New window on strange quark matter as the ground state of strongly interacting matter

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If strange quark matter is the true ground state of matter, it must have lower energy than nuclear matter. Simultaneously, two-flavor quark matter must have higher energy than nuclear matter, for otherwise the latter would convert to the former. We show, using an effective chiral Lagrangian, that the existence of a new lower energy ground state for two-flavor quark matter, the pion condensate, shrinks the window allowing strange quark matter to be the ground state of matter and sets new limits on the current strange quark mass.

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I. INTRODUCTION

The hypothesis that the true ground state of baryonic matter may have a roughly equal fraction of u, d, and s quarks, termed strange quark matter (SQM), is of recent origin [1]. This is based on the fact that, at some density, when the down quark chemical potential is larger than the strange quark mass, conversion to strange quarks can occur. This reduces the energy density by having three (u, d, and s) Fermi seas instead of just two (u and d), and can yield a state of energy lower than nuclear matter. It is also possible to explain why such a state has escaped detection.

This involves at least two puzzles.

(i) Why doesn't ordinary two-flavor nuclear matter, the observed ground state of baryonic matter, decay into strange quark matter? The answer is that this decay is not like the radioactive decay of unstable nuclei. The nucleons cannot decay one by one, as it is not energetically favorable for the nucleon to change into a Λ, but only for the entire nuclear matter to transmute into strange quark matter, and this requires a high order of the flavor changing weak interaction which renders the cross section exponentially and unobservably small.

(ii) Why wasn't this matter created in the evolution of the Universe? This is due to the fact that, as the Universe cooled past a temperature equivalent to the strange quark mass, strange quark matter was not the chosen state of high entropy. Since the u and d quarks have almost neglible masses at this scale, as the temperature dropped further the strange quarks were Boltzmann suppressed, leaving just the u and d quarks, which as we know converted largely into nucleons. For details we refer the reader to [1–3].

It is really quite remarkable that the ground state cannot be realized easily. Only if we can produce high baryon density by compression can SQM be realized—for example, in the interior of neutron stars.

We now turn to the theoretical underpinning of the case for SQM being the potential ground state of matter.

We already know, empirically as well as theoretically, the ground state energy per nucleon of saturation nuclear matter—930 MeV for the ^{56}Fe nucleus. However, for calculating quark matter we have to take recourse to phenomenological models, which are pointers but foundationally inadequate, and here lies the uncertainty.

The usual ground state calculation for SQM treats the quarks as a free Fermi gas of current quarks. The quark matter is in a chirally restored state (CRQM). The volume in which these quarks live comes at the cost of a constant energy density that provides "confinement." It is equivalently the same constant value of negative pressure and hence is often called the bag pressure term. This is a simple extension of the MIT bag philosophy, where the origin of the constant energy density is the fact that quarks are confined. The bag pressure sets the equilibrium or ground state energy density and the baryon density. It can be fixed from the nucleon sector. Further structure can be introduced by adding interaction between the quarks, e.g., one-gluon exchange. Such a phenomenological model has been used by Witten [1] and later by Farhi and Jaffe [2] and others for SQM (see [3] for a review).

II. CHIRAL SYMMETRY

It is clear that such a model is phenomenological and does not, for example, address the issue of the spontaneous breaking of chiral symmetry—an essential feature of strong interactions. We know that for strong interactions the vacuum is a state with spontaneously broken chiral symmetry. On the other hand, the quark matter in the bag, in the state above, is in a chirally restored state.

This means that as in the case of superconductivity it costs energy to expel the chiral condensate which characterizes the true vacuum state. Clearly, this will act just like the bag energy density/pressure. However, its value will be determined by the energy density of the chiral condensate. Such a term binds but does not confine. Confinement thus requires further input than just a bag pressure.

All results for the SQM state will depend on the model that is used to describe it and the ground state thereof. In a chiral model we find that there is a plurality of ground states.

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Of these, we find that one particular ground state has the property of chiral restoration at high density and parallels the MIT bag state used in most previous estimates, where the ground state is a Fermi sea of current quarks (chirally restored quark matter) with the bag pressure provided by the absence of the chiral condensate. This regime sets the connection between the parameters of the chiral model and the MIT bag model used in [1–3].

Unlike for the MIT bag case where the bag pressure is a parameter, in our formulation, it is the chiral condensate energy with a negative sign and is given in terms of the parameters of low energy phenomenology—the pion decay constant f_π, which is precisely known, and the scalar coupling or σ mass, which is rather poorly "known."

There are, however, other ground states for this model, in which the pattern of symmetry breaking is different at high density, for example, the pion condensed (PC) ground state in which the chiral symmetry is still spontaneously broken at high density. Such a state has lower ground state energy than the previous one and thus needs to be considered in the description of quark matter. As we show, it is found to have important influence on the regime of existence of SQM.

A. Effective chiral Lagrangian

We consider this issue in the framework of an intermediate chiral symmetric Lagrangian that has chiral SSB. Such an effective Lagrangian has quarks, gluons, and a chiral multiplet of $[\vec{\pi},\sigma]$ that flavor couples only to the quarks. For $SU(2)_L\times SU(2)_R$ chiral symmetry, we have

$$L=-\frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}-\sum\ \overline{\psi}[D+g_y(\sigma+i\gamma_5\vec{\tau}\vec{\pi})]\psi-\frac{1}{2}(\partial\mu\sigma)^2-\frac{1}{2}(\partial\mu\vec{\pi})^2-\frac{\lambda^2}{4}[\sigma^2+\vec{\pi}^2-(f_\pi)^2]^2. \tag{1}$$

The masses of the scalar (PS) and fermions follow on the minimization of the potentials above. This minimization yields

$$\langle\sigma\rangle^2=f_\pi^2 \tag{2}$$

where f_π is the pion decay constant. It follows that

$$m_\sigma^2=2\lambda^2(f_\pi)^2,\quad m_q=m=g\langle\sigma\rangle=gf_\pi. \tag{3}$$

This theory is an extension of QCD by additionally coupling the quarks to a chiral multiplet ($\vec{\pi}$ and σ) [4–6].

This Lagrangian has produced some interesting physics at the mean field level [6,7].

(1) It provides a quark soliton model for the nucleon in which the nucleon is realized as a soliton with quarks being bound in a Skyrmion configuration for the chiral field expectation values [5,6].

(2) Such a model gives a natural explanation for the "proton spin puzzle." This is because the quarks in the background fields are in a spin, isospin singlet state in which the quark spin operator averages to zero. On the collective quantization of this soliton to give states of good spin and isospin, the quark spin operator acquires a small nonzero contribution [8].

(3) Such a Lagrangian also seems to naturally produce the Gottfried sum rule [9].

(4) Such a nucleon can also yield from first principles (but with some drastic QCD evolution) structure functions for the nucleon that are close to the experimental ones [10].

(5) In a finite temperature field theory, such an effective Lagrangian also yields screening masses that match those of a finite temperature QCD simulation with dynamical quarks [11].

(6) This Lagrangian also gives a consistent equation of state for strongly interacting matter at all density [6,12].

We shall first briefly establish the parameters of the above effective Lagrangian and the specific connection with the MIT bag model of confinement used in previous treatments of SQM.

As already pointed out above, the nucleon in this model is realized as a soliton in a chiral symmetry broken background with quark bound states [5–7]. This sets the value of the Yukawa coupling g required to fit the nucleon mass in a mean field theory (MFT) treatment to be $g=5.4$.

For the nucleon the dependence on the scalar coupling λ is marginal as long as it is not too small. λ determines the scalar mass and also determines the chiral condensate energy density/pressure which is an important parameter for the quark matter phase.

Further, in MFT, the QCD coupling does not play a role; only if one-gluon exchange is included does the QCD coupling enter.

There are no other parameters except f_π, the pion decay constant, which is set to 93 MeV.

III. PRELIMINARIES FOR SQM

The connection to the MIT bag description of quark matter is set as follows. The last term in the above Lagrangian, the potential functional

$$\frac{\lambda^2}{4}[\sigma^2+\vec{\pi}^2-(f_\pi)^2]^2$$

is minimized by the vacuum expectation values (VEVs)

$$(\langle\sigma\rangle=f_\pi,\quad\langle\vec{\pi}\rangle=0)$$

and is equal to zero at the minimum.

In MFT at high density (as we shall see), when chiral symmetry is restored ($\langle\sigma\rangle=0$, $\langle\vec{\pi}\rangle=0$), this term reduces to a constant energy density term equal to

$$\frac{\lambda^2}{4}(f_\pi)^4.$$

In addition, due to chiral symmetry restoration the constituent mass of the quarks also vanishes, leaving free massless quarks. This reduced Lagrangian for high density is no different from MIT bag quark matter with

$$B=\frac{\lambda^2}{4}(f_\pi)^4.$$

This completes the identification of the bag pressure term in this model. It shows that bag pressure is automatically generated by chiral restoration and is controlled simply by the scalar coupling or equivalently the sigma mass.

We first briefly describe the logical basis for the investigation of the QM ground states compared with the usual nuclear matter ground state at saturation density. Here we follow Farhi and Jaffe [2].

(1) We fix coordinates by noting that SQM can be the true ground state only if its energy per baryon, E_B, is lower than the lowest value found in nuclei, 930 MeV for iron, as done by Farhi and Jaffe [2].

(2) We calculate the two-flavor quark matter ground states and fix a lower bound for the only free parameter in our Lagrangian, the scalar coupling; or equivalently we get a lower bound on the chiral condensate pressure (or bag pressure) from the condition that the two-flavor quark matter state must have higher E_B than nuclear matter—otherwise nuclear matter would be unstable to conversion to the two-flavor QM. As pointed out in [2] this condition is that bulk two-flavor quark matter must have $E_B > 934$ MeV.

(3) We calculate the SQM with the parameters established in (2) above and see if for SQM E_B is smaller than that given in (1) above. If this is the case, and as E_B increases monotonically with the scalar coupling (or the chiral condensate pressure), we get an upper bound on the chiral condensate pressure (or bag pressure) when E_B crosses beyond 930 MeV. SQM can then exist, as the true ground state, in this interval between the two bounds.

IV. TWO-FLAVOR QUARK MATTER

We shall now consider in mean field theory the phases of two-flavor quark matter in the $SU(2)_L\times SU(2)_R$ chiral model above. We shall then extend the model to three flavors ($u,d,$ and s) to describe SQM.

A. The space uniform phase

We now turn to the phase in which the pattern of symmetry breaking is such that the expectation values of the meson fields are uniform. At zero density they are just the VEVs

$$\langle\sigma\rangle=f_\pi, \tag{4}$$

$$\langle\vec{\pi}\rangle=0. \tag{5}$$

For arbitrary density we allow the expectation value to change in magnitude, as it becomes a variational parameter that is determined by energy minimization at each density:

$$\langle\sigma\rangle=F, \tag{6}$$

$$\langle\vec{\pi}\rangle=0. \tag{7}$$

Such a pattern of symmetry breaking simply provides a constituent mass to the quark $m=g\langle\sigma\rangle=gF$ and the quarks are in plane wave states as opposed to the bound states in the nucleonic phase [12].

The mean field description of this phase is simple.

The energy density

$$\epsilon_\rho=\Sigma_{u,d}\frac{1}{(2\pi)^3}\gamma\int d^3k\sqrt{m^2+k^2}+\frac{\lambda^2}{4}(\langle\sigma^2\rangle-f_\pi^2)^2, \tag{8}$$

where $m=g\langle\sigma\rangle=gF$ and the degeneracy $\gamma=6$. We shall use $g=5.4$ as determined from fixing the nucleon mass in this model at 938 MeV [5,6]. The integral above runs up to the u and d Fermi momenta.

For neutron matter (without β equilibrium) we have the relations

$$k_u^f=(\pi^2 n_u)^{1/3}=(\pi^2\rho_B)^{1/3}, \tag{9}$$

$$k_d^f=(2\pi^2\rho_B)^{1/3}, \tag{10}$$

$$E_B=\frac{\epsilon_\rho}{\rho_B}, \tag{11}$$

where ρ_B is the baryon density. At any density the ground state follows from minimizing the free energy with respect to $\langle\sigma\rangle=F$.

As shown in the figures of [12,13], this phase begins at zero ρ_B, with $E_B=3gf_\pi$, which then falls until chiral restoration occurs at some ρ_X. After this, as density is increased E_B continues to drop and goes to a minimum and then starts rising corresponding to a massless quark Fermi gas.

In the chirally restored phase the equation of state is very simple and parallels the MIT bag description of [3]

$$\rho_B>\rho_X, \tag{12}$$

$$\epsilon_\rho=\frac{3}{4\pi^2}\pi^2{\rho_B}^{4/3}\alpha+\frac{\lambda^2}{4}f_\pi^4. \tag{13}$$

The last term above is just the bag energy density, and

$$\alpha=(1+2^{4/3}). \tag{14}$$

This phase has two features, a chiral restoration at ρ_X followed, with increasing density, by an absolute minimum in E_B, at $\rho_C>\rho_X$

Since E_B decreases monotonically with increasing density until the chiral restoration density ρ_X and then continues to decrease until the minimum is reached at ρ_C, this implies that the density regime until ρ_C is unstable and has negative pressure. This was recently conjectured [14] as the density at which self-bound droplets of quarks form, which may be related to nucleons. Further, since at this density chiral symmetry is restored, these ''nucleons'' will be like those in the MIT bag model in which chiral symmetry is unbroken inside the nucleons.

We would like to clarify this issue.

From the comparison of this phase with the nucleon and nucleonic "phase" arising from the same model (see [6,12]), it is clear that the nucleonic phase is always of lower energy than the uniform phase above, up to a density of roughly three times the nuclear density, which is above the chiral restoration density in the uniform phase. Further, the minimum in the nucleonic phase occurs very much below the minimum in the uniform phase.

The chiral restoration density in the uniform phase is thus not of any physical interest as matter will always be in the lower energy nucleonic phase, and so the identification of the nucleon as a quark droplet at the density at which the minimum occurs in the uniform phase is not viable. Clearly, the nucleon is a quark soliton of mass $M=938$ MeV and falls at the zero density limit in the nucleonic phase.

B. The pion condensed phase

Here we shall consider another realization of the expectation values of $\langle\sigma\rangle$ and $\langle\vec{\pi}\rangle$ corresponding to pion condensation. This phenomenon was first considered in the context of nuclear matter.

Such a phenomenon also occurs with our quark based chiral σ model and was first considered at the mean field level by Kutschera et al. in an important paper [13]. Working in the chiral limit, they found that the neutral pion condensed state has lower energy than the uniform symmetry breaking state (phase 2) we have just considered for all density. This is expected, as the ansatz for the PC phase is more general than that for phase 2.

The expectation values now carry a particular space dependence:

$$\langle\sigma\rangle=F\cos(\vec{q}\cdot\vec{r}), \tag{15}$$

$$\langle\pi_3\rangle=F\sin(\vec{q}\cdot\vec{r}), \tag{16}$$

$$\langle\pi_1\rangle=0, \tag{17}$$

$$\langle\pi_2\rangle=0. \tag{18}$$

Note that when $|\vec{q}|$ goes to zero we recover the uniform phase 2.

The Dirac equation in this background is solved in [13] and reduces to

$$H\chi(k)=\left(\vec{\alpha}\cdot\vec{k}-\frac{1}{2}\vec{q}\cdot\vec{\alpha}\gamma_5\tau_3+\beta m\right)\chi(k)=E(k)\chi(k) \tag{19}$$

where $m=gF$.

The extra term has been recast in terms of the relativistic spin operator $\vec{\alpha}\gamma_5$. It is evident that if the spin is parallel to $\vec{q}$ and $\tau_3=+1$ (up quark) this term is negative and if $\tau_3=-1$ (down quark) it is positive. For spin antiparallel to $\vec{q}$ the signs for $\tau_3=+1$ and -1 are reversed.

The spectrum for the Hamiltonian is the quasiparticle spectrum and can be found to be

$$E_{(-)}(k)=\sqrt{m^2+k^2+\frac{1}{4}q^2-\sqrt{m^2q^2+(\vec{q}\cdot\vec{k})^2}}, \tag{20}$$

$$E_{(+)}(k)=\sqrt{m^2+k^2+\frac{1}{4}q^2+\sqrt{m^2q^2+(\vec{q}\cdot\vec{k})^2}}. \tag{21}$$

The lower energy eigenvalue $E_{(-)}$ has spin along $\vec{q}$ for $\tau_3=1$, or has spin opposite to $\vec{q}$ for $\tau_3=-1$. The higher energy eigenvalue $E_{(+)}$ has spin along $\vec{q}$ and $\tau_3=-1$, or has spin opposite to $\vec{q}$ and $\tau_3=+1$.

In this background the Fermi sea in no longer degenerate in spin but gets polarized into the states above. The quasiparticles are, however, states of isospin. We describe matter at a given Fermi energy of u and d quarks set by their respective densities and by charge neutrality (corresponding to, say, neutronlike matter).

First we fill up all the lower energy, $E_{(-)}(k)$, states and then we have a gap and start filling up the $E_{(+)}(k)$ states until we get to E_F^i, the Fermi energy corresponding to a given density for each flavor:

$$\rho_i=\frac{1}{(2\pi)^3}\gamma\left(\int d^3k\Theta(E_F^i-E_{(-)}(k))+\int d^3k\Theta(E_F^i-E_{(+)}(k))\right), \tag{22}$$

$$\rho_B=(\rho_u+\rho_d)/3, \tag{23}$$

$$\epsilon_i=\frac{1}{(2\pi)^3}\gamma\left(\int d^3kE_{(-)}(k)\Theta(E_F^i-E_{(-)}(k))+\int d^3kE_{(+)}(k)\Theta(E_F^i-E_{(+)}(k))\right), \tag{24}$$

$$\epsilon_\rho=\epsilon_u+\epsilon_d+\frac{1}{2}F^2q^2+\frac{\lambda^2}{4}(F^2-f_\pi^2)^2. \tag{25}$$

We can now write down the equation of state as in Ref. [13]. It is found that the PC state is always lower in energy then the uniform phase 2. For the explicit numbers and figures we refer the reader to [13].

We briefly remark on some features of this phase.

(1) The two-flavor PC state is quite different from the uniform phase: unlike the two-flavor CRQM states considered in [2], it cannot be recovered from three-flavor CRQM by taking the strange quark mass to infinity. As we shall see in the next sections, this gives a new feature—a maximum strange current quark mass for SQM to be the true ground state.

(2) The reason that the PC phase has energy lower than the uniform $\langle\sigma\rangle$ condensate is perhaps best understood in the language of quarks and antiquarks. To make a condensate a quark and antiquark must make a bound state and condense. For a uniform $\langle\sigma\rangle$ condensate the q and $\bar{q}$ must have equal and opposite momenta. Therefore, as the quark density goes

up the system can only couple a quark with $k>k_f$ and a $\overline{q}$ with the opposite momentum. This costs much energy, so the condensate can occur only if k_f is small, at low density. On the other hand, the pion condensed state is not uniform. So at finite density, if we take a quark with $k=k_f$, the $\overline{q}$ can have momentum $k=|\vec{k}_f-\vec{q}|$, which is a much smaller energy cost.

(3) Since the pion condensate is a chirally broken phase, the chiral restoration shifts from very low density in the uniform phase to very high density $\sim 10\rho_{nuc}$. This is a signature of this phase.

(4) Since this phase is always lower in energy than the uniform phase we go directly from the nucleonic phase to the PC phase, completely bypassing the uniform phase, and thus all the interesting features and conjectures for the uniform phase are never realized.

(5) Another feature of this $\vec{\pi}$ condensate is that, since we have a spin isospin polarization, we can get a net magnetic moment in the ground state.

V. THE THREE-FLAVOR STATE

The extension of the above to three flavors or SU(3) chiral symmetry needs some clarification.

The generalized Dirac equation for the SU(3) case is considerably more complicated and involves a singlet ξ_0 and an SU(3) octet ξ_a of scalar fields and a singlet ϕ_0 and an SU(3) octet ϕ_a of pseudoscalar fields, which interact with the quarks as shown in [15]:

$$H\psi(k)=\{-i\vec{\alpha}\cdot\vec{\partial}-g\beta[\sqrt{2/3}(\xi_0+i\phi_0\gamma_5)+\lambda^a(\xi_a+i\phi_a\gamma_5)]\}\psi=E\psi. \tag{26}$$

In the chiral limit, the spontaneous symmetry breaking pattern is not unique. We choose the pattern in which the $SU(3)_L\times SU(3)_R$ chiral symmetry breaks down to a vector SU(3). For the uniform case, we have

$$\langle\xi_0\rangle=\sqrt{3/2}f_\pi, \tag{27}$$

$$\langle\xi_a\rangle=0, \tag{28}$$

$$\langle\phi_0\rangle=0, \tag{29}$$

$$\langle\phi_a\rangle=0. \tag{30}$$

This gives a constituent mass $m=gf_\pi$ for all (u, d, and s) quarks. The explicit symmetry breaking strange quark mass term with mass m_s is then added to H. The strange quark mass M_s then turns out to be the sum of the constituent and explicit masses $M_s=gf_\pi+m_s$.

A. The three-flavor pion condensed phase

For describing strange quark matter we use the three-flavor pion condensed state. This is a more versatile state than the one used in [2] (three-flavor CRQM), the latter being a subset of the former.

Next, we formulate the symmetry breaking in the presence of the pion condensate. This is given as follows:

$$\langle\xi_0\rangle=\sqrt{3/2}F[1+2\cos(\vec{q}\cdot\vec{r})]/3, \tag{31}$$

$$\langle\xi_8\rangle=-\sqrt{3}F[1-\cos(\vec{q}\cdot\vec{r})]/3, \tag{32}$$

$$\langle\phi_0\rangle=0, \tag{33}$$

$$\langle\phi_3\rangle=F[\sin(\vec{q}\cdot\vec{r})], \tag{34}$$

and all other fields have expectation value zero.

This gives exactly the PC Hamiltonian equation for the u,d sector and yields the simple mass relation above for the strange quark: $M_s=gF+m_s$; when $q=0$ and $m_s=0$ we recover the chiral limit above.

We may now simply add the two-flavor PC results for the energy density and density derived above to the strange quark energy density that arises from the single particle relation

$$E_s=\sqrt{M_s^2+k^2}.$$

The strange quark energy density is given by Baym [16], [Eq. (8.20)],

$$\epsilon_s=\frac{3}{\pi^2 8}M_s^4[x_s n_s(2x_s^2+1)-\ln(x_s+n_s)], \tag{35}$$

where $x_s=k_s^f/M_s$ and $n_s=\sqrt{1+x_s^2}$; k_s^f is the Fermi momentum for the strange quarks.

The total energy density of the quarks for the three-flavor PC is given by

$$\epsilon_\rho=\epsilon_u+\epsilon_d+\epsilon_s+\frac{1}{2}F^2q^2+\frac{\lambda_1^2}{4}(F^2-f_\pi^2)^2. \tag{36}$$

From the effective potential given in [15] for the SU(3) case, there is an extra factor of 3/2 that multiplies the last term. This can be absorbed, as we have done, by a redefinition: $\lambda_1=A\lambda$, where $A=\sqrt{3/2}$.

B. β equilibrium in the PC phase

We have the following general chemical potential relations for quark matter:

$$E_F^u=\mu_u, \tag{37}$$

$$E_F^d=\mu_d=\mu_s, \tag{38}$$

$$\mu_e=\mu_d-\mu_u, \tag{39}$$

$$n_e=\frac{\mu_e^3}{3\pi^2}. \tag{40}$$

The charge neutrality condition, below, further reduces the number of independent chemical potentials to 1:

$$\frac{2n_u(\mu_u,q,F)-n_d(\mu_d,q,F)-n_s(\mu_s)}{3}-n_e=0. \tag{41}$$

TABLE I. Ground states with energy of 930 MeV/nucleon for the three-flavor condensed state. m_s is the assumed strange quark mass, B the bag pressure, μ_u the u-quark chemical potential, n_s/n_u the ratio of the density of strange quarks to that of u quarks, and $\langle\sigma\rangle$ the expectation value of the σ field. These results are without one-gluon exchange

m_s (MeV)	$B^{1/4}$ (MeV)	μ_u (MeV)	n_s/n_u	$\langle\sigma\rangle$ (MeV)
0	162.7	309	0.96	9.55
50	161.4	307	0.96	1.45
100	158.9	304	0.92	0.01
150	154.9	298	0.80	0.01
200	150.6	290	0.63	0.01
250	147.7	260	0.00	26.31

The baryon density is

$$\rho_B = \frac{n_u(\mu_u, q, F) + n_d(\mu_d, q, F) + n_s(\mu_s)}{3}, \tag{42}$$

$$n_s = (k_s^f)^3/(\pi^2). \tag{43}$$

For matter in β equilibrium we need to add the electron energy density to the quark energy density above:

$$\epsilon_e = (1/4\pi^2)\mu_e^4.$$

The total energy density is

$$\epsilon = \epsilon_\rho + \epsilon_e.$$

The energy per baryon, $E_B = \epsilon/\rho_B$, then follows.

For the pion condensed state, the ground state energy and the baryon density depend on the variational parameters, the order parameter or the expectation value $F = \sqrt{\langle\vec{\pi}\rangle^2 + \langle\sigma\rangle^2}$, and the condensate momentum $|\vec{q}|$. To define the free energy at a fixed baryon density then requires some care. However, we are interested in only the absolute minimum of the energy per baryon, E_B, for all density. We can then simply minimize E_B with respect to the variational parameters F and $|\vec{q}|$ and the one independent chemical potential to get the absolute minimum. Note that this is for a given value of

$$B = \lambda_1^2 (f_\pi)^4/4. \tag{44}$$

VI. RESULTS FOR THE MEAN FIELD THEORY THREE-FLAVOR PC STATE (PCSQM)

A. PCSQM without one-gluon exchange

The new window for SQM is established thus.

We maintain the minimum permissible limit on E_B for two-flavor quark matter to be 934 MeV. Since the PC is a lower energy state than the chirally restored QM considered in Ref. [2], we find that the lower bound on $B^{1/4}$ goes up, closing the window on SQM. The results are given in Table I. This lower bound for two-flavor PC is found to be

$$B_<^{1/4} = 148 \text{ MeV},$$

whereas, for the case of two-flavor CRQM considered by Farhi and Jaffe, it was

$$B_<^{1/4} = 145 \text{ MeV}.$$

We then calculate the minimum E_B for three-flavor QM. For this we use our generalized PCSQM as the state. It is important to note that the rather particular three-flavor CRQM lies within its variational reach. This provides us with the upper bound on the bag pressure $B_>$. The maximum upper bound on B naturally occurs for the $m_s = 0$ case and is found to be almost the same as in Ref. [2]:

$$B_>^{1/4} = 162.5 \text{ MeV}.$$

This is because the minimum occurs in the three-flavor CRQM state, as given in [2], with $F = 0$.

We note some new features compared to Ref. [2]. Whereas in [2], the two-flavor CRQM threshold is virtually the limit of $m_s \to \infty$ for three-flavor CRQM, in our case it is not, as our two-flavor PC ground state is of lower energy and different. In this case the three-flavor CRQM becomes of higher energy than the two-flavor PC ground state at a finite m_s.

We thus find the limit

$$m_s < 250 \text{ MeV}$$

for SQM to exist as the ground state, simply from the constraint on two-flavor QM.

We also find that for this limiting m_s the absolute minimum of E_B is the two-flavor PC state with zero strange quark density. But, for masses somewhat below the limiting mass (with the condition that SQM be the actual ground state), the absolute minimum occurs in the three-flavor CRQM state as given in [2], with $F = 0$. A summary of the results appears in Fig. 1.

B. Results for the three-flavor pion condensed phase (with one-gluon exchange interaction and $\alpha_{QCD} = 0.6$)

We next consider the case in which one-gluon exchange interaction is included, following Farhi and Jaffe. This is with a view to estimating the effects of including such "perturbative" interactions. No attempt will be made at rigor. Our approximate scheme for this case is best regarded as an estimate.

One reason it is difficult to do an analytic calculation of the interaction energy for the PC is that the quark propagators in the presence of the pion condensate [13] are far more complicated than in the case of free Fermi sea quarks.

We first note that in the limit of the condensate, $F \to 0$, all the PC results go smoothly to the free Fermi sea results. We find that the condensate expectation values F in the regime of interest to us are such that the value of F at the minimum makes $m = gF$ fall below the relevant quark chemical potentials. So, as a first approximation, we use the free Fermi sea results for the given chemical potential and mass.

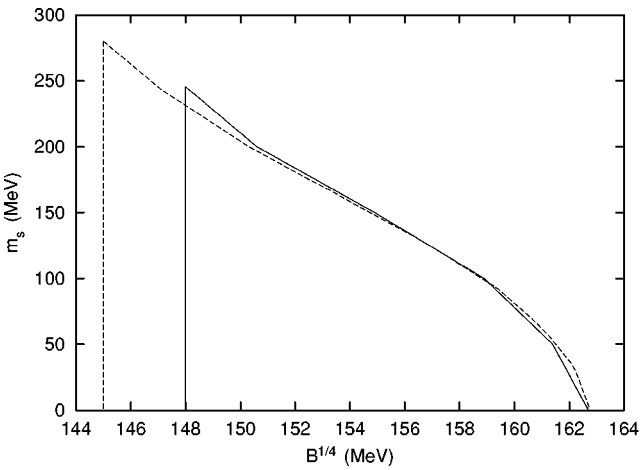

FIG. 1. Allowed region in the bag constant−strange quark mass parameter plane for the strange quark matter ground state to be the absolute ground state of matter. Gluon exchange interaction is not included. The solid line shows the allowed window, taking into account the pion condensed phase reported in this paper. The region included within the curve is the allowed region. The dashed line shows the result for chirally restored quark matter from Ref. [2]. The vertical left boundary for either case represents $E_B = 934$ MeV in two-flavor matter and the curved line represents $E_B = 930$ MeV in three-flavor matter. The curve for CRQM shown here is a linear interpolation to 930 MeV from the results for 919 MeV and 939 MeV presented in Ref. [2].

The contribution from the one-gluon exchange interaction to the free energy at given density or the thermodynamic potential (TP), including renormalization group corrections, (a), is given in [2]. The expression for the one-gluon exchange or interaction energy, for free Fermi seas of quarks, (b), is given by Baym [16], [Eq. (8.20)]. Although we expect the interaction contribution (in the absence of renormalization group corrections) to both (a) and (b) to be the same, we find that the two expressions are different. Results for these two cases are summarized in Table II.

We briefly give a comparison with the results of the case without the one-gluon exchange interaction. In our notation the results for (a) are given first (without parentheses) and the results for (b) appear alongside in parentheses.

The lower bound for two-flavor PC is found to be

$$B_<^{1/4} = 141.5 \quad (156.7) \quad \text{MeV}.$$

For (a) this is down from the case without one-gluon exchange, where it was 148 MeV. This indicates that gluon exchange is repulsive, even with the constituent quark mass generated by the condensate, whereas for (b) it is up and the one-gluon exchange is attractive.

We note that for the case of two-flavor CRQM (with gluon exchange) considered by Farhi and Jaffe we find

$$B_<^{1/4} = 132 \quad \text{MeV},$$

which is even more reduced from the case without one-gluon exchange, where it was 145 MeV.

This clearly shows that the effect of gluon exchange is more repulsive for this case, as the quarks are massless, as opposed to the PC case when they have a mass $m = gF$.

We note that our value $B_<^{1/4} = 132$ MeV is more than that given in Fig. 1(c) in [2]. This is due to the difference in the way energy density and density are defined by us and in [2]. The authors of [2] begin with the TP to order α, derive a density that includes interaction to order α, and use this density to define the energy density. The difference between our case and theirs is $O(\alpha^2)$.

The maximum upper bound on B naturally occurs for the $m_s = 0$ case,

$$B_>^{1/4} = 150.3 \quad (166.1) \quad \text{MeV}.$$

Interestingly, in this case the minimum in E_B comes from a *new ground state* and is genuinely different. It does not occur either in the two-flavor PC state or in the three-flavor CRQM state as given by Farhi and Jaffe, with $F = 0$, as was the case in the absence of one-gluon interaction. In this case the minimum is lower than either of these states and comes from a true merger of the two; it has a nonzero value of $F = 22.1$ (36.5) MeV, and also a ratio of the strange quark density to the u quark density of 0.83 (0.69).

For comparison, for the three-flavor CRQM state of [2],

$$B_>^{1/4} = 144.5 \text{ MeV}.$$

We find the maximum allowed limit on m_s, with gluon exchange included, for both (a) and (b), moves down to

$$m_s < 150 \quad \text{MeV}$$

for SQM to be the absolute ground state.

TABLE II. Same as Table I, but including interaction energies due to one-gluon exchange.

	(a) Farhi-Jaffe model				(b) Baym model			
m_s (MeV)	$B^{1/4}$ (MeV)	μ_u (MeV)	n_s/n_u	$\langle\sigma\rangle$ (MeV)	$B^{1/4}$ (MeV)	μ_u (MeV)	n_s/n_u	$\langle\sigma\rangle$ (MeV)
0	150.3	274	0.83	22.08	166.1	302	0.69	36.53
50	147.7	274	0.96	0.33	161.6	292	0.54	34.45
100	144.6	280	0.89	0.01	157.2	282	0.28	35.54
150	140.8	244	0.00	32.91	155.8	272	0.00	40.67
200	140.8	244	0.00	32.91	155.8	272	0.00	40.68
250	140.8	244	0.00	32.91	155.8	272	0.00	40.67

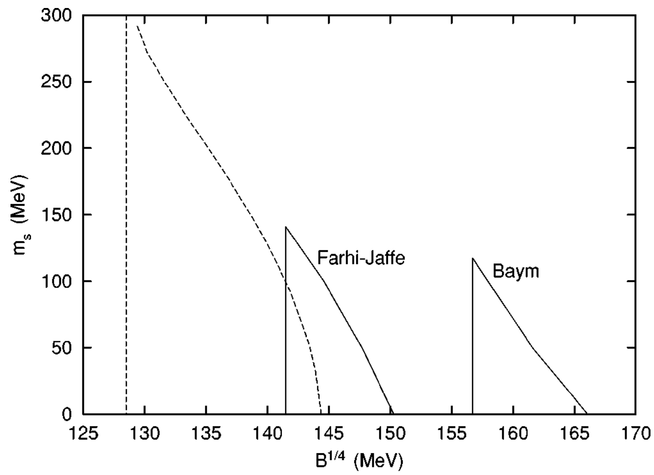

FIG. 2. Similar to Fig. 1, but with one-gluon exchange included. The exchange energy is computed using two alternative prescriptions, by Farhi-Jaffe [2] and Baym [16], as indicated. The solid lines represent the constraints imposed by taking into account the PC phase in these two cases. The dashed line is for CRQM, obtained from the data presented by Farhi and Jaffe [2]. α_{QCD} is set to 0.6. The PC phase places a strong constraint on the strange quark mass if SQM should represent the absolute ground state of matter.

We remark that the cases (a) and (b) show the same trends. The difference is that the allowed values of B are shifted up in (b).

Some of these results are summarized in Fig. 2.

VII. CONCLUSIONS

We have found that the existence of a new and lower energy state of two-flavor quark matter, the pion condensed state, has a significant effect on the window of opportunity for SQM to be the true ground state of matter.

We work with an effective chiral Lagrangian. Unlike the MIT bag case [2], where the bag pressure is a parameter, in our formulation it is the chiral condensate energy and is given in terms of the parameters of low energy phenomenology—the pion decay constant f_π, which is precisely known, and the scalar coupling or σ mass, which is rather poorly known. However, it is of interest that SQM is related to parameters of low energy phenomenology. It requires more detailed work to firm up this connection. Furthermore, we have found a new and interesting constraint on the existence of SQM as the true ground state that comes from the current mass of the strange quark.

Our findings are as follows.

(1) Without including one-gluon exchange the new PC ground state limits the bounds on the bag pressure B, allowing

$$148 \text{ MeV} < B^{1/4} < 162.5 \text{ MeV}$$

instead of the result of [2],

$$145 \text{ MeV} < B^{1/4} < 162.5 \text{ MeV},$$

and cuts down the allowed parameter space of the explicit or current strange quark mass to

$$m_s < 250 \text{ MeV}.$$

(2) With $\alpha_{QCD} = 0.6$ and including one-gluon exchange, the new PC ground state, with some simplifying approximations, strongly limits the bounds on the bag pressure B, allowing

$$141.5 \ (156.7) \text{ MeV} < B^{1/4} < 150.3 \ (166.1) \text{ MeV}$$

instead of the result of [2]:

$$128.5 \text{ MeV} < B^{1/4} < 144.5 \text{ MeV},$$

and it further cuts down the allowed parameter space of the explicit strange quark mass to

$$m_s < 150 \text{ MeV}.$$

This is a rather severe constraint.

For all cases considered by us the maximum value of $B^{1/4}$ is 166 MeV, which corresponds to $m_\sigma = 680$ MeV. Such a value for the sigma mass may be a little too low, in the context of the linear sigma model employed by us, to allow SQM to be the true ground state.

We note that recently Schechter *et al.* [17] made fits to the scalar channel scattering data to see how it may be fitted with increasing range in $\sqrt{s}$ by chiral perturbation theory and several resonances. They further looked at this channel using just a linear sigma model. Their results indicate that for $\sqrt{s} < 800$ MeV a reasonable fit to the data can be made using the linear sigma model with a sigma mass between 700 and 800 MeV.

For the chiral model used this means that SQM is unlikely to be the true ground state implying that strange stars are also very unlikely.

Our results are obtained in the two-flavor chiral limit with some approximations and also assuming that all the bag pressure comes from the chiral condensate. Adding a confinement pressure will raise the energy E_B for SQM and may shrink the window further.

This is by no means the last word on possible ground states even in this model—for example, we may have a kaon condensate. However, we shall not investigate this here.

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