THE INTERFERENCE OF POLARIZED LIGHT AS AN EARLY EXAMPLE OF BERRY'S PHASE

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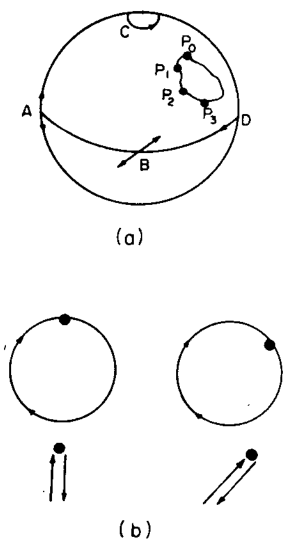
ABSTRACT

The term "Berry's phase" is now commonly used to describe the change in the phase of a quantum state vector under a sequence of adiabatic changes which returns the system to its original state. The purpose of this note is to point out that a particular case of this concept, as applied to the interference of polarized light, was studied both theoretically and experimentally in a series of papers published in the fifties by S. Pancharatnam. The phase term is proportional to a solid angle on the Poincaré sphere which represents the states of polarization of light.

In a recent paper which has excited much interest, Berry[1] drew attention to a property of the phase factors which accompany adiabatic changes in a quantum mechanical system. The overall phase of the wave function is, of course, subject to an arbitrary convention for each state of the system. However, the phase change occurring when the system returns to its original state is independent of this convention and a genuine physical property, measurable in principle. Simon[2] has emphasized the geometric aspects of Berry's phase while Chiao and Wu have suggested[3] an elegant realization of this concept using changes in the state of polarization of a beam of light travelling along a twisted space curve defined by an optical fibre. This has been demonstrated by Tomita and Chiao[4].

We undoubtedly owe the clarification of this concept in its full generality to Berry's paper[1]. Nevertheless, a few instances of the phenomenon can be found in earlier work—notably that of Longuet-Higgins[5] on the changes of the electronic wavefunction during the motion of nuclei in a molecule. Remarkably, Schiff's well known textbook of quantum mechanics[6] also contains hints of the idea. We wish to add one more item to this list—the work by Pancharatnam[7] on the interference of polarized light.

The states of polarization of light, for the purpose of this discussion (and many others!) are conveniently represented on a sphere[8], following Poincaré. As shown in figure 1a and reviewed in references 7 and 8, the two senses of circular polarization lie at the poles of the sphere and the states of linear polarization oriented at angles lying between 0° and 180° (with respect to some fixed axis) are arranged along the equator of the sphere. For concreteness, consider an experiment in which there are two linear vibrations in directions inclined at 45° to each other, represented by the points A and B on the sphere. A natural definition[7] of these two vibrations being in phase would be the situation when the intensity of the resultant is a maximum. In the present case of two linear vibrations, this occurs when they both pass through a maximum at the



Figures 1a and b. a. The Poincaré sphere representation of the states of polarization of light. C is a circular state, represented at the pole, while A, B and D represent linear vibrations respectively oriented vertically, at 45°, and horizontally. **b.** The lower half of the figure shows two linear states inclined at 45°. The filled circles indicate a situation when the two linear vibrations may be said to be in phase. The upper part of the figure shows the result of analyzing these two states with a circular analyzer. The phase difference is now 45°.

same time with the vectors making an acute angle (figure 1b). Now let both these linear vibrations pass through a circular analyzer, represented by the pole C of the sphere. One might naively expect that since A and B were 'in phase' to start with, their components after transmission through C would remain in phase. A little thought or calculation shows that in fact the two circular vibrations produced by analysis of A and B through C differ in phase by 45° (figure 1b). This example has been chosen to illustrate the general result (Section 8 of Pancharatnam's paper) that an additional phase, equal to half the area of the spherical triangle ABC, must be introduced in calculations of the interference between A and B, as analyzed by C. This is fully borne out by Pancharatnam's experiments[9] on interference figures in absorbing biaxial crystals.

Although, A, B and C do not appear symmetrically in the above example, the following restatement makes the situation clear. Consider any closed curve on the Poincaré sphere, starting and ending at a point P_0. The state of polarization P_0 can be analyzed along a sequence of states P_1, P_2, P_3 along the curve, with the last step being the analysis of P_n along P_0 (figure 1). In the limit $n \to \infty$ with the separations P_0P_1, P_1P_2, ... etc, all tending to zero, the final state P_0 differs from the initial by a phase equal to half the solid angle subtended by the curve $P_0P_1 \ldots P_n$ at the centre of the sphere. The analogue of this for a spin half particle would be an arrangement often used in polarized neutron diffraction—a guide field which the spin direction follows adiabatically. The phase changes and interference effects resulting from such rotations indeed constitute one of the examples in the original paper[1] and the excess phase is directly the solid angle swept out by the magnetic field vector.

The alert reader will have noticed that the entire discussion has involved the phases of classical electromagnetic waves, while Berry's argument concerns the phases of quantum mechanical state vectors. Berry[1] has in fact commented that the phase factors which he discusses are applicable to classical wave phenomena as well. Further, Mukunda and Sudarshan[10] have set up a clear correspondence between state vectors in the one photon subspace of quantum electrodynamics and a set of solutions of the classical Maxwell equations. This justifies our identification of Pancharatnam's excess phase[7] on the Poincaré sphere as an early example of what is now widely known and discussed as Berry's phase.

ACKNOWLEDGEMENT

The authors would like to thank J. Samuel and C. Shukre for discussions on Berry's phase.

24 November 1986

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