In the context of the formation of a naked singularity, therefore, it is essential that we examine this issue carefully and investigate the nonradial spectrum of null geodesics.

A. Basic geodesic equations

The tangents to the outgoing geodesics are given by

$$K^t = \frac{dt}{d\lambda} = \frac{P}{R}, \quad K^r = \frac{dr}{d\lambda} = \frac{\sqrt{P^2 - l^2 + BR^2}}{RR'}, \tag{9}$$

$$K^\phi = \frac{d\phi}{d\lambda} = \frac{l}{R^2}, \tag{10}$$

where λ is the affine parameter and P satisfies the differential equation

$$\frac{dP}{d\lambda} - \frac{PR'}{R} - (P^2 - l^2 + BR^2)\left[\frac{\dot{R}}{R^2} - \frac{\dot{R}'}{RR'}\right] - \sqrt{P^2 - l^2 + BR^2}\,\frac{P}{R^2} + B\dot{R} = 0. \tag{11}$$

We can work in the equatorial plane $\theta = \pi/2$, with $K^\theta = (d\theta/d\lambda) = 0$, and due to spherical symmetry this can then be rotated to recover the same qualitative features for the full spectrum of null geodesics. In the above equations $B=0$ for null geodesics, $B=-1$ for timelike geodesics, and $B=1$ for spacelike geodesics.

Using Eq. (3), these equations can be written in the (r,R) plane as

$$\frac{dR}{dr^\alpha} = \frac{R'}{\alpha r^{\alpha-1}}\left[1 - \sqrt{\frac{F}{R}}\, f \frac{1}{\sqrt{1 - l^2/P^2 + BR/P^2}}\right], \tag{12}$$

where α is a constant fixed by demanding that $R'/r^{\alpha-1}$ remains finite. For the quantities ϕ and P we have

$$\frac{d\phi}{dr} = \frac{l}{P}\,\frac{R'}{R\sqrt{P^2 - l^2 + BR^2}} \tag{13}$$

and

$$\frac{dP}{dr} = \frac{PR'}{R} - \sqrt{P^2 - l^2 + BR^2}\,\frac{1}{2}\sqrt{\frac{F}{R}}\left[\frac{3R'}{R} - \frac{F'}{F}\right] + \frac{BRR'}{\sqrt{P^2 - l^2 + BR^2}}\sqrt{\frac{F}{R}}, \tag{14}$$

respectively.

The above equations are a set of three coupled first order ordinary differential equations. For a given value of the parameter l one has to specify three initial conditions (starting values of r, R, and ϕ) and solve them simultaneously to get the geodesic trajectories.

It follows from the above equations that for $t < t_0(0)$ and $r=0$ the tangent equations give $l=0$ for self-consistency. Thus no NRGs will be radiated by the center before the singularity is formed at the center at $r=0$.

We now turn our attention to nonradial geodesics in TBL models near the central singularity. Defining $u = r^\alpha$, $X = R/u$, Eq. (12) can be written as

$$\frac{dR}{du} = H(X,u)\left[1 - \sqrt{\frac{\Lambda}{X}}\frac{1}{\sqrt{1 - l^2/P^2 + BXu/P^2}}\right] \equiv U(X,u) \tag{15}$$

where we have put $R' = H(X,u) r^{\alpha-1}$ and $\Lambda = F/u$. $H(X,u)$ is positive and nonzero for all $r>0$. In order to understand the nature of the singularity in the context of nonradial geodesics, one has to study the detailed behavior of the characteristic curves of the above first order equation in the vicinity of the singularity. If these curves terminate in the past at a singularity with a definite tangent then this is determined by the limiting value of $X = R/r^\alpha = R/u$ at the singularity $R=0, u=0$. If the future directed nonspacelike geodesics meet the singularity in the past with a definite value of tangent, then using Eq. (12) and l'Hôpital's rule along the geodesics we can write

$$X_0 \equiv \lim_{R\to 0, u\to 0} X = \lim_{R\to 0, u\to 0}\frac{R}{u} = \lim_{R\to 0, u\to 0}\frac{dR}{du} \equiv U(X_0, 0)$$

or

$$U(X,0) - X \equiv V(X) = 0, \tag{16}$$

where $U(X,u) = dR/du$ (along the geodesics), as defined above.

If the above equation has a real positive root $X = X_0$ then the singularity is at least locally naked [8]. In such a case there is at least one geodesic that comes out of the singularity with $R \equiv X_0 r^\alpha$. To check whether a family of nonspacelike geodesics comes out along the given direction, one needs to examine the higher order terms to get a constant of integration. It turns out that for radial geodesics we have two solutions, and along the larger solution (root) direction, which is the Cauchy horizon (the first ray coming out of the singularity), only one RNG comes out, while along the other direction a family of infinitely many RNGs comes out of the singularity. In the $\alpha<3$ case, to lowest order this family has the same behavior as that of the apparent horizon near the singularity.

We want to solve these equations in a self-consistent manner near the center, requiring that the outcoming null geodesics have a well-defined tangent at the singularity in a suitable plane. For simplicity and clarity, we assume the mass function to have the form

$$F = F_0 r^3 + F_n r^{3+n} + \text{higher order terms},$$

and we first consider only the null geodesics $(B=0)$. Later we generalize the results to other geodesics.

For clarity we divide our further analysis into three main cases.

1. $n<3$, i.e., $\alpha<3$

In this case, assuming that $l/P<1$ and checking for self-consistency, there are two kinds of behaviors possible for R, as given by (a) $R \approx X_0 r^{1+2n/3}$ [where $X_0 = (-F_n/2F_0)^{2/3}$] (this is the direction of the Cauchy horizon) or (b) R

$\approx X_0 r^3 \approx F_0 r^3$ [4,8]. In case (a), from Eq. (14) (assuming that P remains nonzero) we get

$$P \approx P_0 r^{\alpha} = P_0 r^{1+2n/3}, \tag{17}$$

where P_0 is a constant of integration. So we get a contradiction, because $1-l^2/P^2 \to -\infty$. It follows that we cannot have NRNGs coming out of the singularity along this direction.

In case (b) ($R \approx X_0 r^3 \approx F_0 r^3$) [4,8] assuming $1-l^2/P^2 > 0$ we get, from Eq. (14),

$$P = P_0 e^{-[nF_n/6(3-n)F_0^{5/2}]r^{n-3}}. \tag{18}$$

Thus in the limit of the singularity P blows up exponentially so we get $1-l^2/P^2 \to 1$ and we have a self-consistent solution for the nonradial geodesics. One can also check that there are no self-consistent solutions for NRNGs along any other directions.

2. $n=3$, i.e., $\alpha=3$

We can uniquely fix $\alpha=3$ and assuming that along the geodesics $R=X_0 r^3$, we get from Eq. (14)

$$P \approx P_0 r^{-3(2b_0^2-2b_0-1)/2b_0(b_0-1)}, \tag{19}$$

where $b_0=\sqrt{X_0/F_0}$. For P to blow up we need $(2b_0^2-2b_0-1)>0$. This gives

$$b_0=\sqrt{X_0/F_0}<b_{0crit}=\frac{1+\sqrt{3}}{2}. \tag{20}$$

Assuming that P blows up, the root equation (16) reduces to the same equation as for the radial null geodesics case. Introducing $X=F_0 x^2$ and $\xi=F_3/F_0^{5/2}$ the root equation then becomes

$$2x^4+x^3-\xi x+\xi=0. \tag{21}$$

From the theory of quartic equations, this admits a real positive root for $\xi<\xi_{crit}=-(26+15/\sqrt{3})/2$. Basically, if this condition is satisfied then we have two real positive roots. The smallest value for the larger root x_1 is the same as the largest value of the smaller root x_2 and it is the same as b_{0crit}; this is achieved at $\xi=\xi_{crit}$. So the condition (20) is never satisfied along the larger root x_1 (Cauchy horizon), and is always satisfied along the smaller root x_2. So $P \to 0$ along the larger root, and we cannot get self-consistent real solutions for the set of equations for the null geodesics. That means we cannot have NRNGs coming out along the Cauchy horizon direction. Along the smaller root direction this condition is always satisfied, P blows up, and we have a self-consistent solution for the geodesic differential equation. That means we can have NRNGs coming out of the singularity only along the direction of the smaller root.

3. $n>3$, i.e., $\alpha>3$

In this case we have to fix $\alpha=1+2n/3>3$, so $F/R\to\infty$, and we cannot have any null geodesics coming out of the singularity, which is always within a black hole. The Oppenheimer-Snyder collapse is a special case of this class with $n=\infty$.

B. Non-null geodesics

Now we will check what happens for timelike ($B=-1$) and spacelike ($B=1$) NRGs. Even in these cases, in Eqs. (12)–(14), for all the singular geodesics BR^2 goes to zero as we approach the center ($r=0$) and it is always negligible compared to the other terms. So provided P remains nonzero near the center, the root equation (16) remains the same as that for null geodesics. Now if we also solve the equation for P [Eq. (14)] in a self-consistent manner we see that for the smaller root P (and so also K^t) blows up and we get a self-consistent solution for the geodesic equations near the singularity; the behavior of these geodesics near the singularity is very similar to that of the null geodesics. Similarly, we can easily check that no nonradial geodesics can come out along the larger root direction.

C. Global behavior

We now consider the global behavior of nonradial null geodesics coming out of the singularity in various cases, and check whether there are any NRNGs that can be actually seen by a faraway observer. Basically, we try to check for singular NRNGs (outgoing nonradial geodesics that terminate in the past at the singularity), and examine with what maximum value of l ($l=l_{max}$) they can meet the boundary of the cloud $r=r_c$, when the boundary has area radius $R=R_c$. Here we fix the boundary in such a way that the density goes to zero smoothly at the boundary. The outside solution is Schwarzschild, with total mass $m=F(r_c)/2$. So with our condition we fix

$$r_c=\left(-\frac{3+n}{3}\frac{F_n}{F_0}\right)^{1/n}.$$

Figures 1(a) and 2(a) show the graphs of R_c versus l_{max}/P_c (where $P=P_c$ at $R=R_c$ and we give this as the initial condition). We see that l_{max}/P_c increases as we decrease R_c. This is expected in a way as the geodesics with larger value of l_{max}/P_c revolve more around the center, i.e., they remain in the inner region for longer times, so they are more likely to get trapped before reaching the larger values of R. Figures 1(b) and 2(b) show ϕ versus r while Figs. 1(c) and 2(c) show R versus r for some singular and nonsingular geodesics.

The considerations above point to NRNGs terminating in the past at a naked singularity that could reach a distant external observer. Unlike the regular center of the cloud, which cannot have nonradial geodesics terminating there, we have a distinctly new scenario where an observer could intercept NRNGs from the naked central singularity. Because of the high curvature in the singular regions, nonradial photons revolve around the center and stay for a longer time in the cloud before reaching the surface and hence the distant observer. Therefore, unlike the black hole case, where the center of the cloud disappears rather quietly, in the case of a

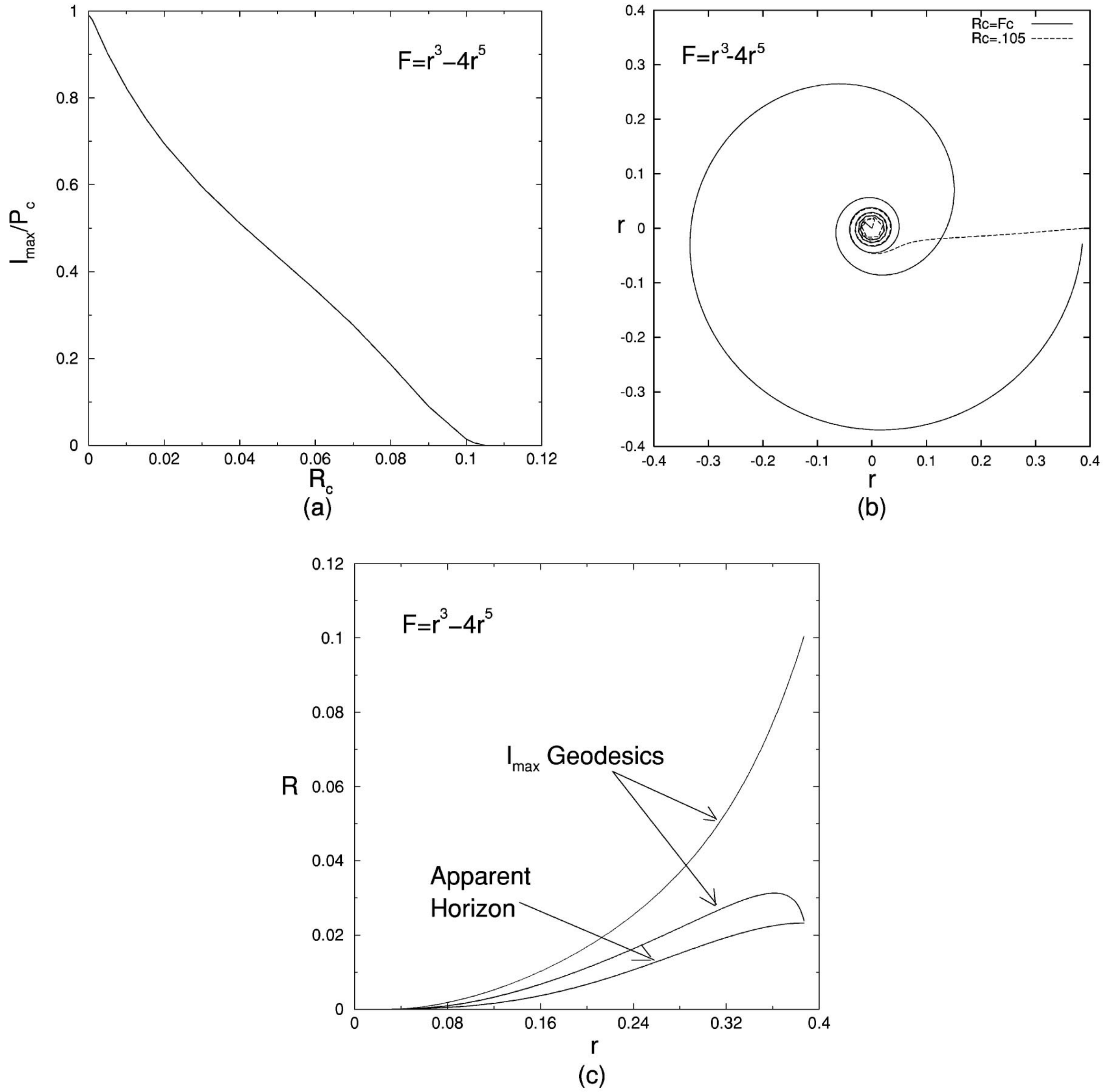

FIG. 1. Various plots for $F=r^3-4r^5$. (a) Plot showing the maximum value of l_{max}/P_c for various values of R_c. (b) Plot showing the geodesics with $l=l_{max}$ in the (r,ϕ) plane for two different values of R_c. (c) Same geodesic in the (r,R) plane.

naked singularity the center will appear as nothing unusual up to the time a singularity is formed. However, once the singularity is formed, it will appear rather differently.

Let us consider this phenomenon in some detail. For a distant observer, until the receipt of the first singular RNG from the naked singularity, the center will be observed through the RNG and will appear as a regular center inside the cloud. After the formation of the naked singularity, however, the observer will also start intercepting NRNGs which arrive later than their RNG counterparts. The observed position through the NRNGs depends on the value of the impact parameter l. The higher the value of the impact parameter the more time is spent by these photons in revolving inside the cloud and therefore they take longer time to come out. The maximum value of the impact parameter $l_{max}(t_0)$ is a function of time. Therefore, at any instant of time, after receiving the first RNG from the naked singularity, the observer will receive NRNGs within the range $l_{max}(t)\geqslant l\geqslant 0$ (for the cases when there are families of singular RNGs coming out). To a distant observer, the photons will appear to be coming not just from the center but from a spherical region (with $r=0$ as the center) having a radius corresponding to l_{max}. In this sense, therefore, the center will appear as a spherical ball with an expanding radius. The expansion continues until a maximum value of l is reached. Apart from the visible appearance of the naked singularity in the electromagnetic spectrum, the observations in terms of high energy particles traveling along nonspacelike curves will also be the same. That is, unlike the black hole scenario, in the case of a naked singularity forming, the timelike particles will also appear to be coming from an expanding ball.

IV. LUMINOSITY FUNCTION FOR THE SINGULARITY

Normally most of the luminosity (energy) of an object comes from the nonradial rays. We have shown above that an

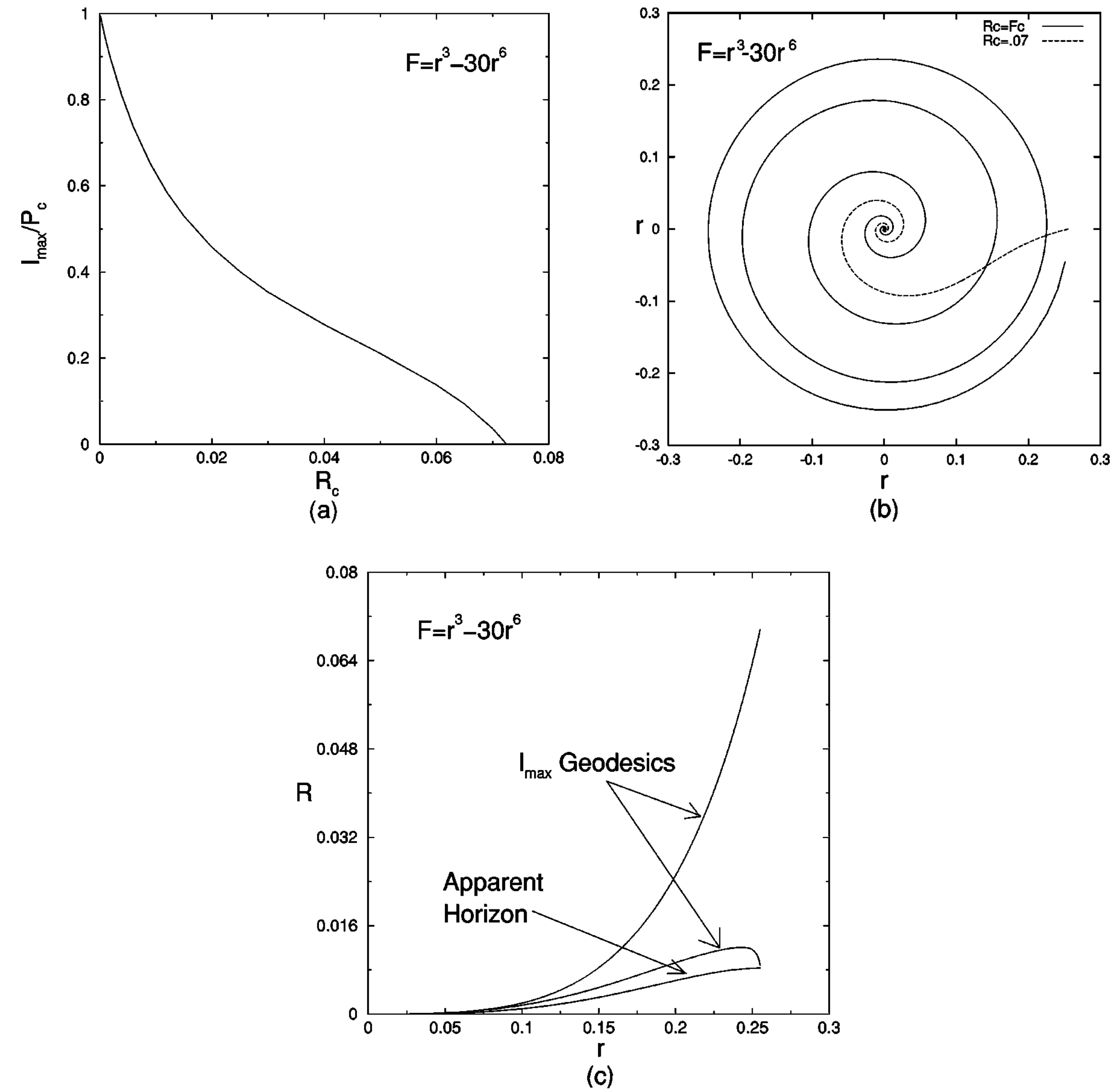

FIG. 2. Various plots for $F=r^3-30r^6$. (a) Plot showing the maximum value of l_{max}/P_c for various values of R_c. (b) Plot showing the geodesics with $l=l_{max}$ in the (r,ϕ) plane for two different values of R_c. (c) Same geodesic in the (r,R) plane.

infinite number of nonradial geodesics come out of the naked singularity, along the non-Cauchy horizon direction. Only one RNG comes out of the singularity along the Cauchy horizon direction (the larger root) [8]. So, if any radiation comes out of the singularity, it will normally be expected to come out along the smaller root direction through rays with various values of the impact parameter l. From such a perspective, we need to consider the luminosity function for these rays for various observers.

Let us consider an observer at comoving coordinate $r=r_o$. The observed intensity I_p of a point source is

$$I_p=\frac{P_0}{A_0(1+z)^2}, \tag{22}$$

where P_0 is the power radiated by the source into the solid angle $\delta\Omega$, and A_0 is the area sustained by the rays at the observer. The redshift factor $(1+z)^2$ appears because the power radiated is not the same as the power hitting the area at the observer. In the case of RNGs $A_0\propto R_0^2$ where R_0 is the area radius of the observer.

Let $u^a{}_{(s)}$ and $u^a{}_{(o)}$ be the four-velocities of the source and the observer and let E_1 and E_2 be two events connecting the source and the observer through the RNG. The redshift factor is given by

$$1+z=\frac{[K_a u^a{}_{(s)}]_{E_1}}{[K_a u^a{}_{(o)}]_{E_2}}, \tag{23}$$

where the numerator and denominator are evaluated at events E_1 and E_2, at the source and observer, respectively, with

$$u^a{}_{(s)}=\delta^a_t, \quad u^a{}_{(o)}=\delta^a_t. \tag{24}$$

Taking the source as the naked center at $r=0$ and the observer at $r=r_o$ we have

$$1+z\propto\frac{(K^t)_s}{(K^t)_o}. \tag{25}$$

In the evaluation of the redshift, the behavior of the tangent vector component K^t is important. It is finite at the nonsingular observer who is sufficiently far away from the center, i.e., $r_o \gg 0$. Therefore, the behavior of K^t at the naked singularity determines the behavior of the redshift factor. As discussed earlier, along the trajectories of interest K^t diverges very rapidly (exponentially if $\alpha<3$, and by a large power law if $\alpha=3$). This means that the redshift diverges very rapidly for the singular rays of our interest and for any nonsingular observer it will be infinite. (In a way the exponential divergence of the redshift for $\alpha<3$ is expected as these rays stay very close to the apparent horizon near the singularity and in such a situation the redshift in the Schwarzschild case also diverges exponentially.) As the redshift diverges, and A_0 is finite, classically the luminosity of the naked singularity along such families should vanish.

It is very difficult to get rid of this dominance of rapidly diverging K^t near the singularity on the redshift z, and so also the dominance of this diverging redshift on the luminosity. That means the luminosity of the naked singularity along such NRNGs for an observer will be zero, i.e., no energy will reach the observer along such families from the naked singularity, at least classically. Therefore, in the case above, the naked singularity may not be physically visible to faraway observers directly by means of emitted light. However, for the $\alpha<3$ naked singularity cases, if an enormous amount of radiation is emitted just along the Cauchy horizon, it may be possible that a measurable fraction of the emitted energy will reach an observer. This may happen as along this wave front K^t and so also the redshift remain finite [9]. But this observation of luminosity will only be instantaneous as only a single light ray is allowed to escape.

All the same, the possibility of mass emission via timelike or nonspacelike nongeodetic families of paths coming out of the naked singularity remains open. In the case of such a violent event being visible, particles escaping with ultrarelativistic velocities cannot be ruled out from this neighborhood. It is also to be noted that the classical possibilities such as that above regarding the probable light or particle emission, or otherwise, from a naked singularity may not perhaps offer a serious physical alternative one way or the other. The reason is that in all physical situations classical general relativity will break down once the densities and curvatures are sufficiently high so that quantum or quantum gravity effects become important in the process of an endless collapse. Such quantum effects would come into play much before the actual formation of the classical naked singularity, which may possibly be smeared out by quantum gravity. The key point then is the possible visibility, or otherwise, of these extremely strong gravity regions, which develop in any case, in the vicinity of the classical naked singularity. It is then the causal structure, that is, the communicability or otherwise, of these extremely strong gravity regions that will make the essential difference as far as the physical consequences of naked singularity formation are concerned.

In the black hole case, resulting from the collapse of a finite sized object such as a massive star, such strong gravity regions, or what we may call "fireballs," will be necessarily covered by an event horizon of gravity, well before the curvature conditions becomes extreme (e.g., well before the collapsing cloud goes to the Planck size). In such a situation, the quantum effects, even if they were causing qualitative changes closer to the singularity, will be of no physical consequences, because no causal communications are allowed from such regions. On the other hand, if the causal structure were that of a naked singularity, communications from such a quantum gravity dominated extreme curvature ball would be visible in principle, either directly, or via secondary effects such as shocks produced in the surrounding medium. Then we may have a chance to observe directly the quantum gravity effects from such fireballs generated due to stellar collapse.

V. CONCLUSIONS

By studying NRNGs we have demonstrated that, whenever the singularity is naked, along with RNGs, NRNGs also come out of the singularity. One can also say that if NRNGs came out of the singularity then RNGs will also come out, because we have shown that eventually the root equation for the existence of geodesics does not depend on the value of the impact parameter l. This is similar to the recent paper [10] on null geodesics, which appeared while this was being written. We have also shown that similar results exist for the timelike and spacelike geodesics also. Basically we see that in comoving coordinates all the geodesics have very similar behavior near the central singularity.

The existence of nonradial geodesics coming out can in a way make the naked singularity look as if it has a finite area to an outside observer. This possibly happens because, even though the singularity is at the center of symmetry and geodesics can come out of it, gravity is very powerful and dominant for such trajectories.

When the singularity is naked, RNGs come out of the singularity with two possible tangents (roots). Along the larger tangent (Cauchy horizon) direction only one RNG can come out, and along the smaller tangent direction an infinite family of RNGs come out of the singularity. We have shown here that NRNGs come out only along the direction of the smaller root, i.e., no NRNG comes out along the Cauchy horizon direction which corresponds to the larger root. Together with earlier results [8], this means that only a single radial null geodesic comes out of the naked singularity along the Cauchy horizon direction. In the marginally bound case ($f=0$), if the mass function has the form $F=F_0r^3+F_nr^{3+n}+$higher order terms, then when $n<3$ the Cauchy horizon has the behavior $R\propto r^{\alpha}$ (where $\alpha=1+2n/3<3$), while along the other direction (smaller root) the geodesics have a behavior similar to that of the apparent horizon, i.e., $R\equiv F_0r^3$. In the $n=3$ case, $\alpha=3$, and along both the tangents $R\propto r^3$, but the values of the proportionality constants are different.

By studying the global behavior of geodesic families using numerical methods, we have shown that, even when any value of l is allowed near the singularity, only the geodesics with a certain maximum value of l_{max}/P_c can reach any given outside observer. This value depends on the position of the observer and is larger if the observer's area radius is smaller. This is expected as geodesics with larger values of l will stay near the center for a longer time, and as the cloud is collapsing they are more likely to get trapped. The numerical study also shows that typically all such geodesics undergo a finite number of revolutions while they go out. The main reason for this is that the effects of gravity are very dominant near the center. Typically, these geodesics can revolve around the center at the most a few times before escaping.

Further, by studying the redshift and luminosity along the various geodesics we have shown that, apart from one special RNG (the Cauchy horizon) in the $n<3$ (i.e., $\alpha<3$) case, along all other singular trajectories the redshift diverges for any comoving (nonsingular) observer and so the luminosity reaching the observer from the naked singularity will be zero along such families. One could argue that even if a single photon or a single wave front carrying huge energy escapes from the singularity with a finite redshift, that may destroy the cosmic censorship, because that can alter the qualitative picture considerably. However, normally we do not expect a single wave front to emit an arbitrarily large amount of energy, although one does not know what happens near such extreme regions. What we may say then is that for $\alpha=3$ (i.e., $n=3$) naked singularity energy along null geodesics is censored for all the trajectories. For $\alpha<3$ (i.e., $n<3$) it is censored apart from the first trajectory, i.e., for trajectories along the larger root direction. Thus, for regions of spacetime where the curvature diverges fast enough, the redshift is infinite, thus censoring energy. In a sense this result can be considered to be supporting the cosmic censorship hypothesis, if we mean by the latter a statement such as that general relativity allows the occurrence of naked singularities, but they may not directly radiate away energy to outside observers. What this means exactly is that, although general relativity allows the occurrence of a naked singularity, the radiated energy in the electromagnetic spectrum does not reach the distant observer, at least in the dust case. We need to check such a statement for timelike and nonspacelike nongeodetic paths coming from the naked singularity, and also for equations of state other than dust, when the naked singularity may have a complicated topology. We note, of course, that cosmic censorship is not really a statement about the energy escape, it is essentially a basic postulate about not having outgoing causal curves from the singularity.

Although for simplicity and clarity we have shown these results for the marginally bound case, it would be possible to generalize the same to nonmarginally bound cases using a similar method, and depending on the value of α needed to make $R'/(r^{\alpha-1})$ finite, the results will be similar. This will be the case as, depending on the value of α, various functions involved in this analysis can be expanded in a similar way near the central singularity. Further, we can expect timelike NRGs as well to come out of the naked singularity even in these cases, as the study shows that they also have very similar behavior to that of null geodesics in the vicinity of the singularity.

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